Single and many-particle transport on networks

Andreas Buchleitner

Quantum optics and statistics
Physikalisches Institut, Albert-Ludwigs-Universität Freiburg
www.quantum.uni-freiburg.de

Nonlinear Dynamics in Quantum Systems

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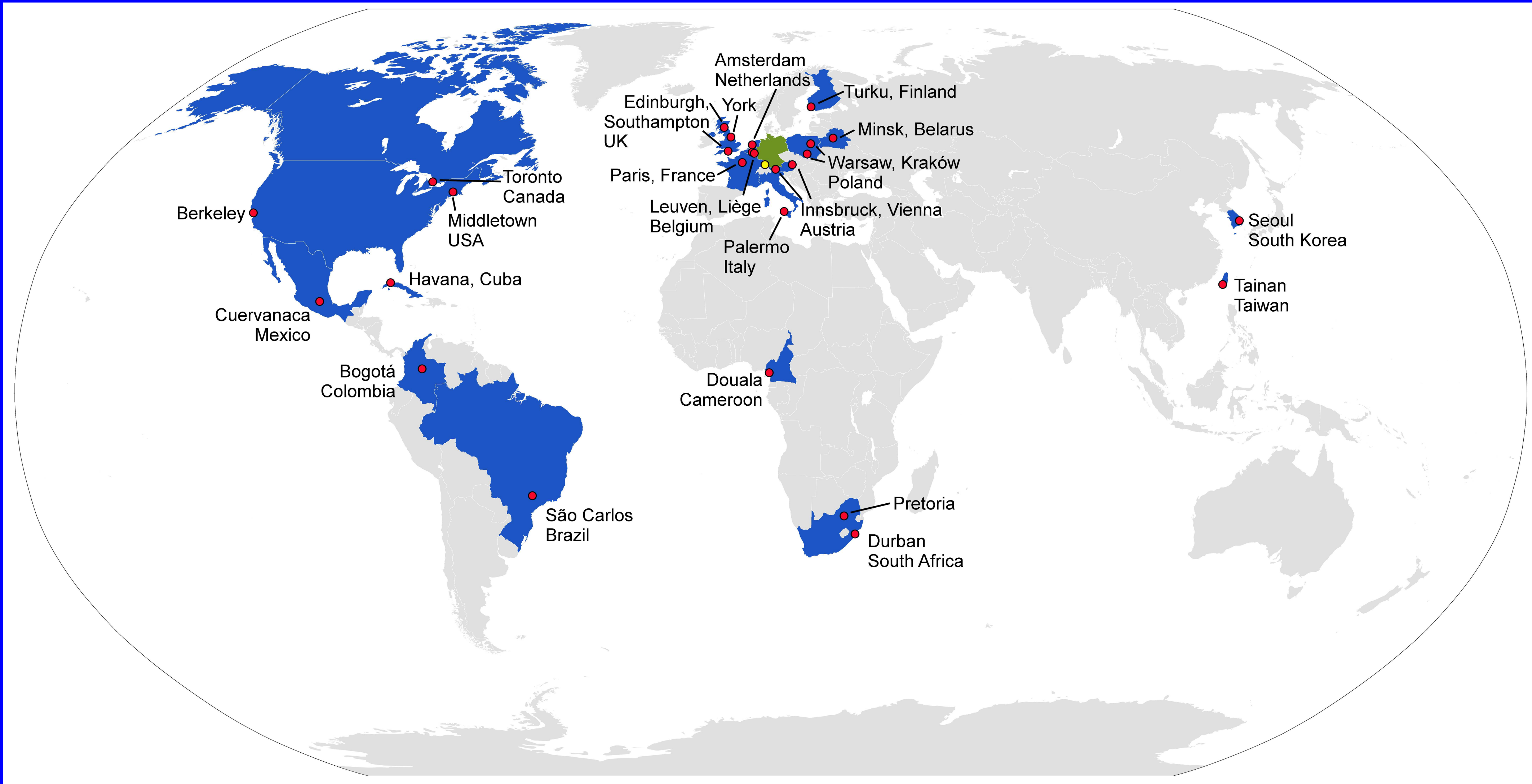
Nonlinear Dynamics in Quantum Systems

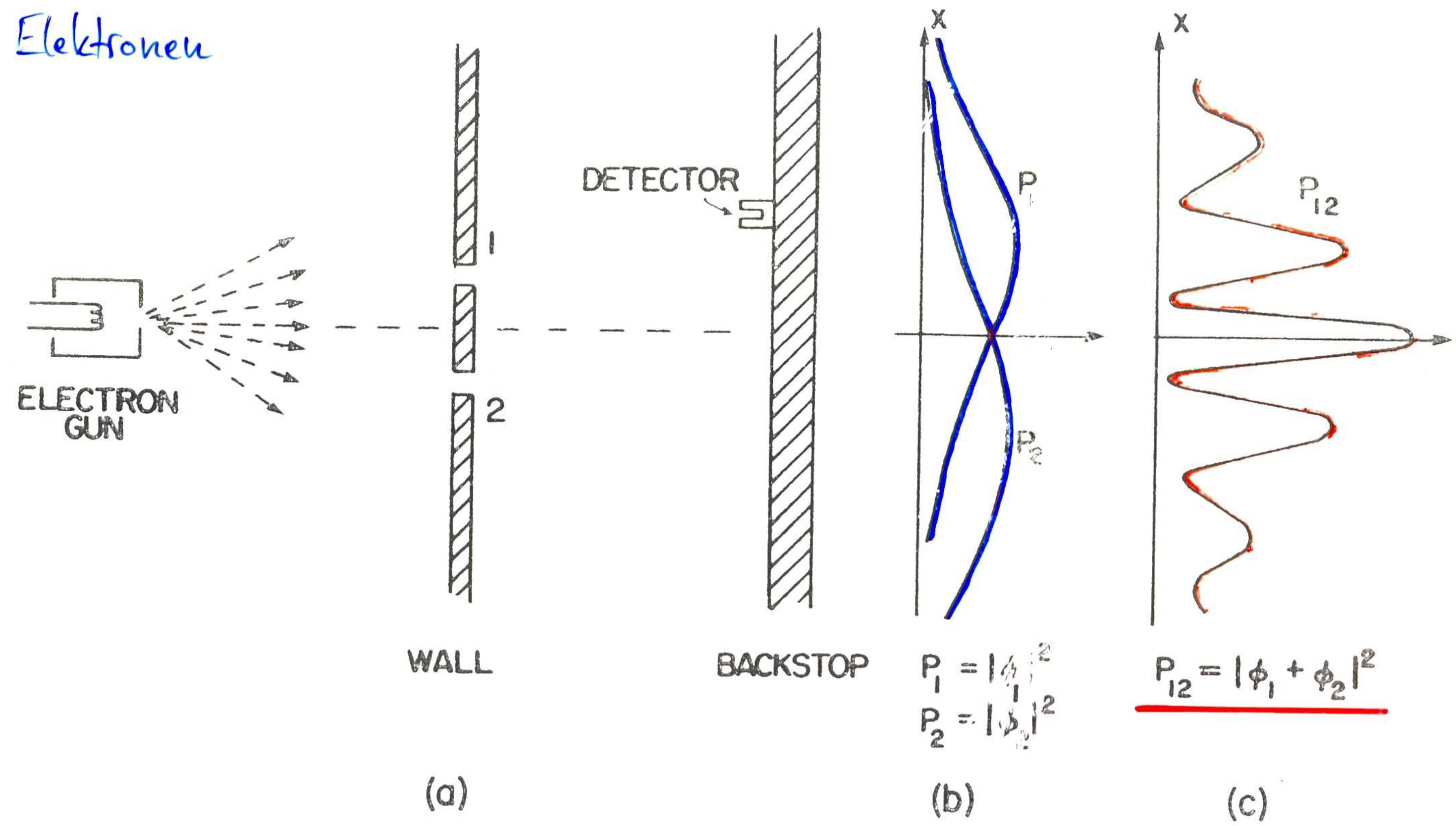
In Collaboration with . . .



AND K. Mayer, F. Mintert, M.C. Tichy, Y.-H. Kim et al., K. Richter et al.

Interference

unterscheidbare Wege

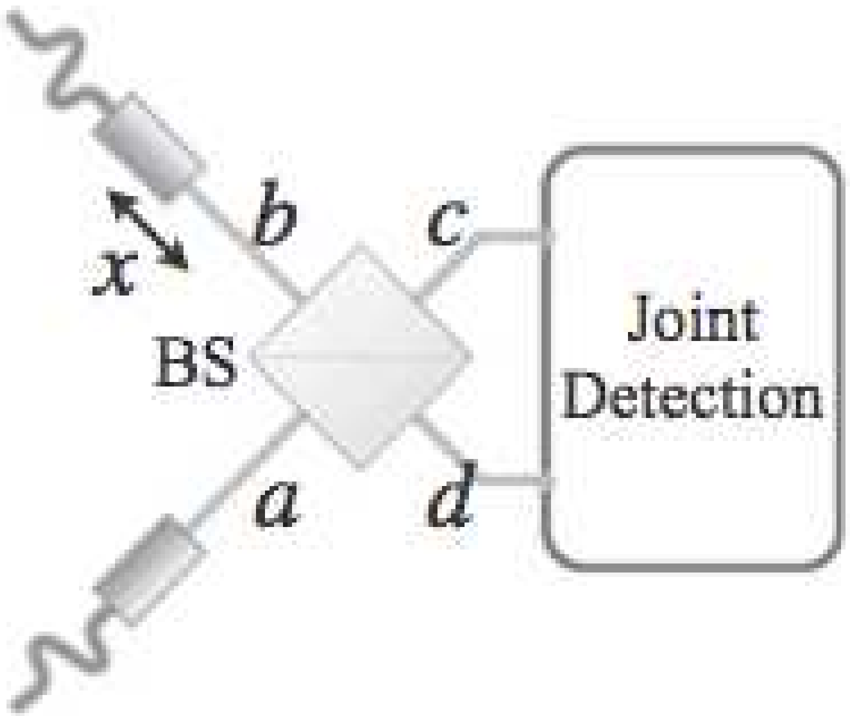


[Feynman, Lecture Notes of Physics]

– generalizations: topology, particle number/distinguishability, NO interactions –

How do (in-)distinguishable particles interfere?

one photon in each mode a and b – path delay x controls distinguishability



coincident detection in c and d

- coincidence probability if **distinguishable:** $P(2;1,1) = 1/2$
- coincidence probability if **indistinguishable:** $P(2;1,1) = 0$

[Shi & Alley (1986, 1988); Hong, Ou & Mandel (1987)]

– <u>if indistinguishable:</u> destructive interference of two two-particle amplitudes –

Tuning the distinguishability

inject (Gaussian) photon wave packet state $|1_{t_1}\rangle$ ($|1_{t_2}\rangle$) with beam splitter arrival time t_1 (t_2) in mode a (b) – then (Gram-Schmidt):

$$|1_{t_2}\rangle = \alpha|1_{t_1}\rangle + \sqrt{1-\alpha^2}\left|\tilde{1}_{t_1}\right\rangle , \; \alpha^2 = \exp\left(-(\Delta\omega)^2(t_2-t_1)^2/2\right)$$

two-photon state impinging on beam splitter:

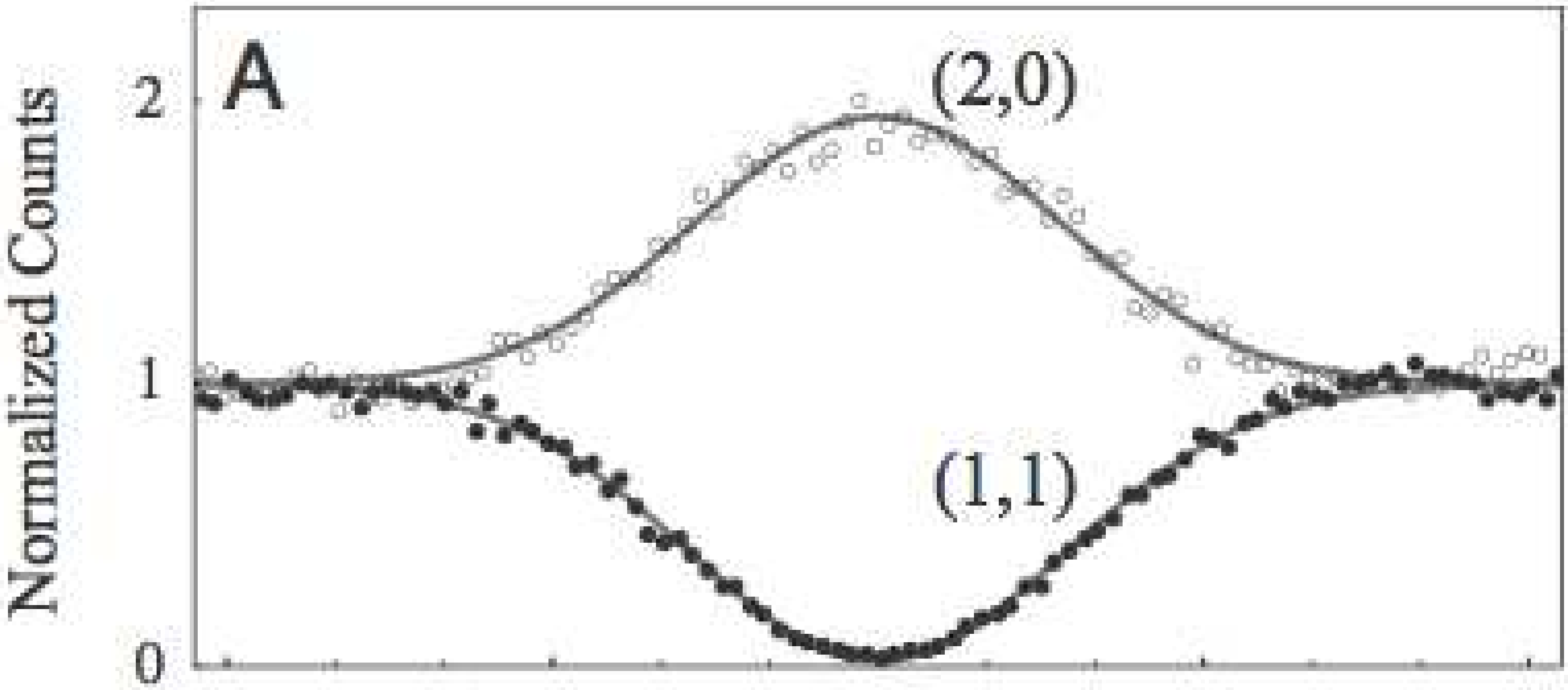
$$a^\dagger_{t_1} b^\dagger_{t_2}|0\rangle = \alpha|1,1\rangle + \sqrt{1-\alpha^2}\left|1,\tilde{1}\right\rangle ,$$

with the photons in $|1,1\rangle$ ($\left|1,\tilde{1}\right\rangle$) in(-/distinguishable) – $\alpha \in \{0,1\}$ quantifies distinguishability (fully indistinguishable for $\alpha = 1$).

Ergo: **Monotonic** disappearance of interference signal in coincident detection as α/distinguishability decreases/increases!

An experimental certification tool: the Hong-Ou-Mandel dip

middle of plot: $x = 0$ – event $(1,1)$ vs. $(2,0)$



– two-particle which-way information vs. two-particle interference [Ra et al., 2013] **–**

Four (i.e., many) photons, one beam splitter

Two photons *per mode* – decomposition with respect to temporal overlap:

$$\frac{1}{2}(a^\dagger_{t_1})^2(b^\dagger_{t_2})^2|0\rangle = \alpha^2|2,2\rangle + \sqrt{2}\alpha\sqrt{1-\alpha^2}\big|2,1,\tilde{1}\big\rangle + (1-\alpha^2)\big|2,\tilde{2}\big\rangle\,,$$

More than one photon per mode – **partially distinguishable contributions**:

$\big|2,1,\tilde{1}\big\rangle$ – **interference of three-particle rather than four-particle paths, with weight $W^{(n;n_c,n_d)}_{\text{type}}$ non-monotonic in α!**

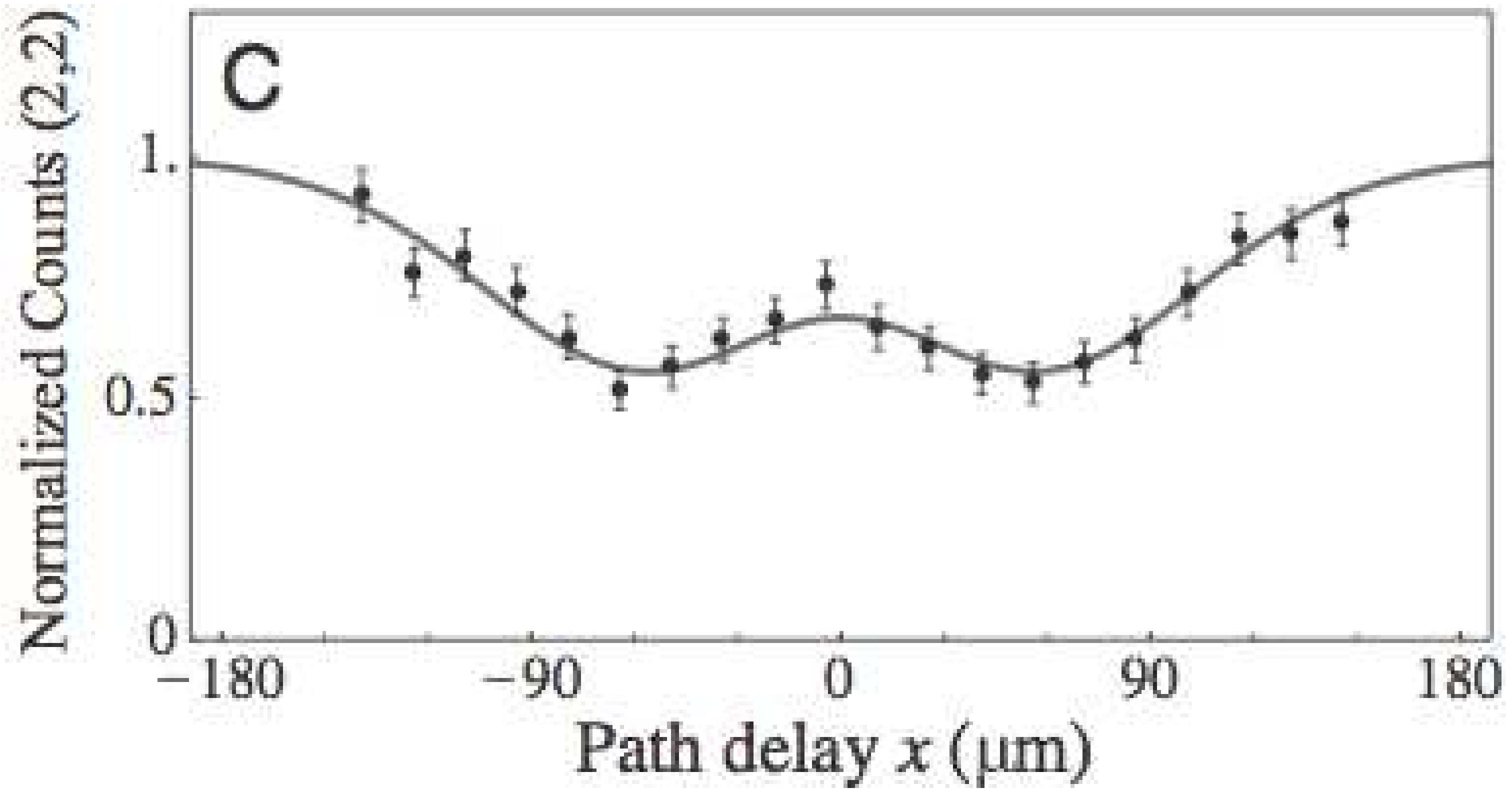
General expression for (n_c,n_d) output event probability **[Tichy et al., 2011]**:

$$P(n;n_c,n_d) = \sum_{\text{type}} p^{(n;n_c,n_d)}_{\text{type}} W^{(n;n_c,n_d)}_{\text{type}}$$

"type" – fully indistinguishable, fully distinguishable, partially distinguishable
p_{type} determined by mode mapping $a^\dagger \to (c^\dagger + d^\dagger)/\sqrt{2}$, $b^\dagger \to (c^\dagger - d^\dagger)/\sqrt{2}$.

Non-monotonic quantum-to-classical transition

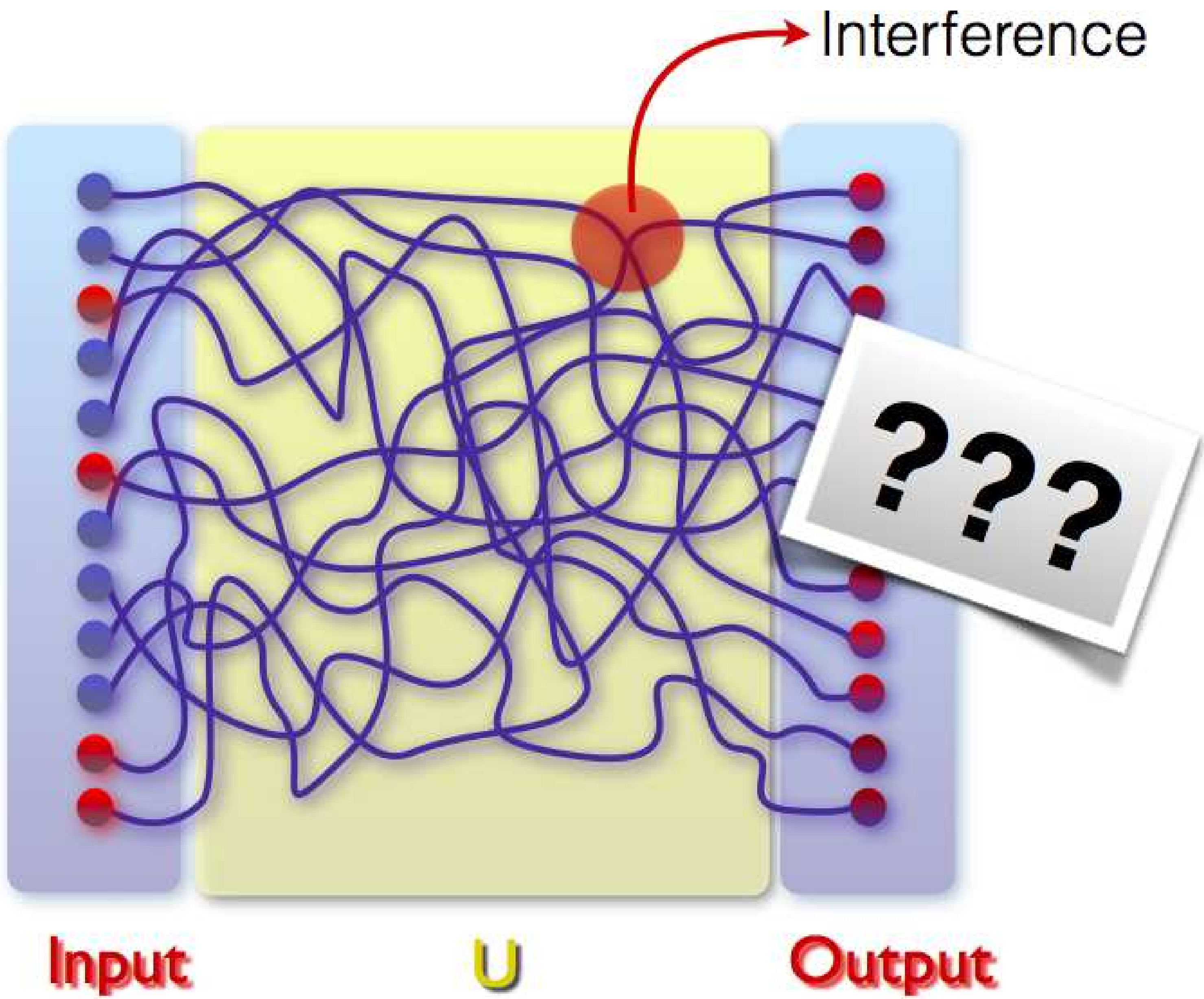
two **particles per input port – event** $(2, 2)$



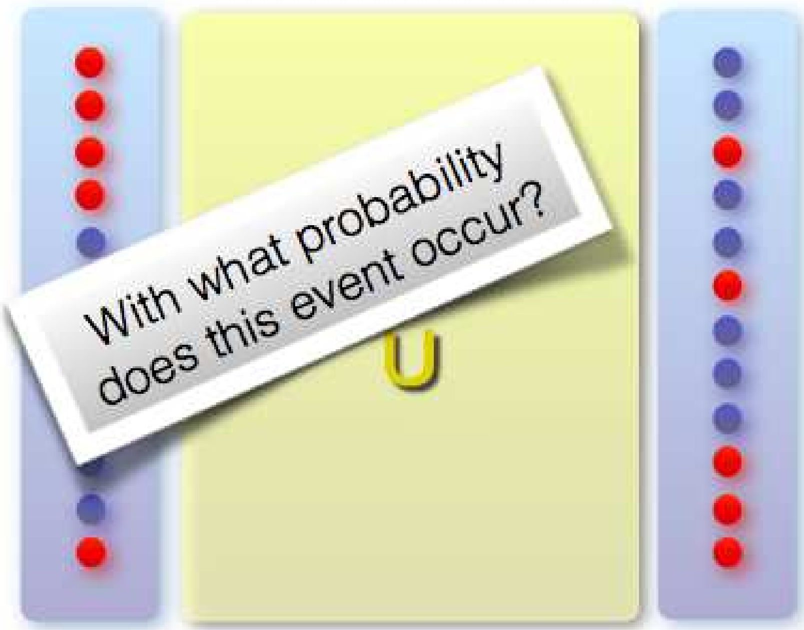
Gaining which-path information (increasing x**)**
generically **leads to a** ***non-monotonic*** **quantum-to-classical transition!**

[Tichy et al., 2011; Ra et al., 2013]

Many particles on a random network



"Complex" transmission signal



$\vec{i}$ $(1,1,1,1,0,0,0,0,0,0,0,1)$

$\vec{o}$ $(0,0,1,0,0,1,0,0,0,1,1,1)$

$$U_{\vec{i},\vec{o}} = \begin{pmatrix} U_{3,1} & U_{3,2} & U_{3,3} & U_{3,4} & U_{3,12} \\ U_{6,1} & U_{6,2} & U_{6,3} & U_{6,4} & U_{6,12} \\ U_{10,1} & U_{10,2} & U_{10,3} & U_{10,4} & U_{10,12} \\ U_{11,1} & U_{11,2} & U_{11,3} & U_{11,4} & U_{11,12} \\ U_{12,1} & U_{12,2} & U_{12,3} & U_{12,4} & U_{12,12} \end{pmatrix}$$

Distinguishable

$$p_{\vec{i}\to\vec{o}} = \mathrm{perm}\ \left|U_{\vec{i},\vec{o}}\right|^2_{\mathrm{comp}}$$

Fermions

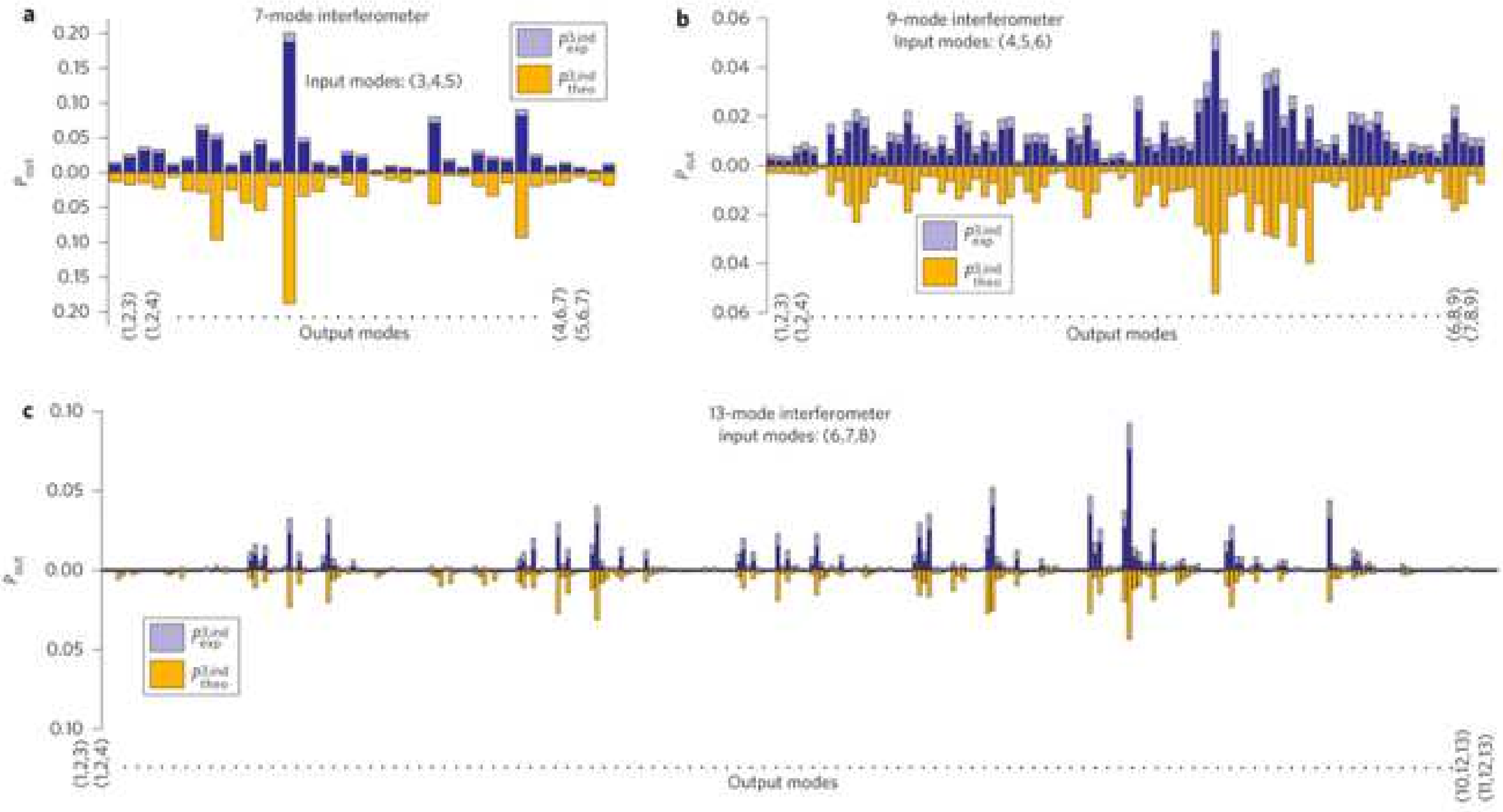
$$p_{\vec{i}\to\vec{o}} = \left|\det U_{\vec{i},\vec{o}}\right|^2$$

Bosons

$$p_{\vec{i}\to\vec{o}} = \left|\mathrm{perm}\ U_{\vec{i},\vec{o}}\right|^2$$

Computationally Complex

Can still handle 3 photons in $7/9/13$ modes

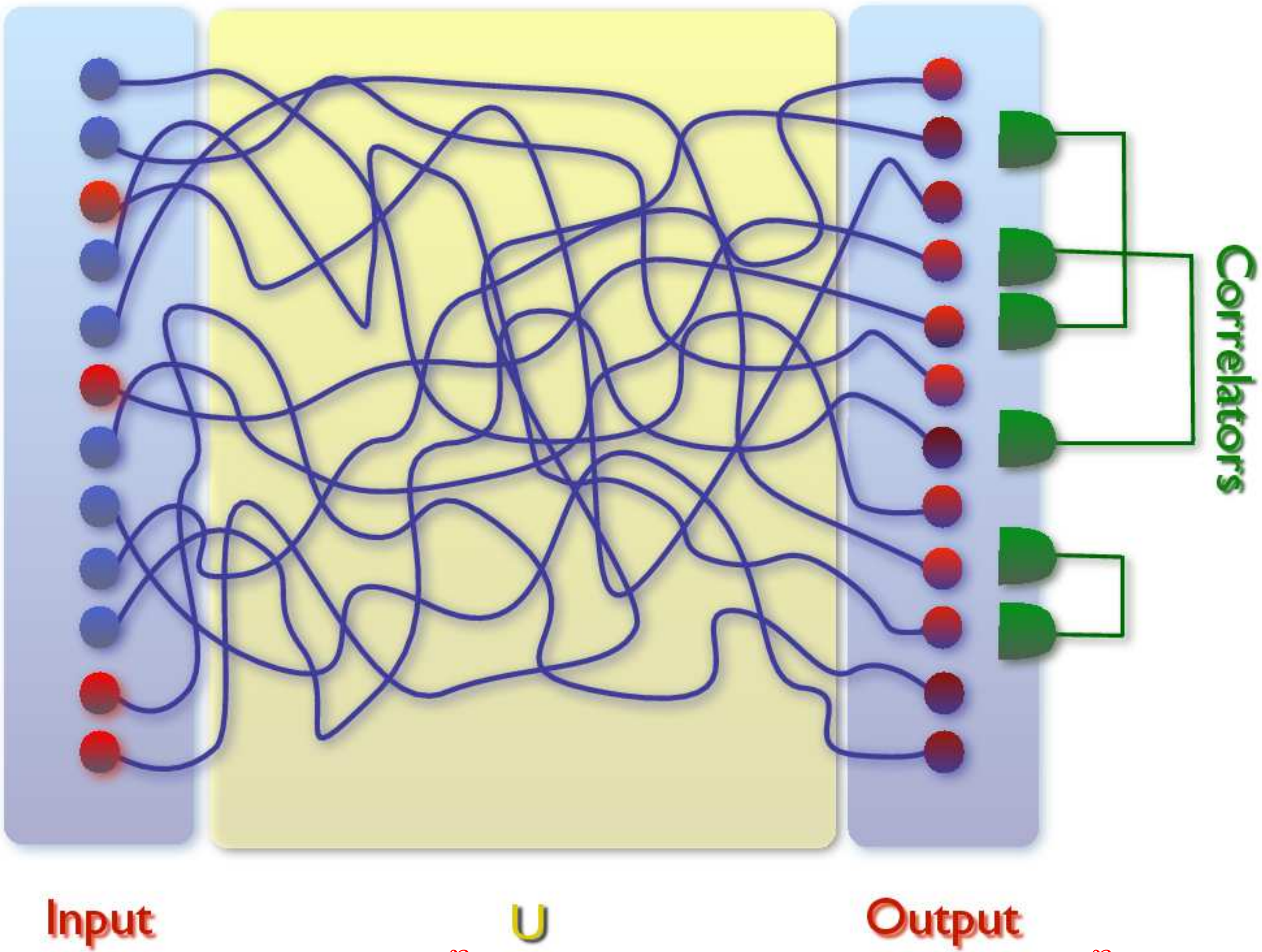


[Spagnolo et al., 2014]

complicated/complex distribution of output events!

can identify any features indicative of U and of particle type?

Rather than evaluating the deterministic output: Statistics!

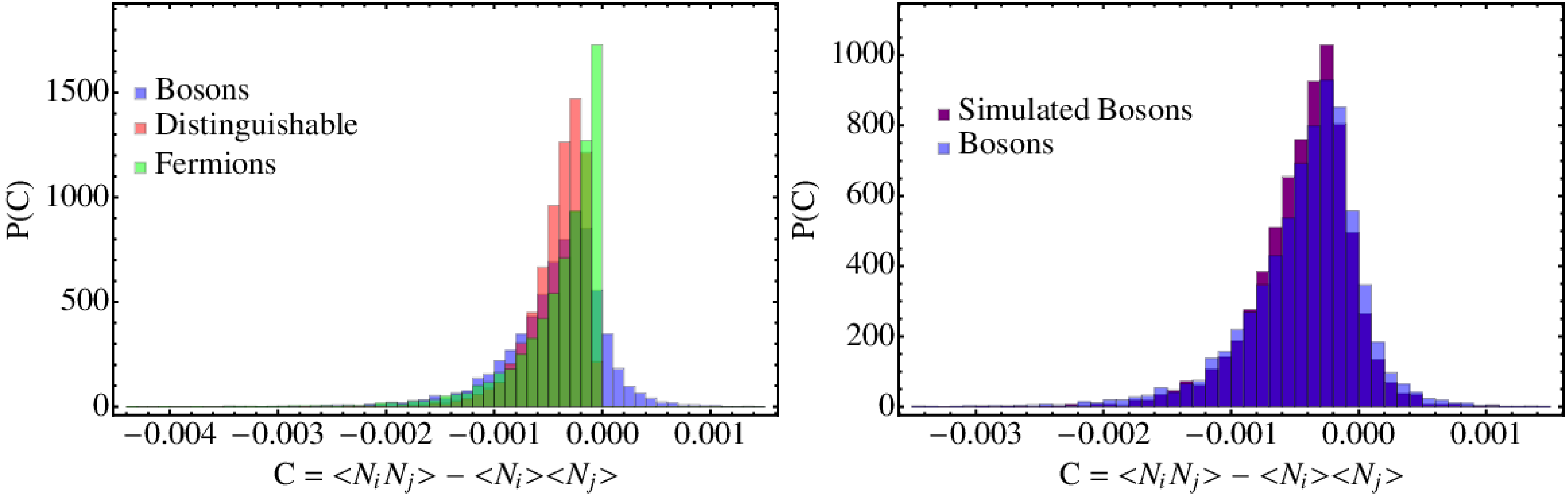


$$C_{ij} = \langle N_i N_j \rangle - \langle N_i \rangle \langle N_j \rangle = -\sum_{k=1}^{n} U_{q_k,i} U_{q_k,j} U^*_{q_k,i} U^*_{q_k,j} + t \sum_{k \neq l = 1}^{n} U_{q_k,i} U_{q_l,j} U^*_{q_l,i} U^*_{q_k,j}$$

$t = 0, \pm 1$ for distinguishable, bosons, fermions

Bunching vs. many-particle interference

sampling C_{ij} over all i, j – the "C-dataset"

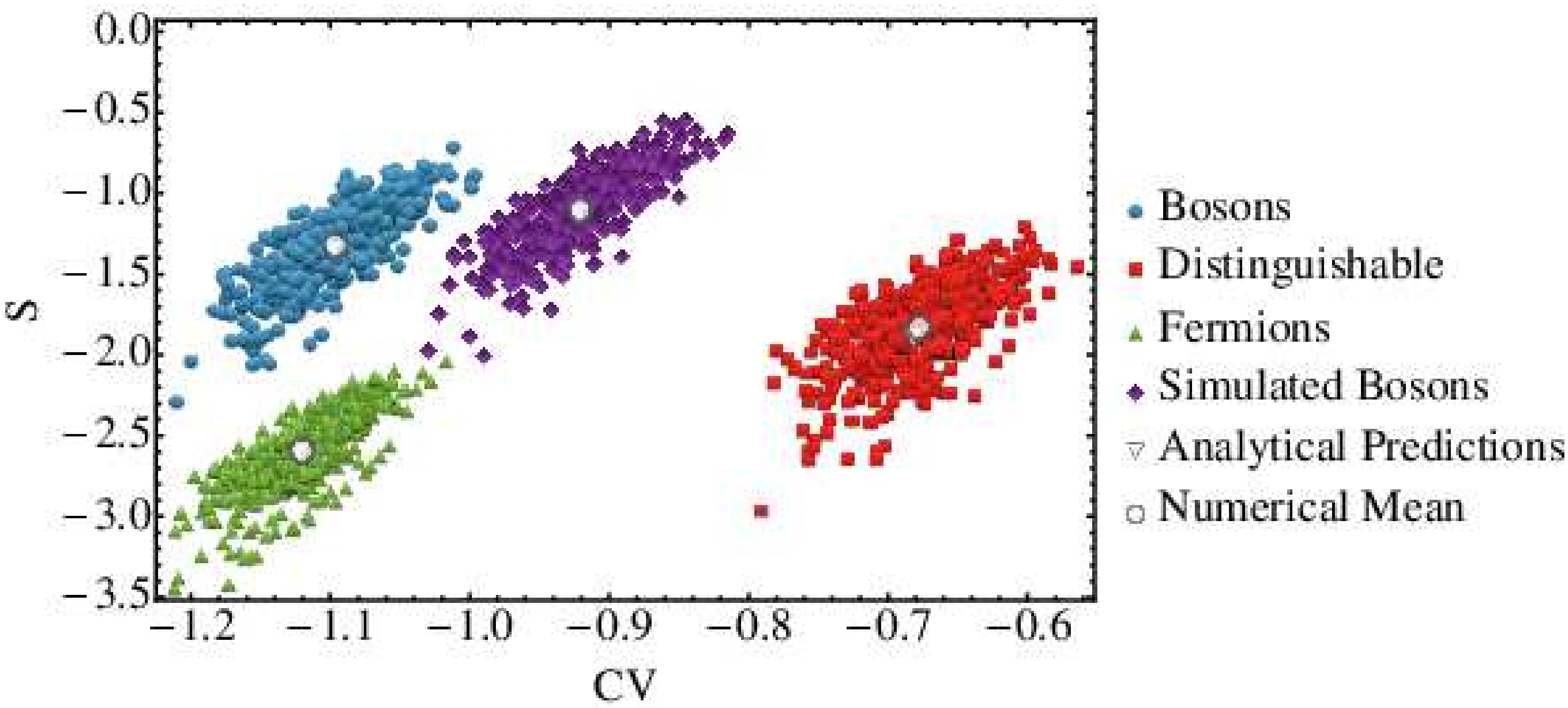


- 6 particles injected into 120 modes
- "simulated bosons" – average over relative phases of interfering many particle amplitudes, thus only keep bunching, kill interference terms

bosons vs. simulated bosons barely distinguishable!

Certification of distinguishable/fermionic/bosonic dynamics

correlate second and third moment – 6 particles in 120 modes



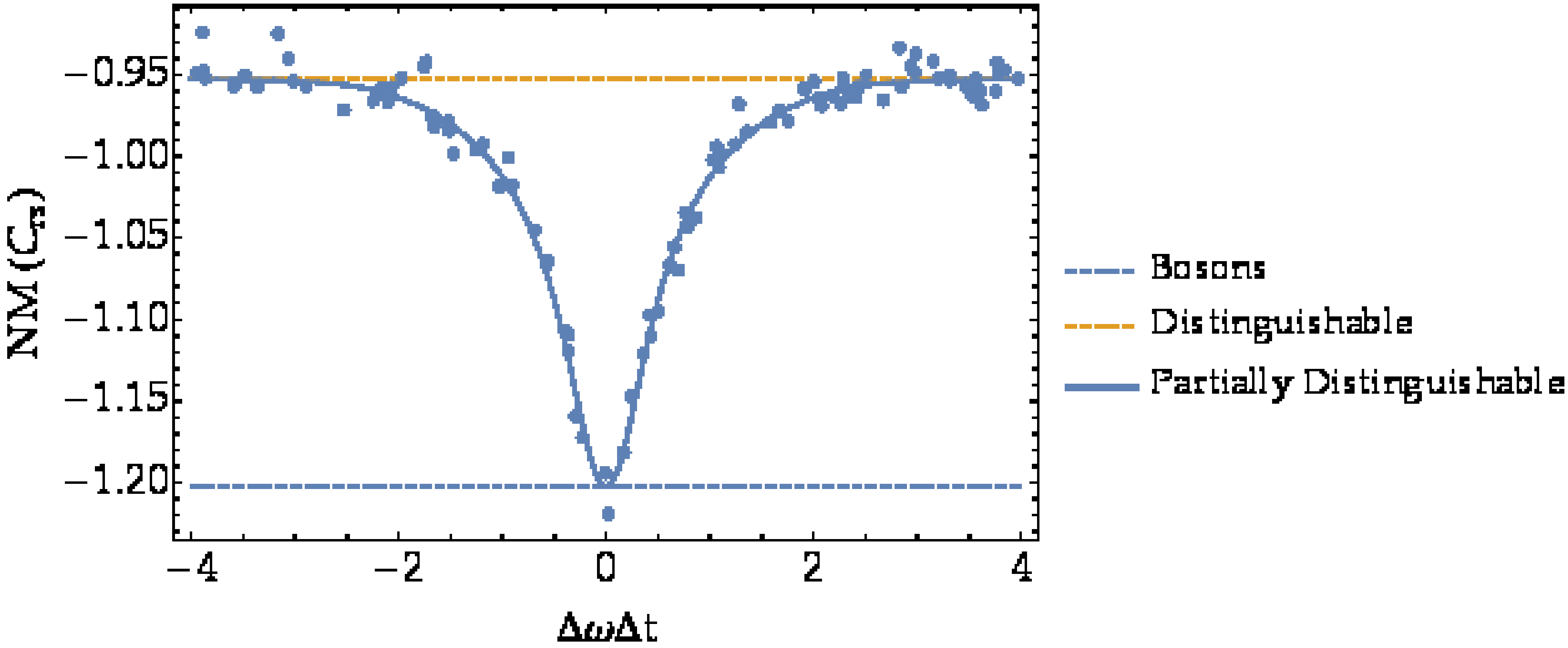
unambiguous distinction of particle type-specific dynamics!

excellent agreement between simulation and RMT prediction!

[Walschaers et al., 2014, 2016]

Multimode correlation spectroscopy

two-point correlators as a diagnostic tool
– pulse train of photons with delay Δt (in units of $1/\Delta\omega$) –



first moment of C-dataset vs. delay of photons i and $i+1$ (6 particles in 20 modes)

[Walschaers, 2016]

Annu. Rev. Condens. Matter Phys. (2016) –
doi:10.1146/annurev-conmatphys-031115-011327

NJP 18, 032001 (2016)

Mattia Walschaers, *Efficient Quantum Transport*, Dissertation,
Freiburg 2016 [under review]