UNI FREIBURG

Quantum Optics and Statistics
Physikalisches Institut

Spectral detection of resonance fluorescence

Vyacheslav N. Shatokhin, Sergei Ya. Kilin, Andreas Buchleitner

Motivation

- **Resonance fluorescence from *artificial* atoms**

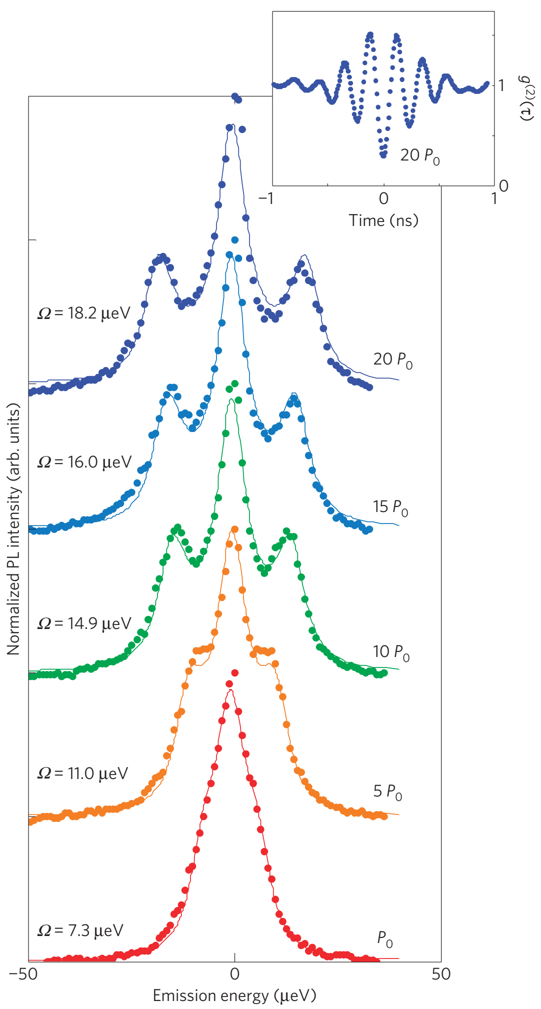
- **Building blocks for controllable, quantum-coherent structures**

- **Applications, e.g. as single- and entangled-photon sources**

Motivation

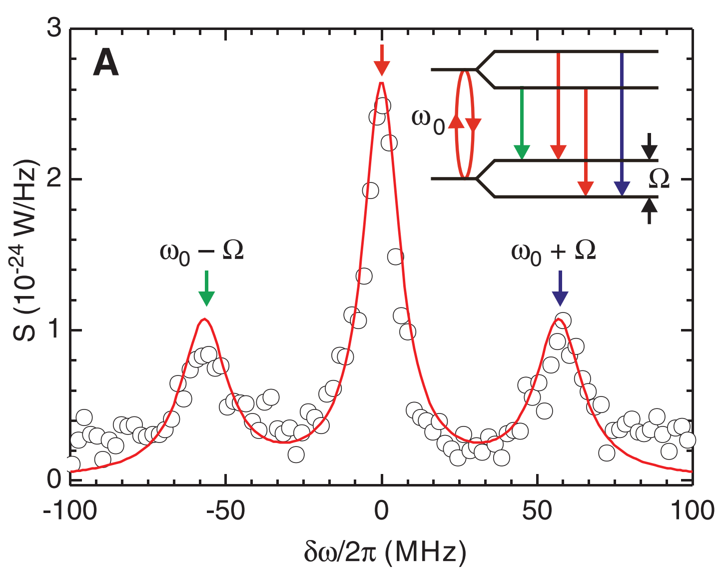
- **Observation of the resonance fluorescence triplet from *artificial* atoms**

with quantum dots



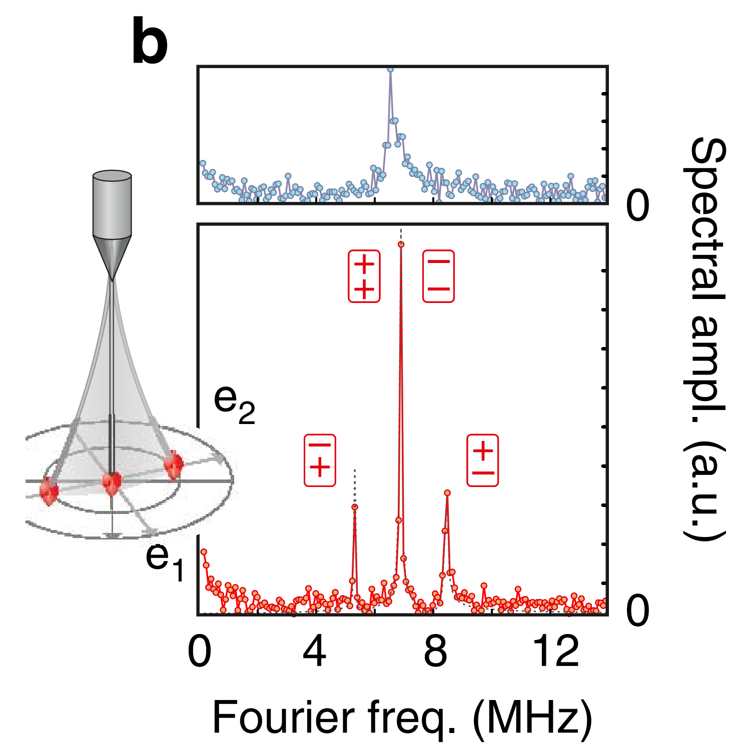
E.B. Flagg et al., Nat. Phys. 5, 203 (2009)

…superconducting qubits



O. Astafiev et al., Science 327, 840 (2010)

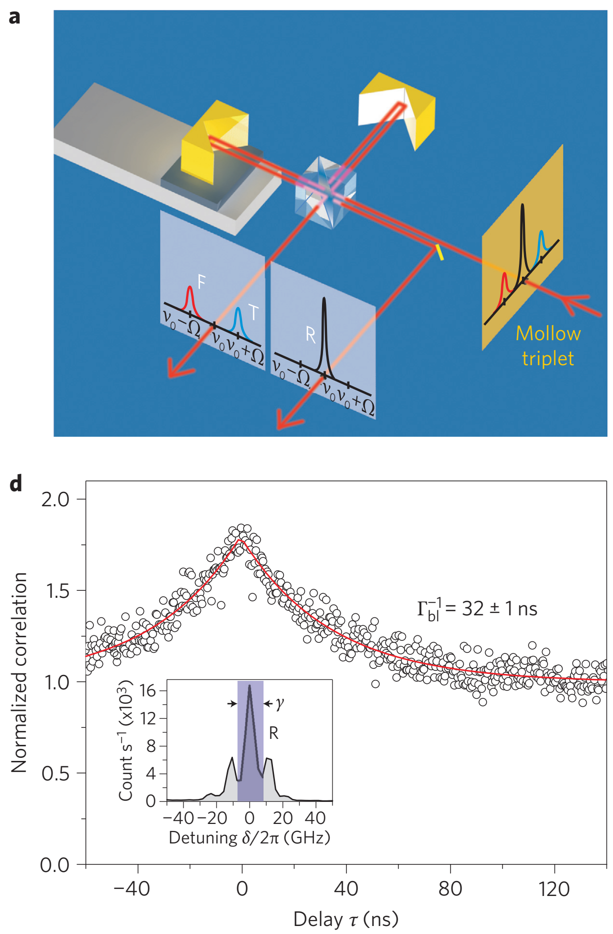
…hybrid spin-nanomechanical systems

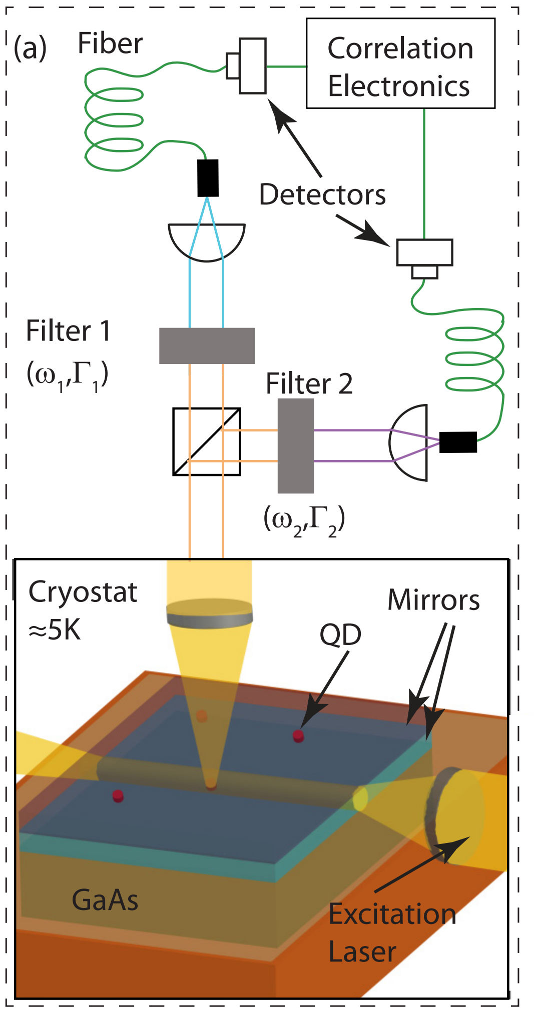


B. Pigeau et al., Nat. Commun. 6, 1038 (2015)

Motivation

- **Spectral correlation measurements with *artificial* atoms**





A. Ulhaq et al. Nat. Photon. (2012)

M. Peiris et al. Phys. Rev. B. (2015)

Motivation

$$\Delta t \Delta \omega \geq 1$$

Motivation

$$\Delta t \Delta \omega \geq 1$$

$$\rho^c(t)$$

conditioned state following detection of a spectrally resolved photon

Outline

- Introduction into resonance fluorescence
- Spectral filtering
- Conditioned atomic state and memory effects
- Conclusion

Single atom resonance fluorescence

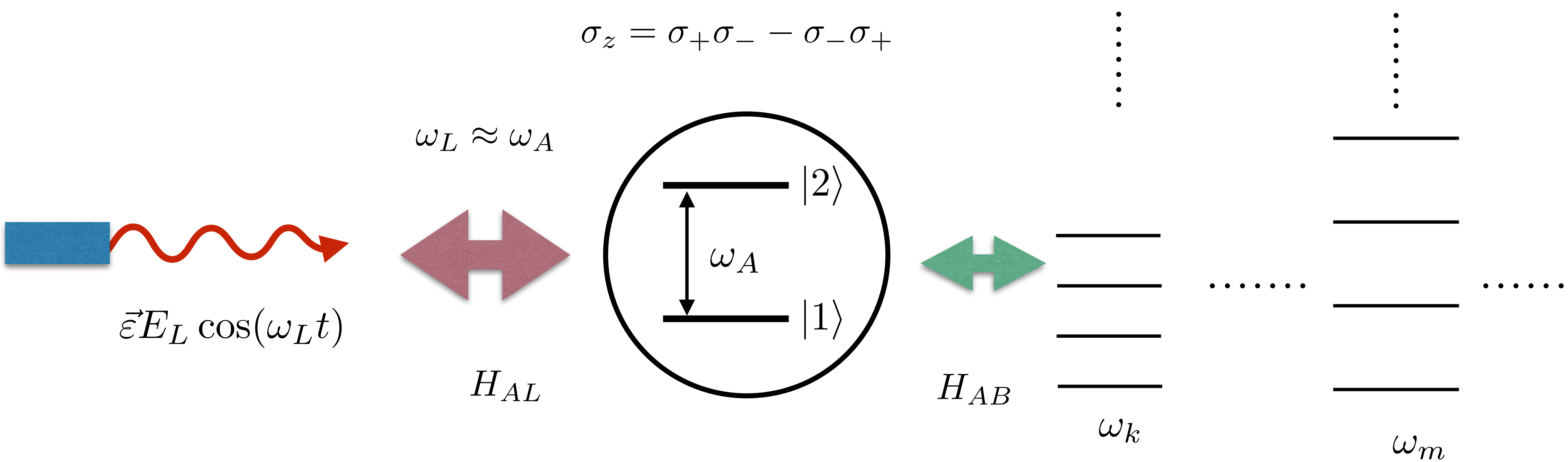
Laser field

Atom

$$\sigma_+ = |2\rangle\langle 1|, \sigma_- = |1\rangle\langle 2|,$$
$$\sigma_z = \sigma_+\sigma_- - \sigma_-\sigma_+$$

Radiation bath

$$[a_k, a^\dagger_{k'}] = \delta_{kk'}$$



$$H_A = \frac{\hbar\omega_A}{2}\sigma_z$$

$$H_B = \hbar \sum_k \omega_k a^\dagger_k a_k$$

$$H = H_A + H_B + H_{AL} + H_{AB}$$

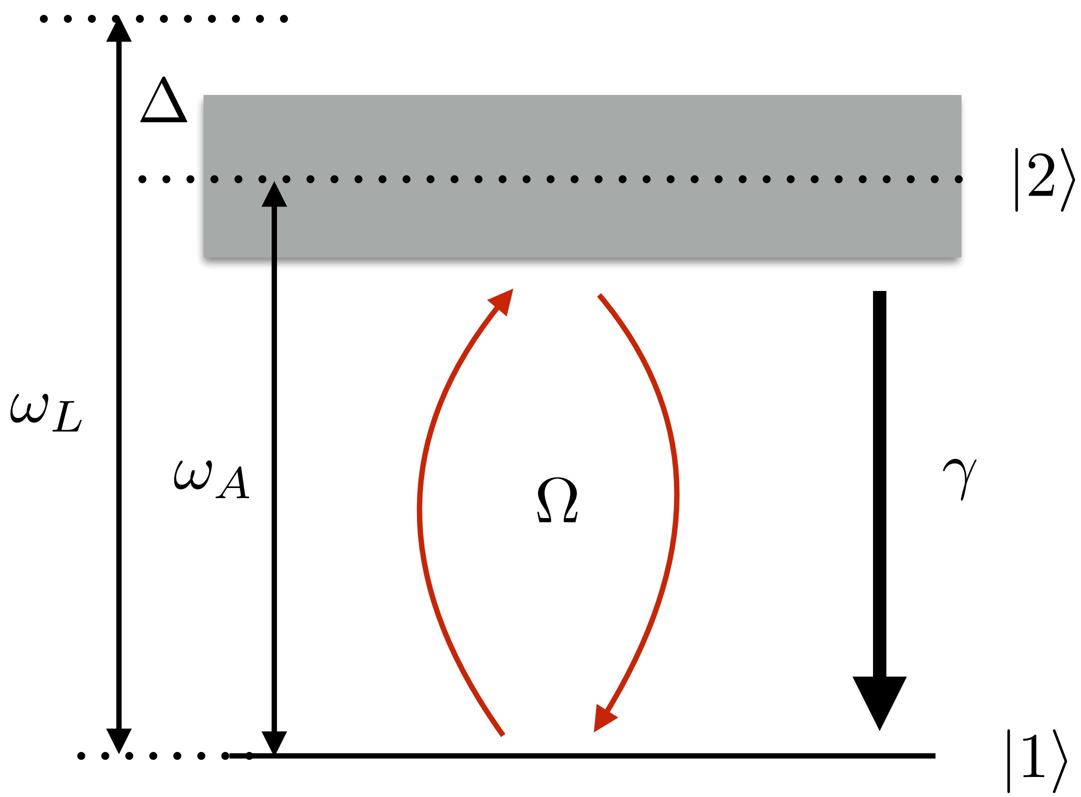
$$H_{AL} = \hbar v(\sigma_+ e^{-i\omega_L t} + \sigma_- e^{i\omega_L t})$$

$v = dE_L/\hbar$ - Rabi frequency

$$H_{AB} = \hbar \sum_k g_k (a_k \sigma_+ + \sigma_- a^\dagger_k)$$

g_k - coupling constant

Single atom resonance fluorescence



von Neumann equation

$$\dot{\rho}_{AB} = -\frac{i}{\hbar}[H, \rho_{AB}]$$

Master equation (Markov type)

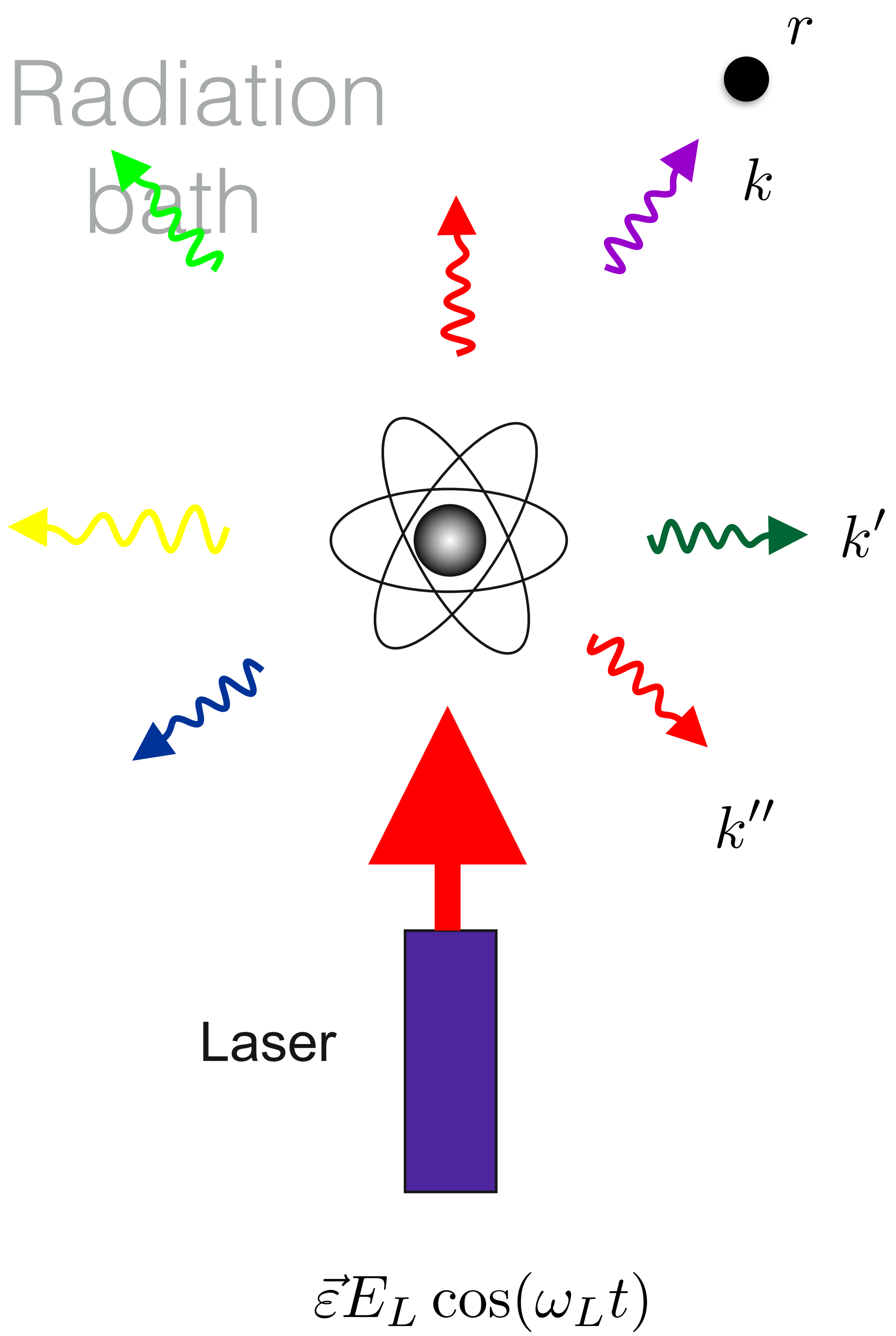
$$\dot{\rho}_A = Tr_B[\dot{\rho}_{AB}]$$

$$\dot{\rho}_A = i\frac{\Delta}{2}[\sigma_z, \rho_A] - i\frac{v}{2}[\sigma_+ + \sigma_-, \rho_A] + \frac{\gamma}{2}(2\sigma_-\rho_A\sigma_+ - \sigma_+\sigma_-\rho_A - \rho_A\sigma_+\sigma_-)$$

$$\Omega = \sqrt{v^2 + \Delta^2}$$ generalised Rabi frequency

$$\gamma = \frac{d^2\omega_A^3}{3\pi\epsilon\hbar c^3}$$ spontaneous decay rate

Single atom resonance fluorescence



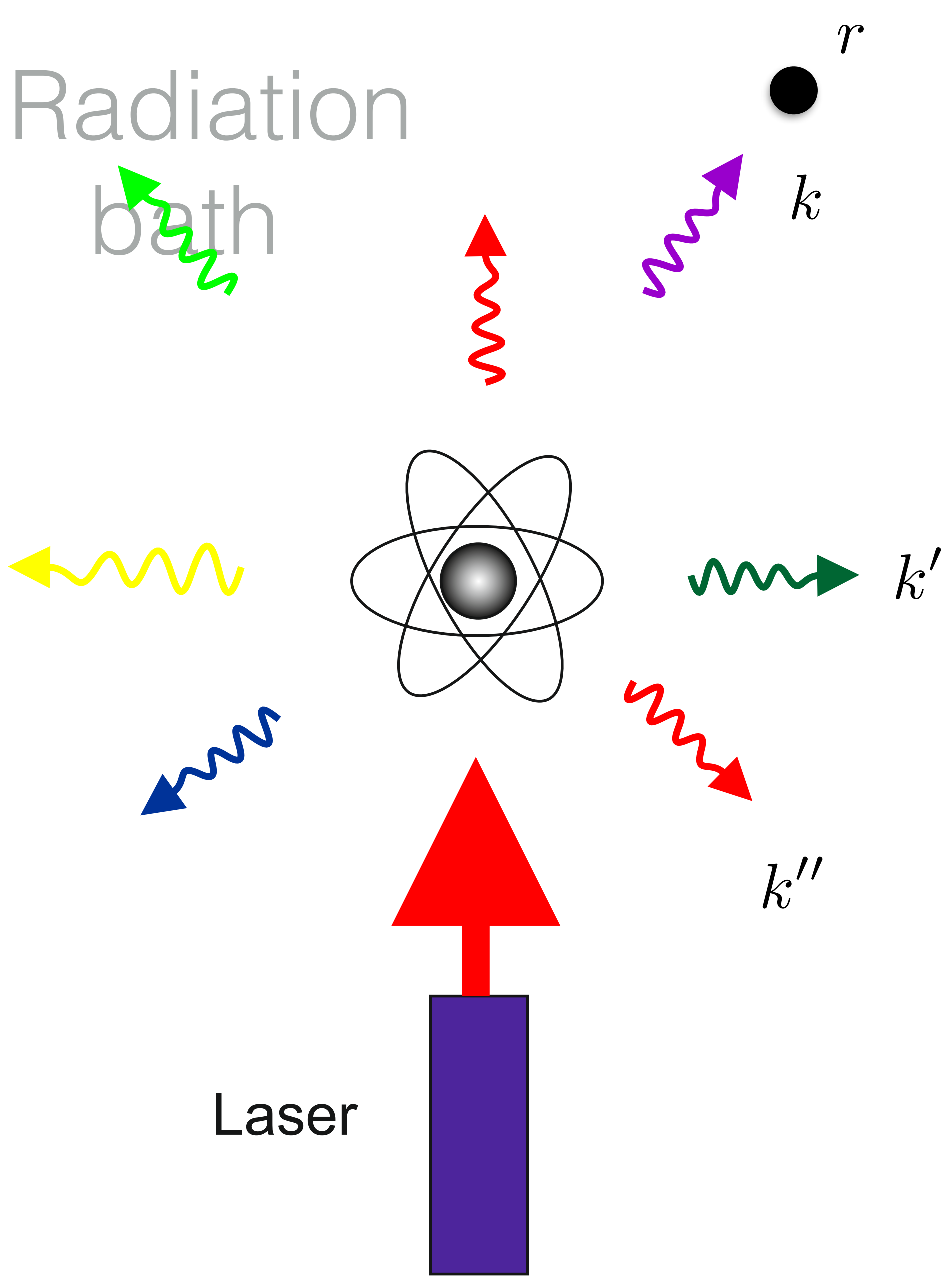
Scattered field in the radiation zone

$$E(t) = E^{(+)}(t) + E^{(-)}(t)$$

$$E^{(+)}(t) \propto g_k \sigma_-(t - r/c),$$
$$E^{(-)}(t) \propto g_k \sigma_+(t - r/c)$$

$$\langle E^{(-)}(t_1) \ldots E^{(+)}(t_n) \rangle \propto \langle \sigma_+(\tilde{t}_1) \ldots \sigma_-(\tilde{t}_n) \rangle$$

Single atom resonance fluorescence



$\vec{\varepsilon}E_L\cos(\omega_L t)$

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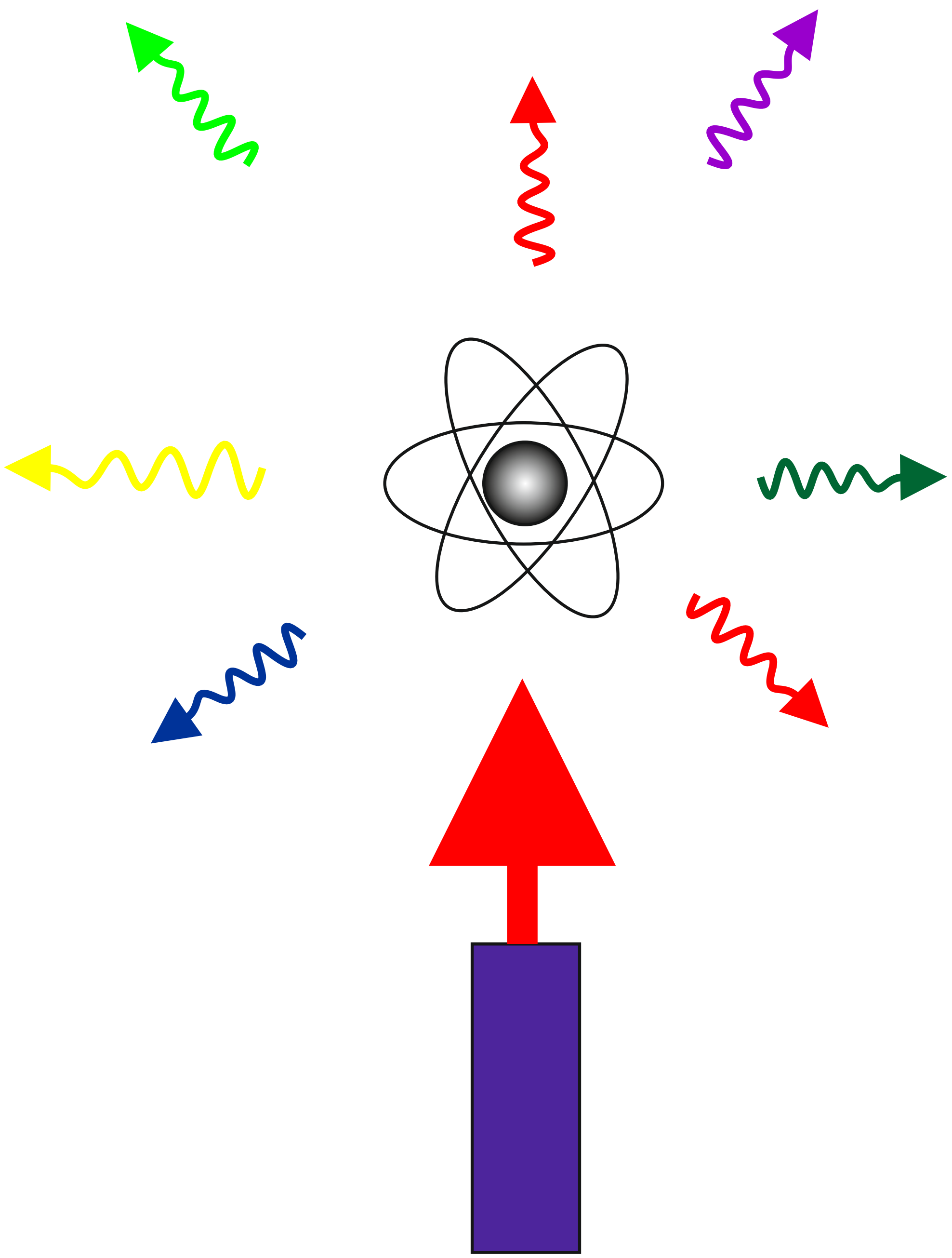
$$\langle E^{(-)}(t_1)\ldots E^{(+)}(t_n)\rangle \propto \langle \sigma_+(\tilde{t}_1)\ldots\sigma_-(\tilde{t}_n)\rangle$$

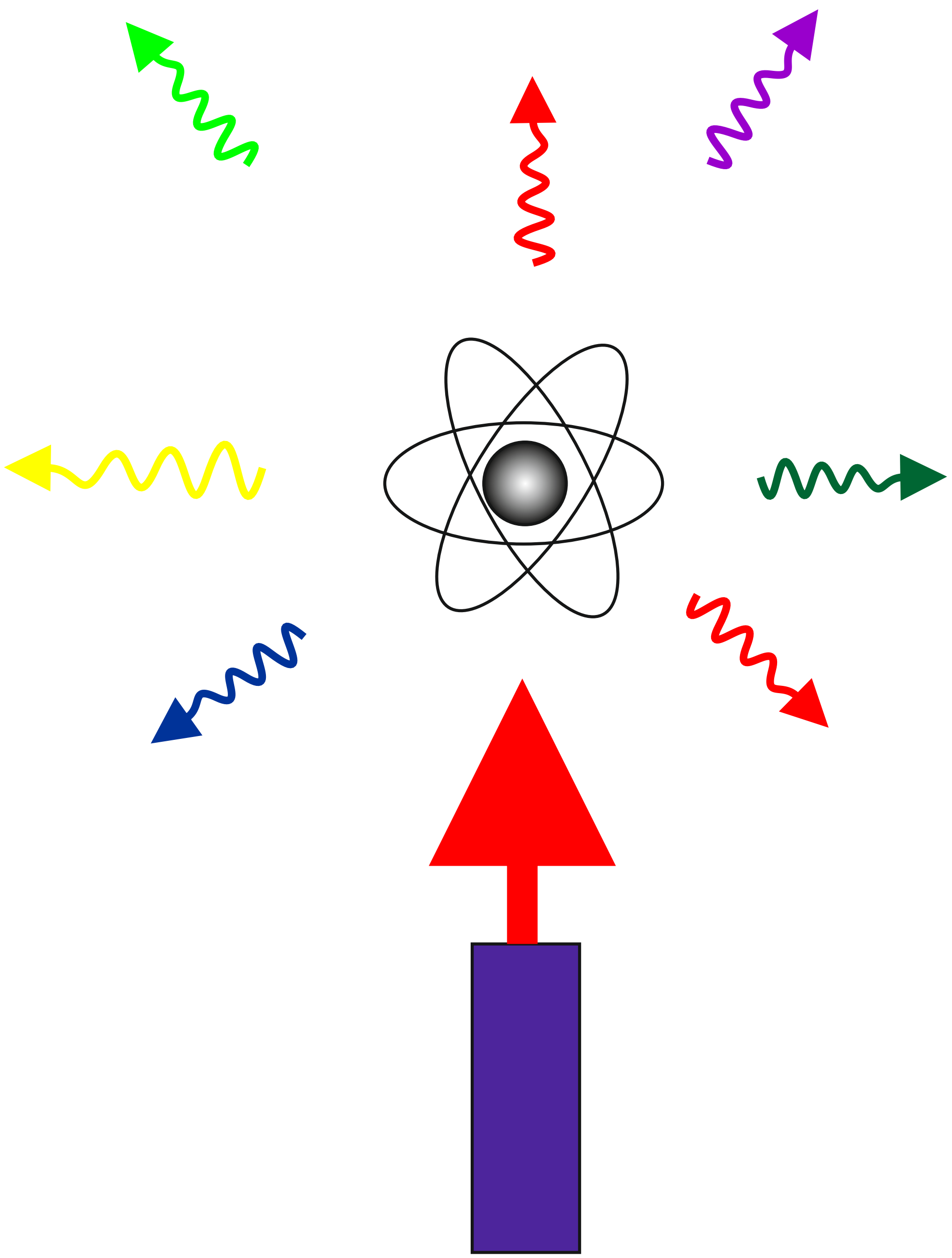
Master equation
+quantum regression theorem

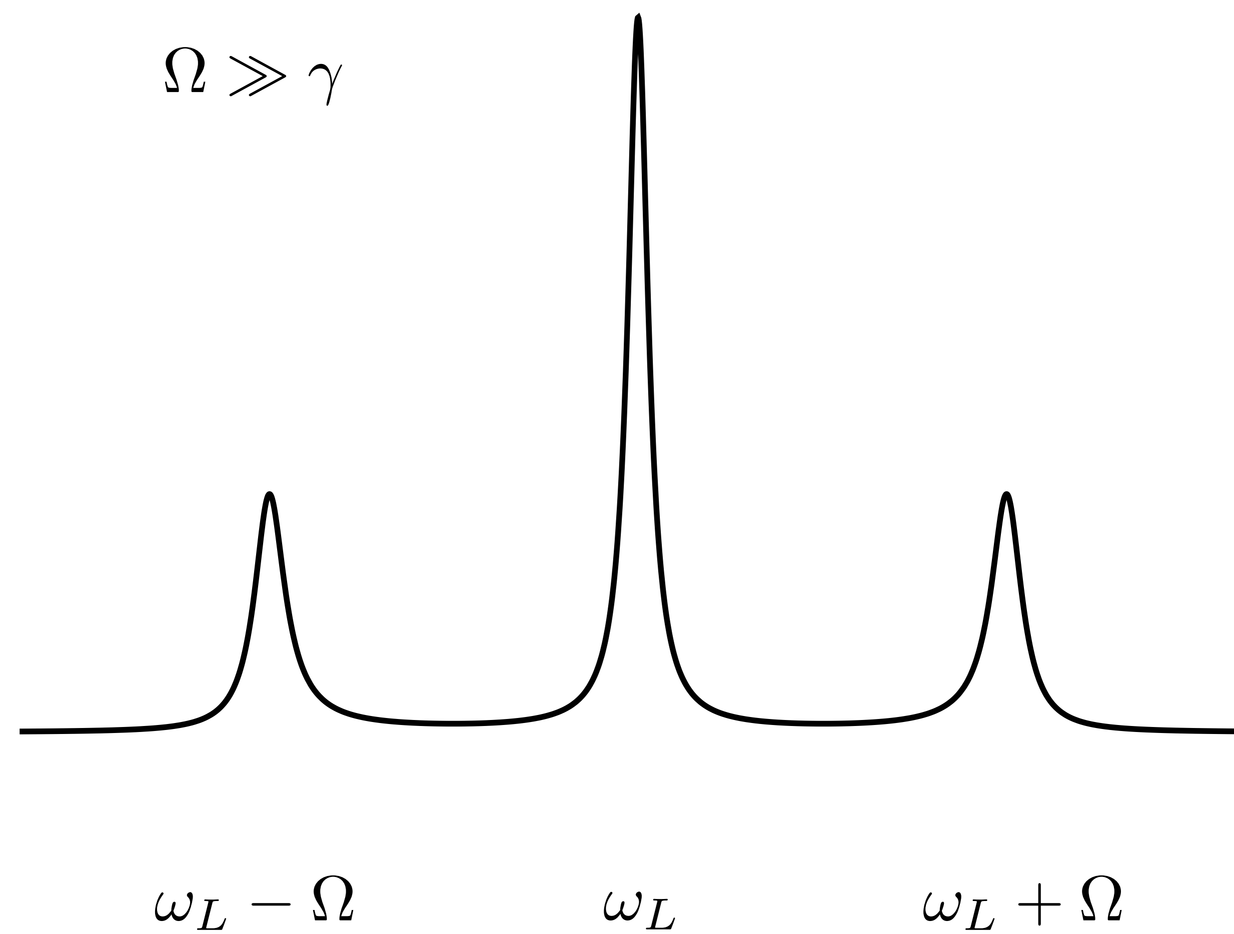
Spectrum



Spectrum

Mollow triplet

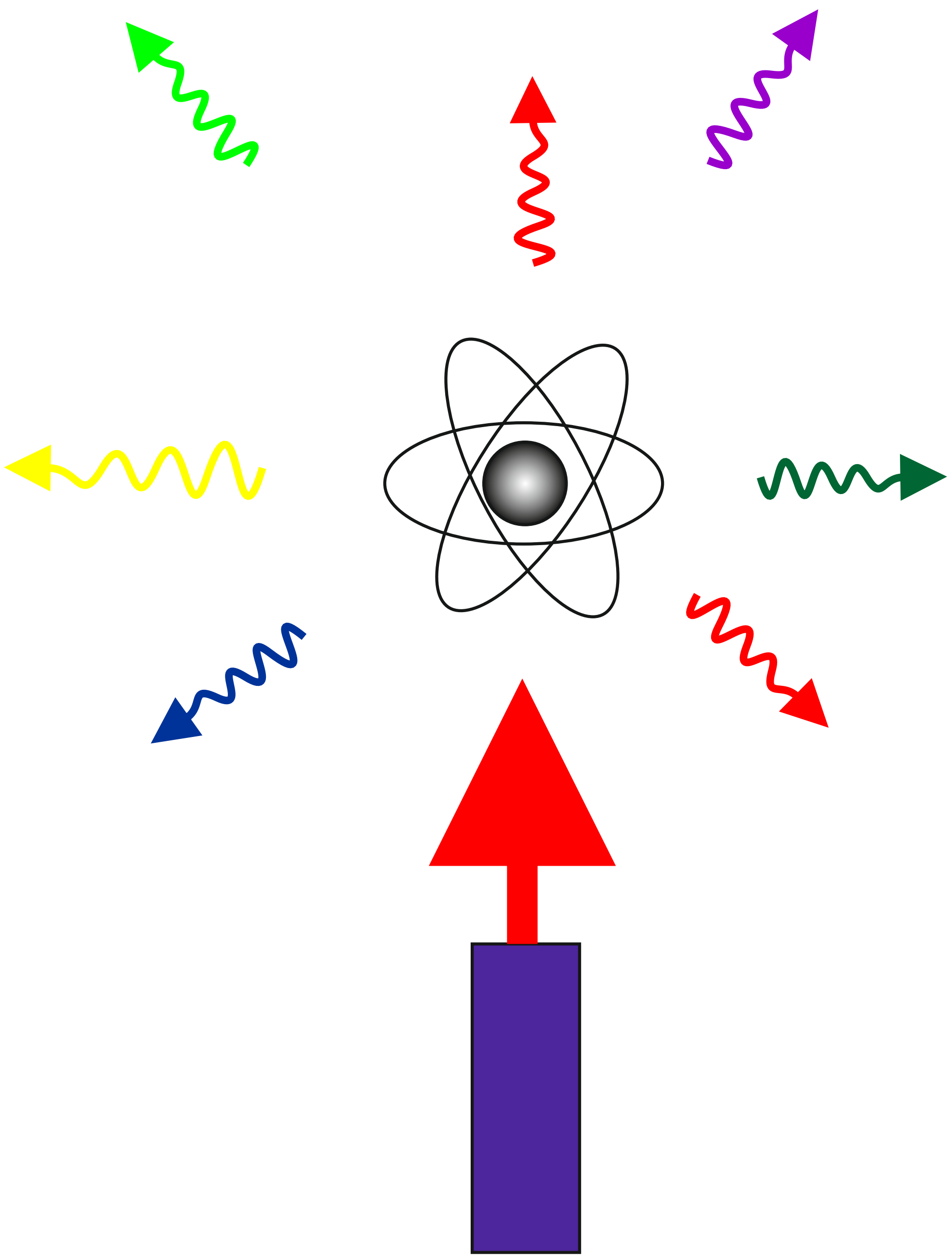




B. R. Mollow, Phys. Rev. 188, 1969 (1969)

Spectrum

- all possible elementary scattering processes of laser photons on the atom



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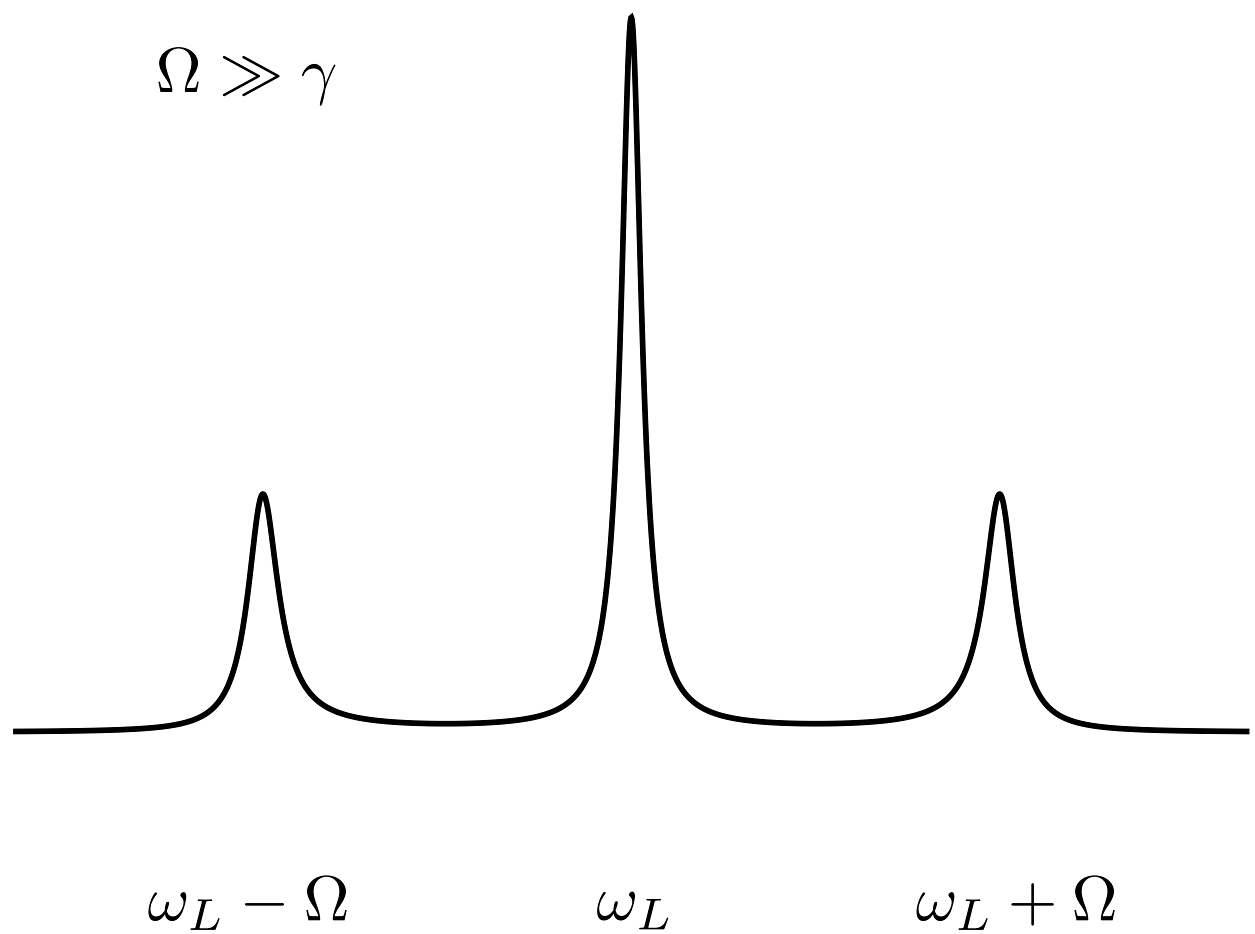

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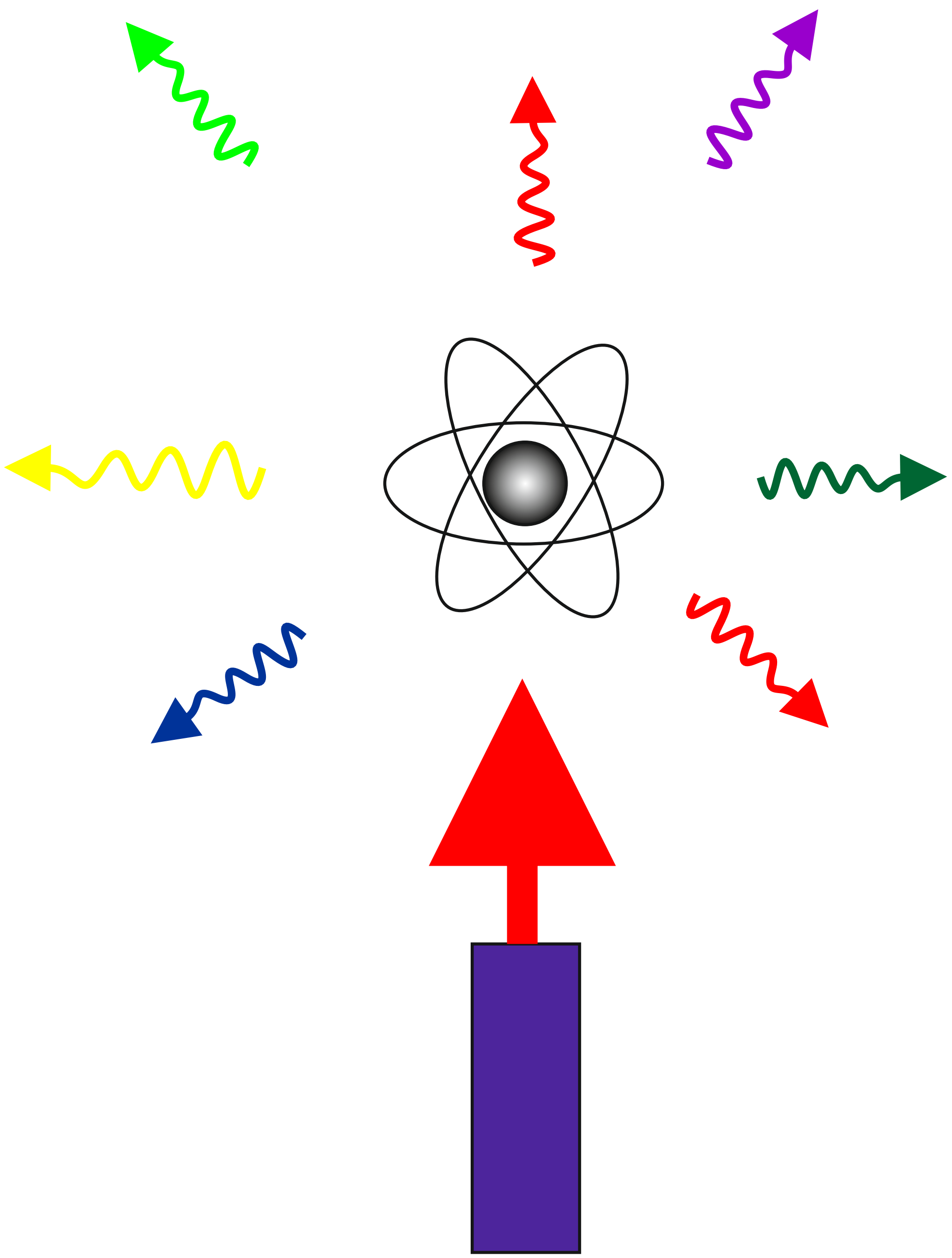
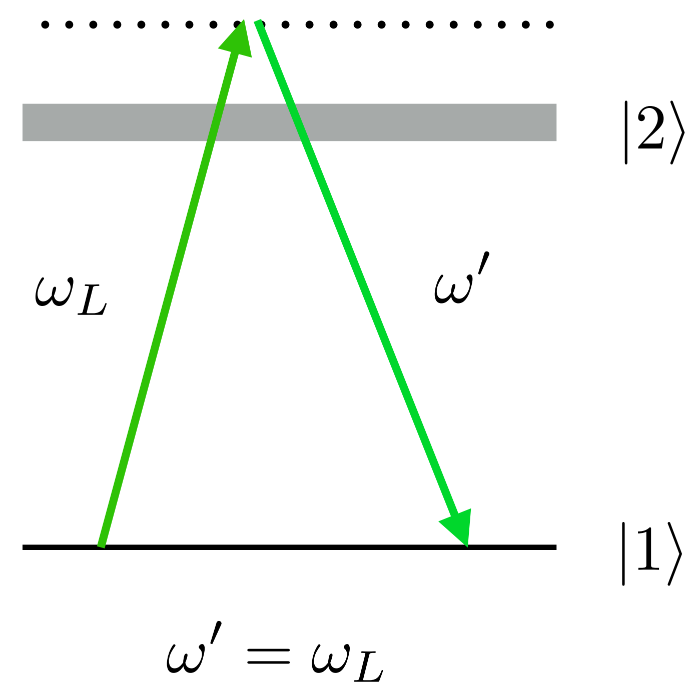
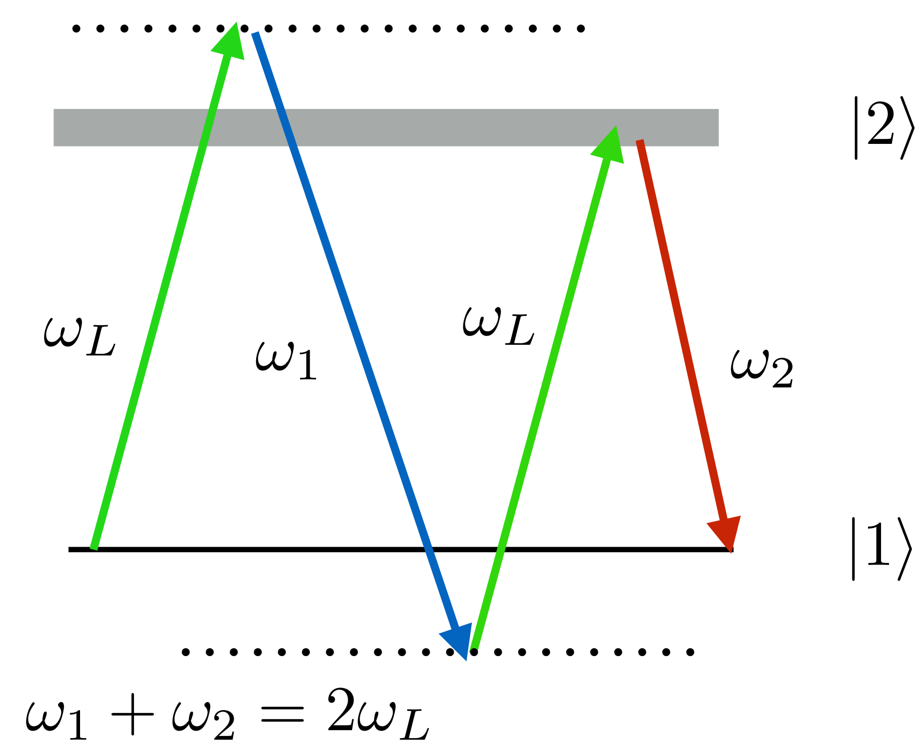
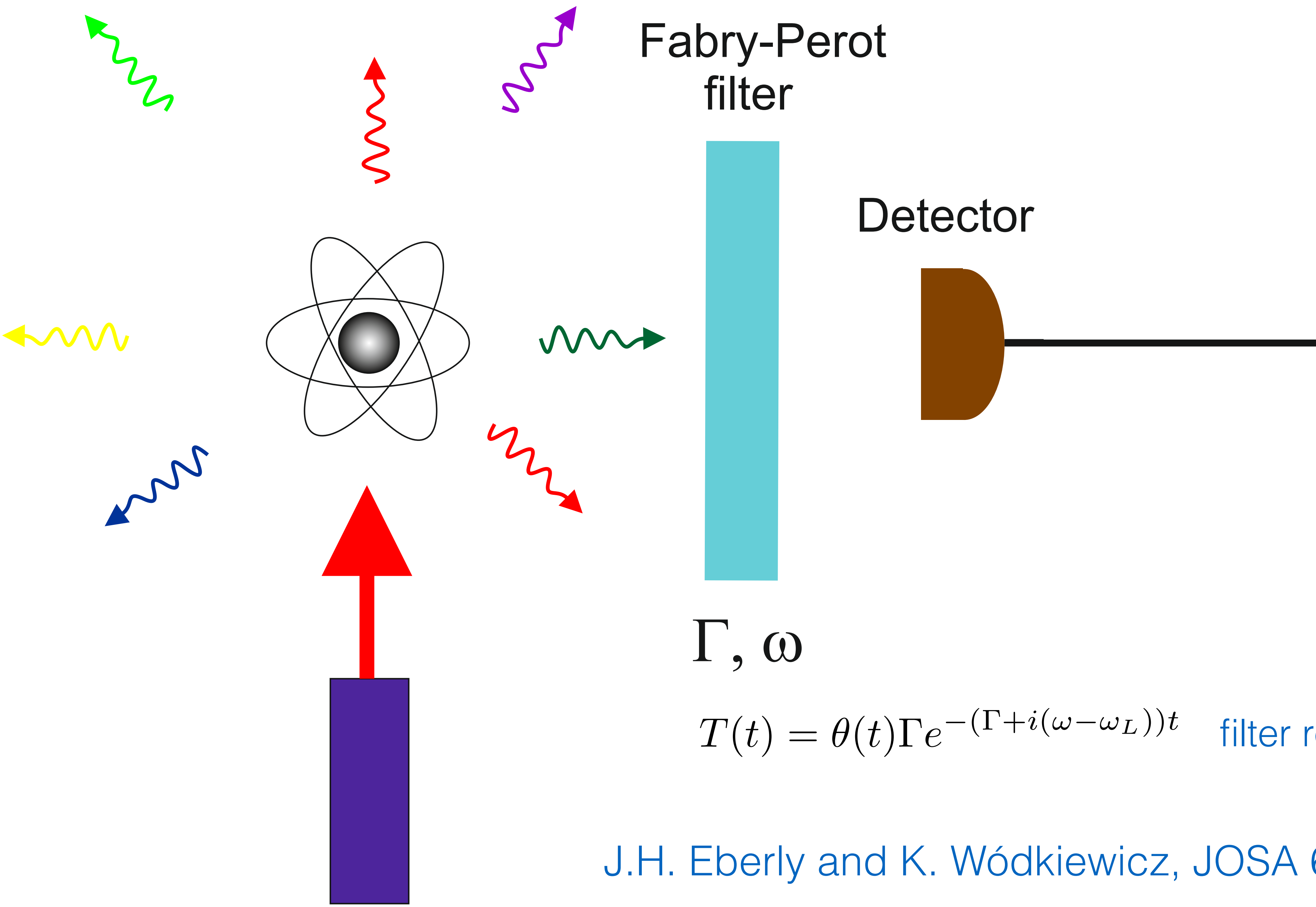






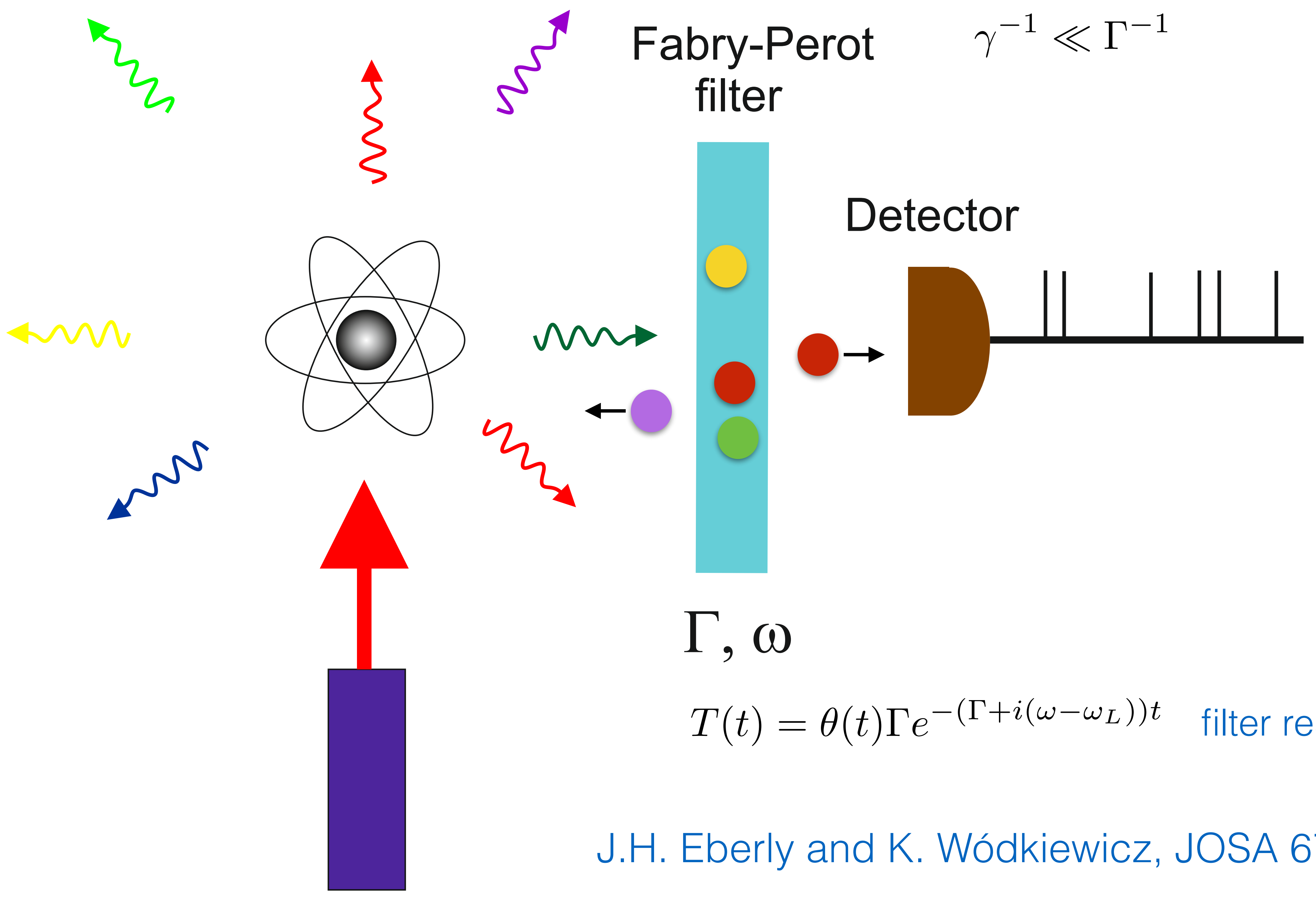
Spectral detection



$$T(t) = \theta(t)\Gamma e^{-(\Gamma + i(\omega - \omega_L))t}$$ filter response function

J.H. Eberly and K. Wódkiewicz, JOSA 67, 1252 (1977)

Spectral detection



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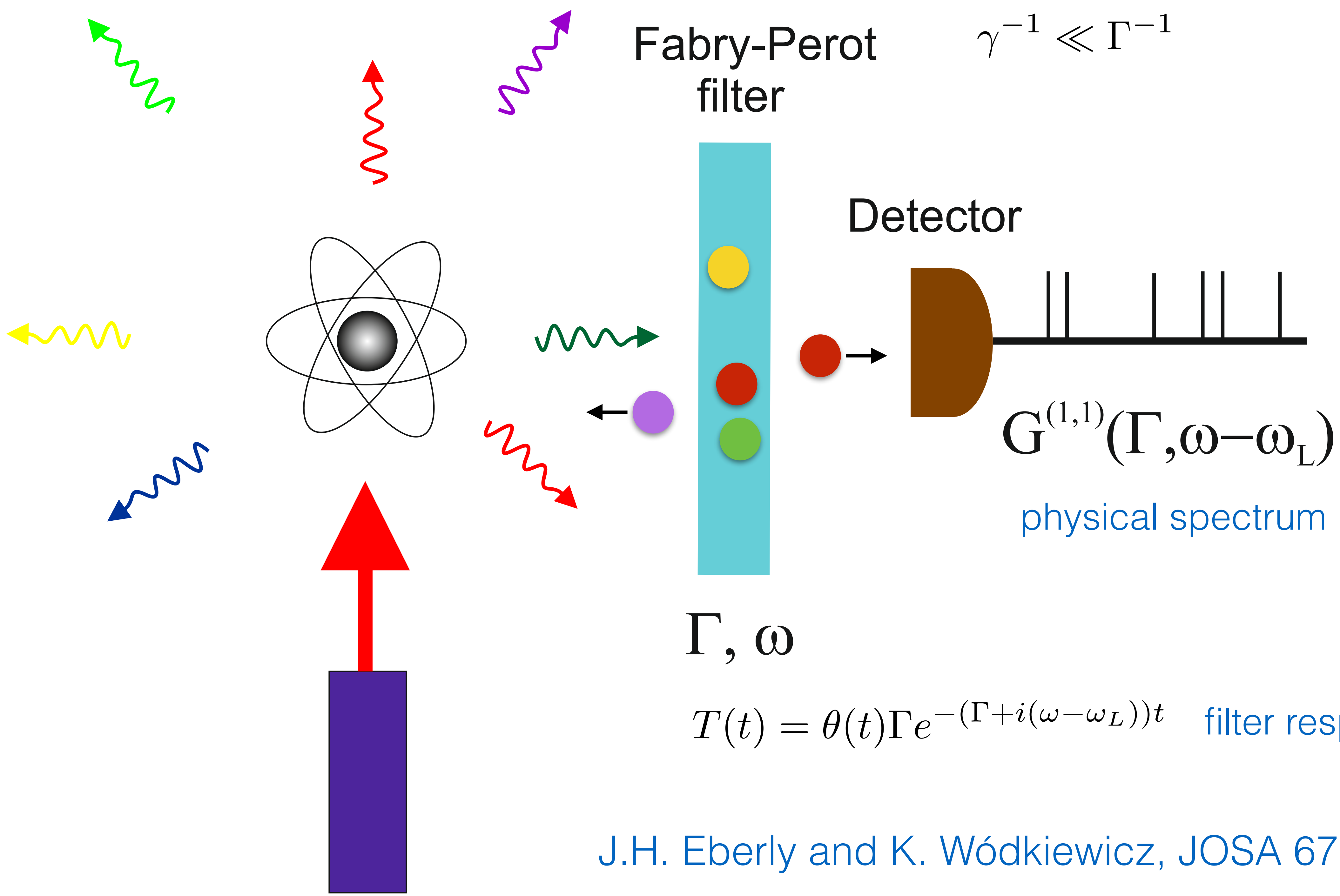

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Physical spectrum

$$G^{(1,1)}(\Gamma, \omega - \omega_L) = \lim_{t \to \infty} \int_0^t dt_1 \int_0^t dt_2 T^*(t - t_1) T(t - t_2) \times \langle \sigma_+(t_1) \sigma_-(t_2) \rangle$$

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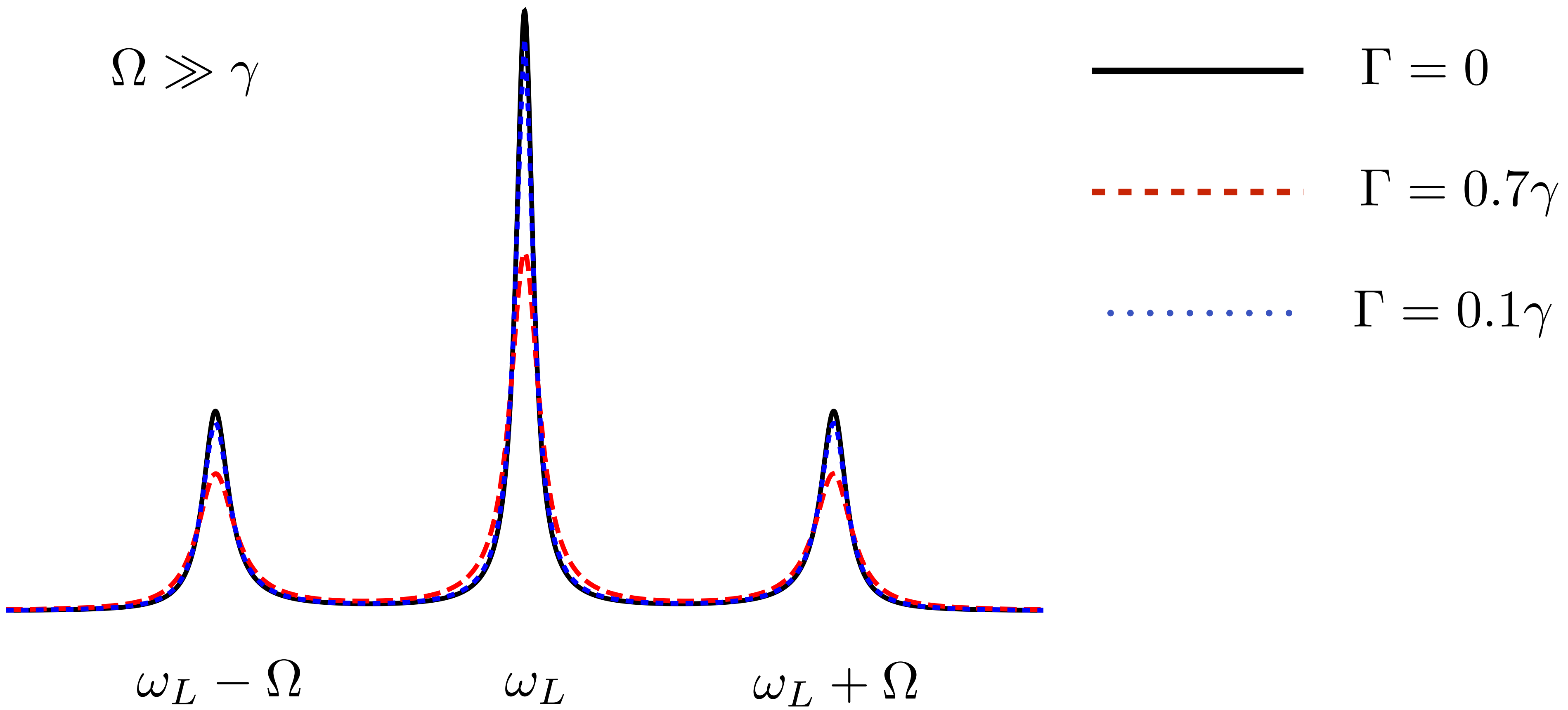
Mollow triplet



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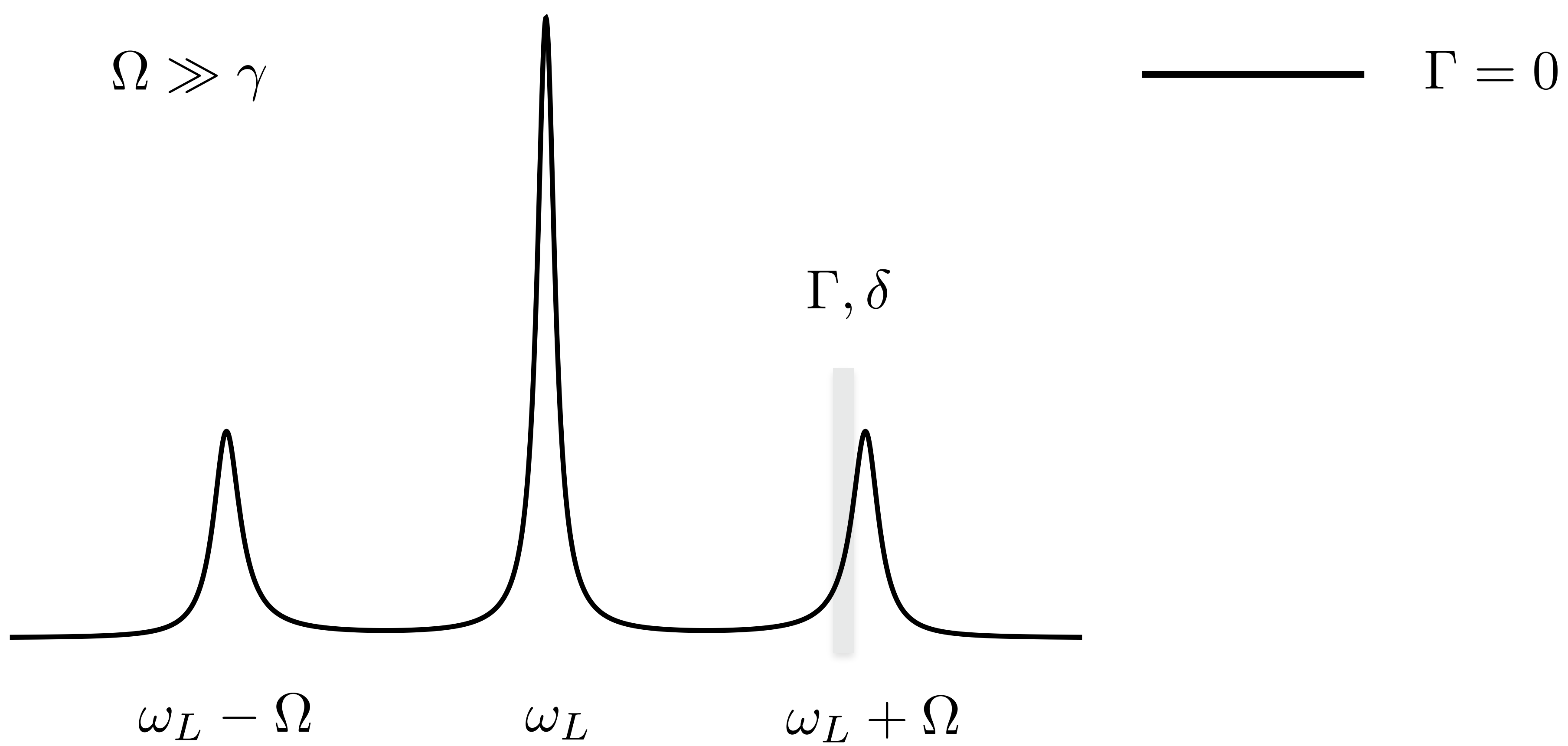

What is the conditioned state of the atom?

Physical spectrum

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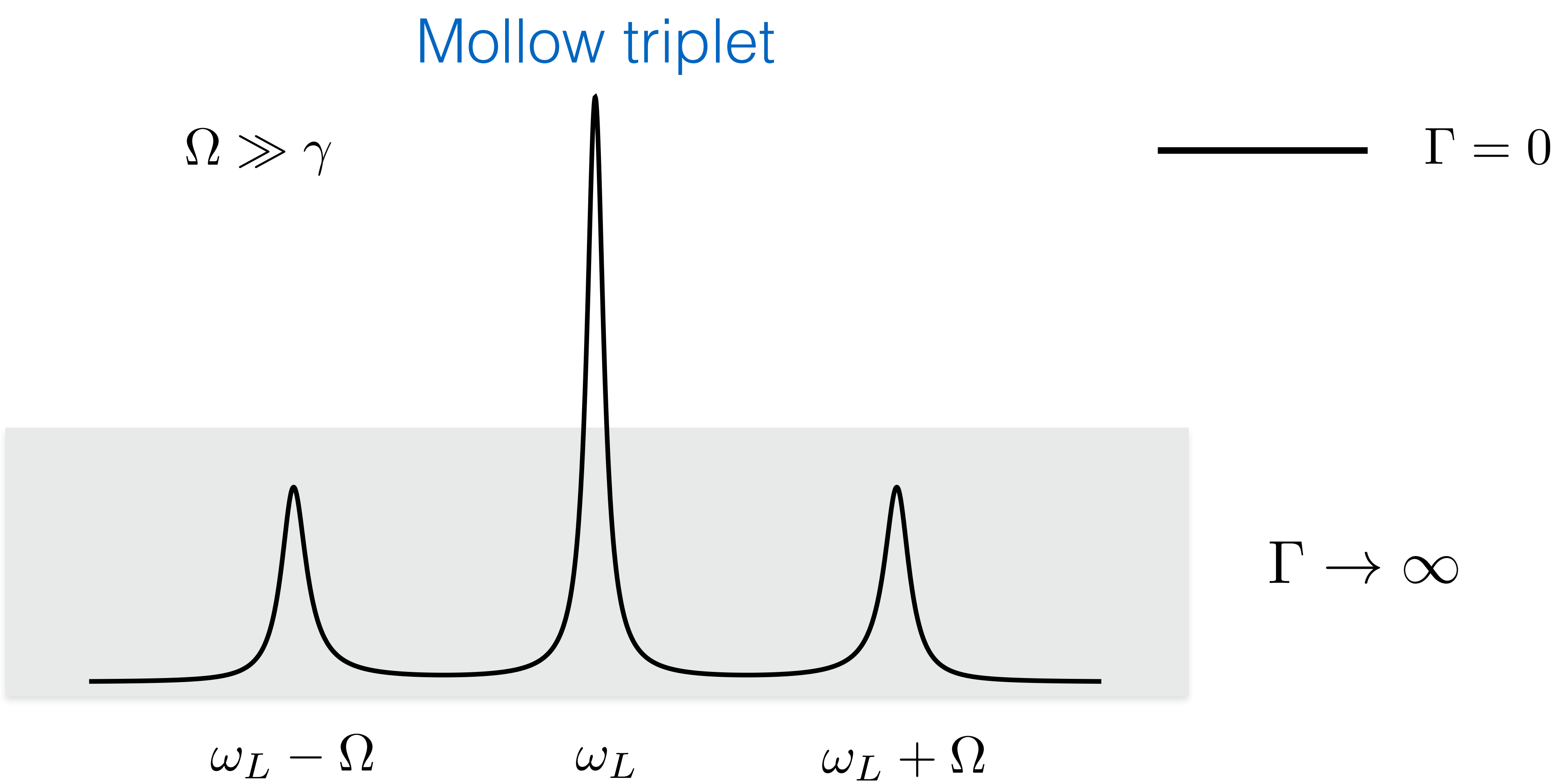
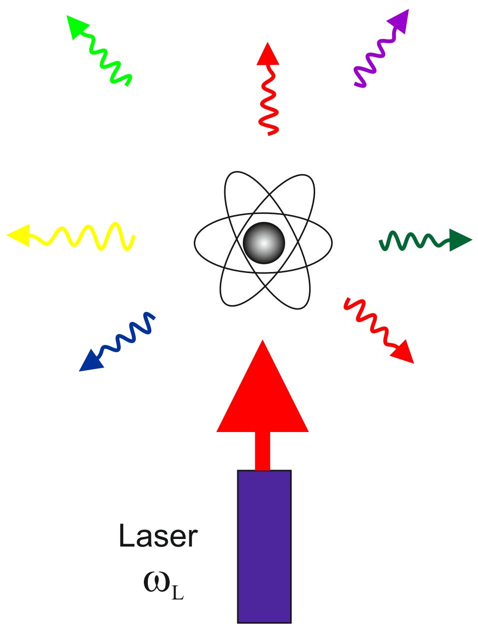
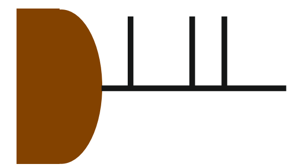


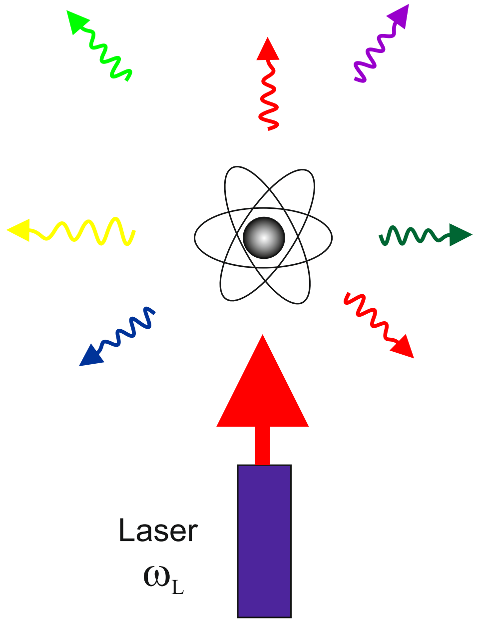
What is the conditioned state of the atom?

Conditioned state: broadband detection

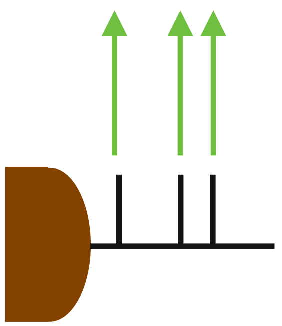




Conditioned state: broadband detection



$$\rho = |1\rangle\langle 1|$$



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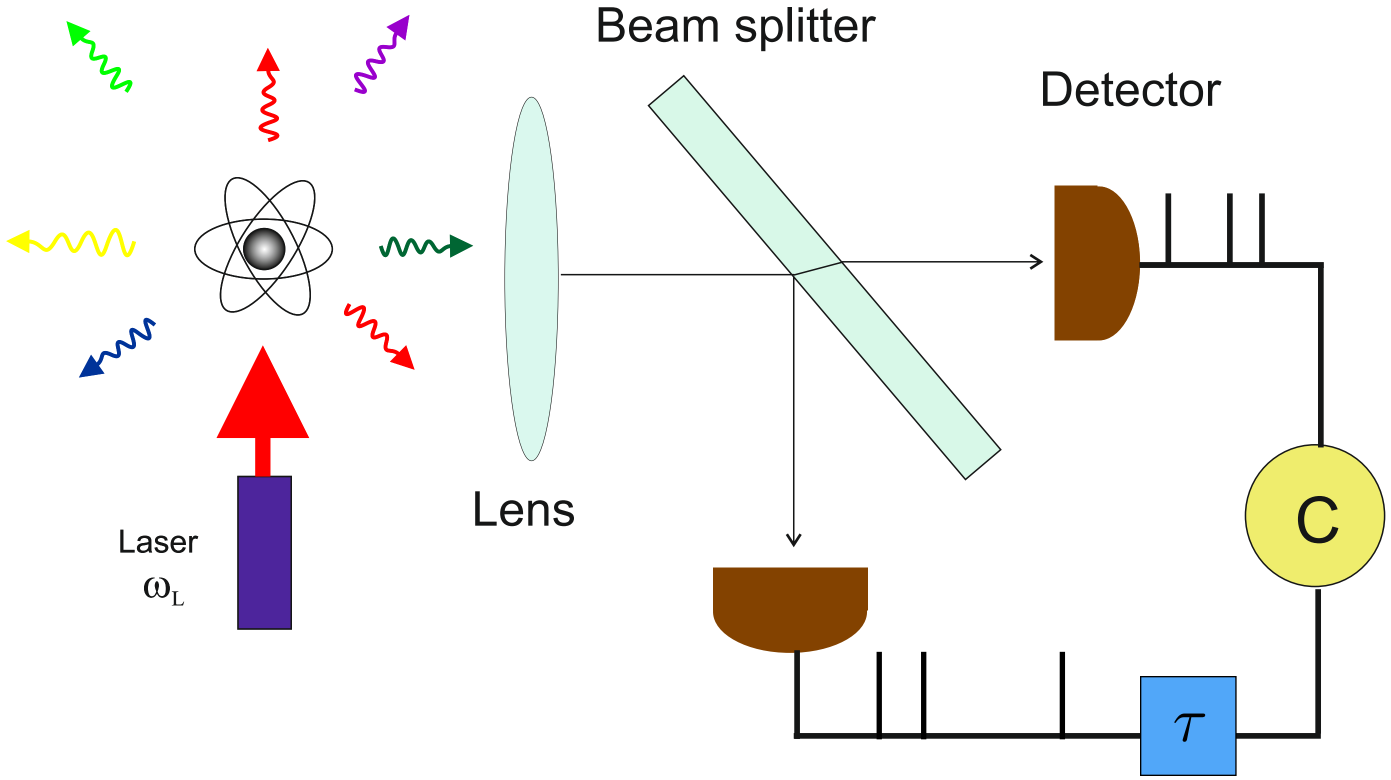

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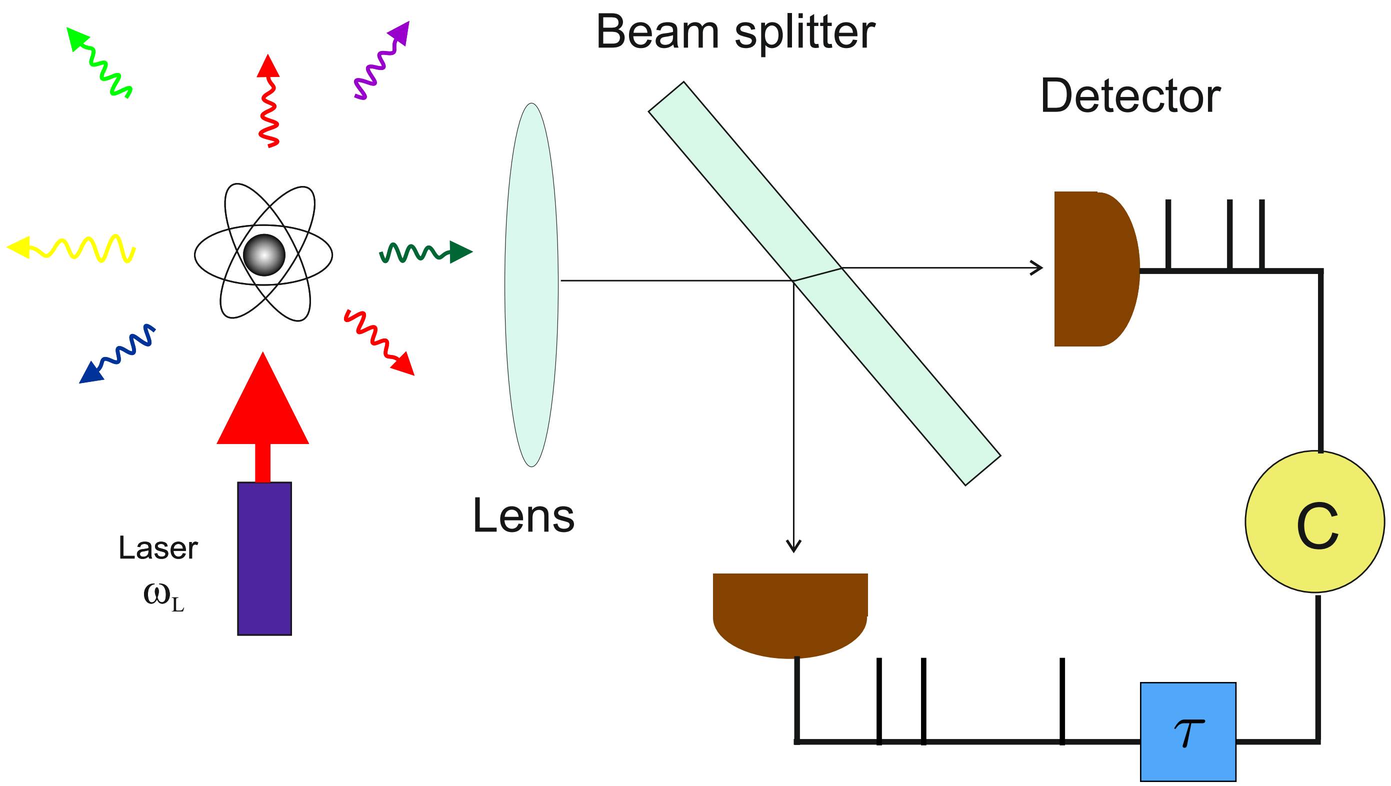
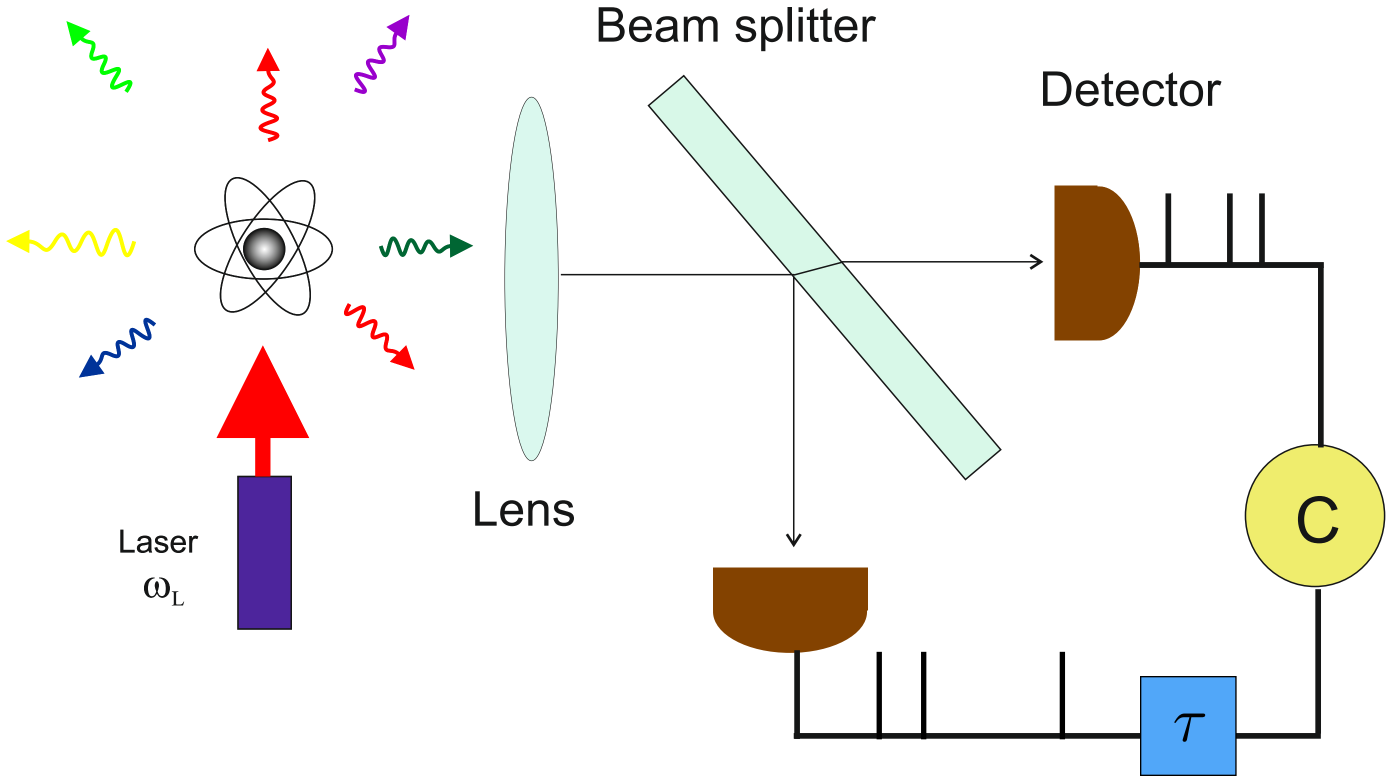


$$g^{(2)}(\tau) = \lim_{t\to\infty} \frac{\langle \sigma_+(t)\sigma_+(t+\tau)\sigma_-(t+\tau)\sigma_-(t)\rangle}{\langle \sigma_+(t)\sigma_-(t)\rangle^2}$$

Conditioned state: broadband detection



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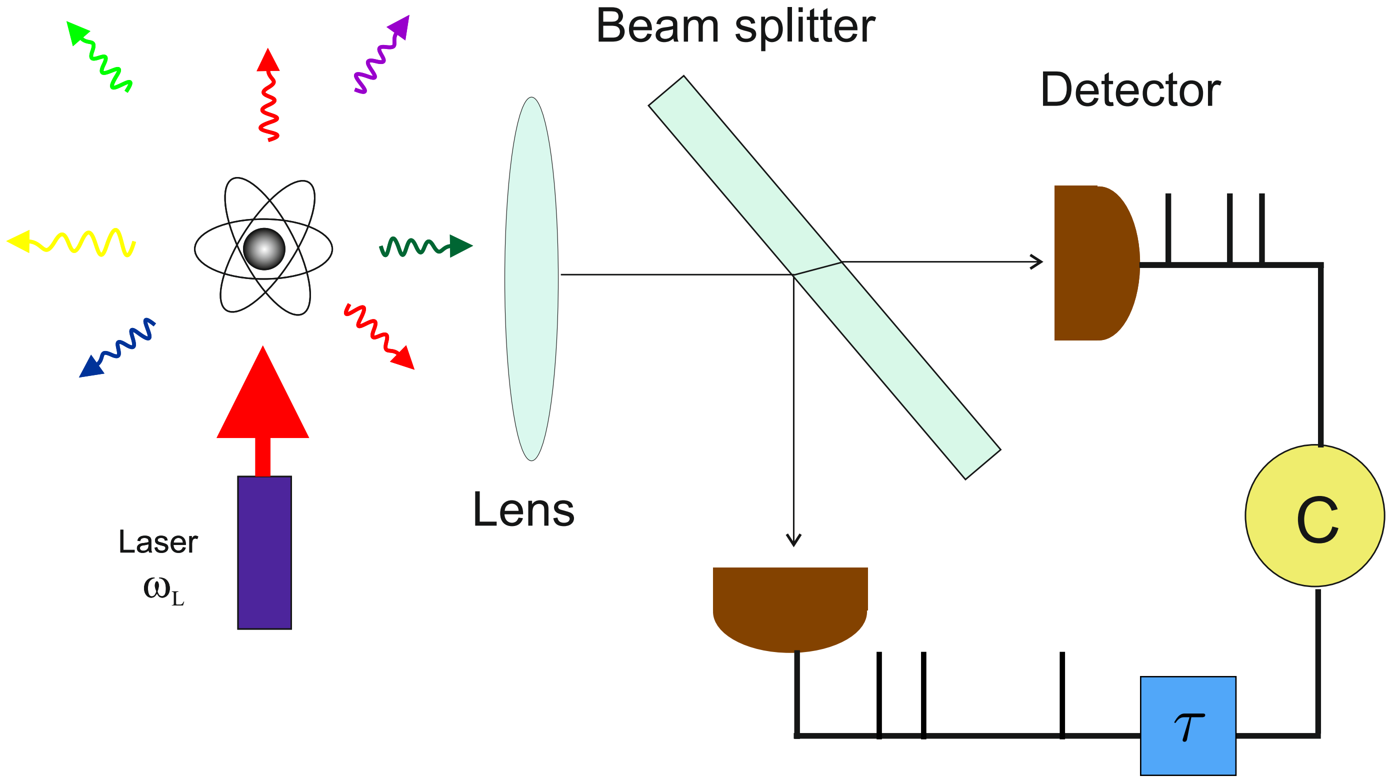
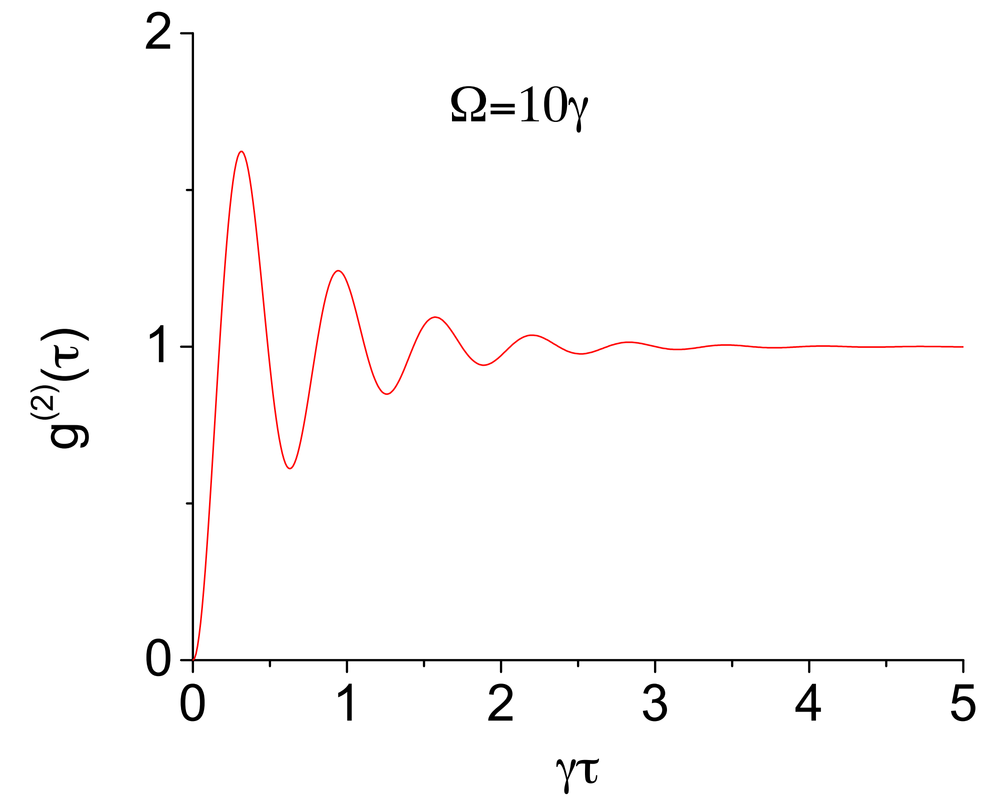
$$g^{(2)}(\tau) = \frac{\rho_{22}(\tau)|_{\rho(0)=|1\rangle\langle 1|}}{\rho_{22}(\infty)}$$

$g^{(2)}(0) = 0$ photon antibunching

Conditioned state: broadband detection





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H.J. Carmichael, D.F. Walls, J. Phys. B (1976) [theory]

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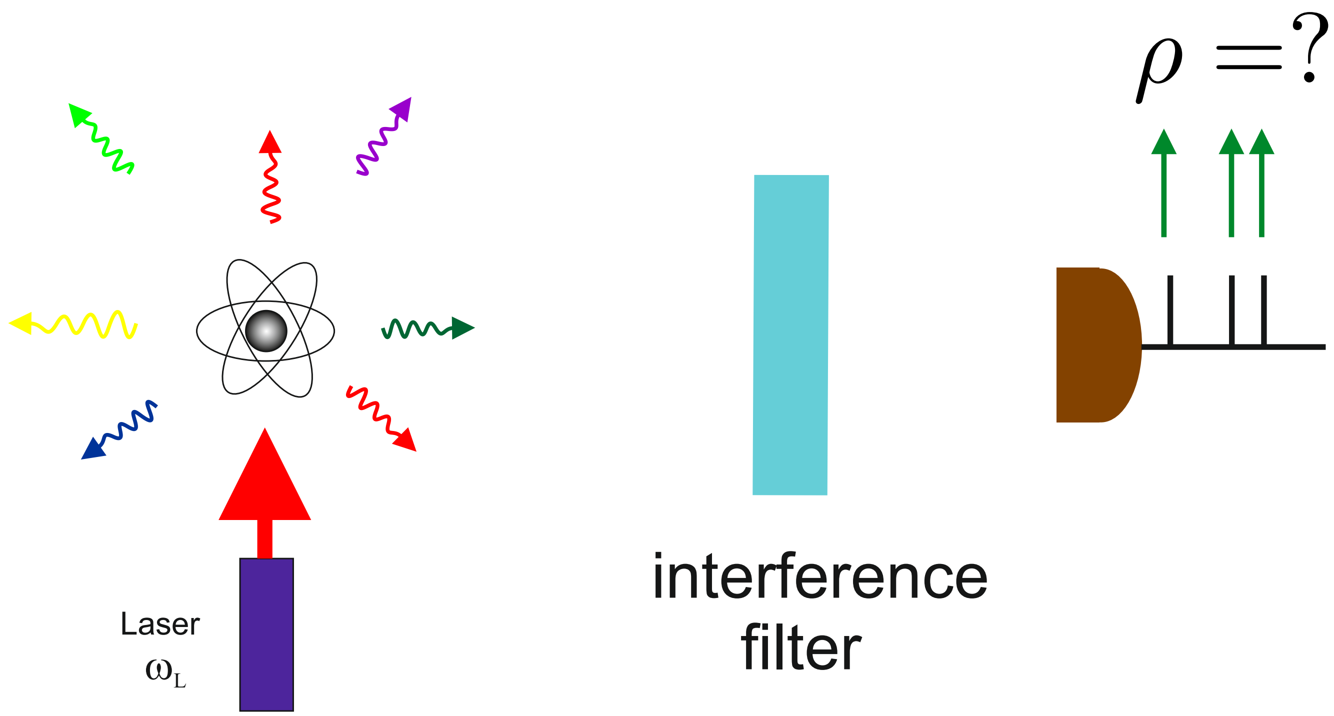
H.J. Kimble, M. Dagenais, L. Mandel, PRL (1979) [experiment]

$g^{(2)}(0) = 0$ photon antibunching

Conditioned state: narrowband detection



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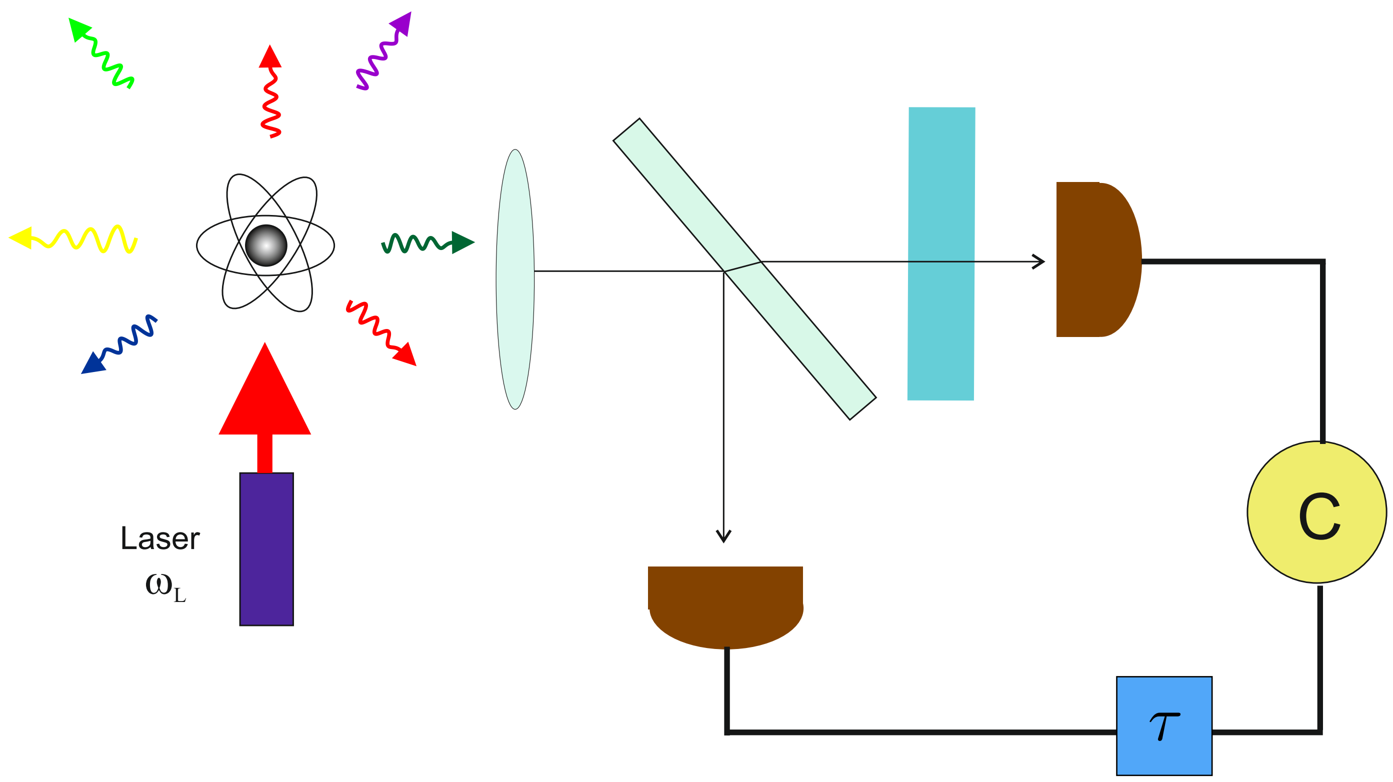
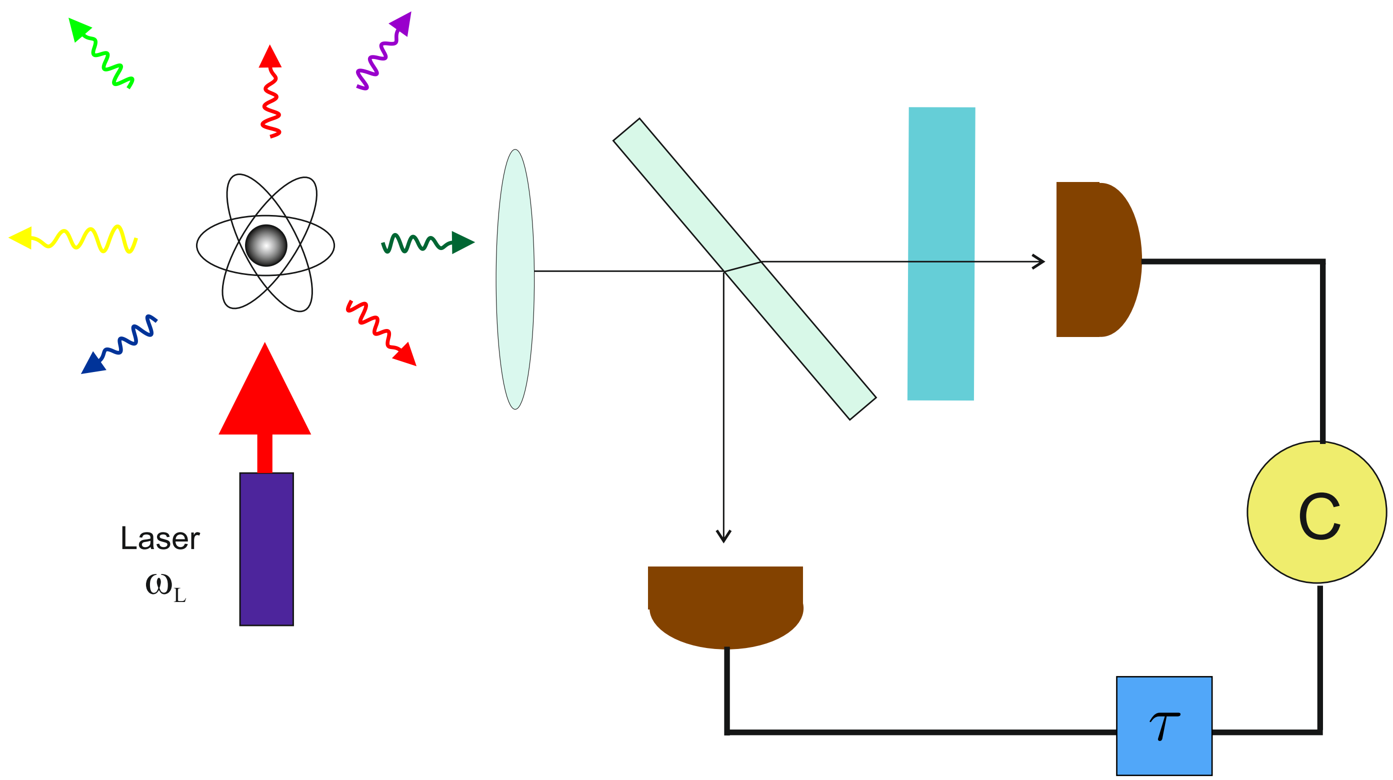


VNS, S. Ya. Kilin, Opt. Commun. (2000)

$$g^{(2)}_{fu}(\tau) = \frac{\rho_{22}(\tau)|_{\rho(0)=\rho^c(\infty)}}{\rho_{22}(\infty)}$$

Conditioned state: narrowband detection



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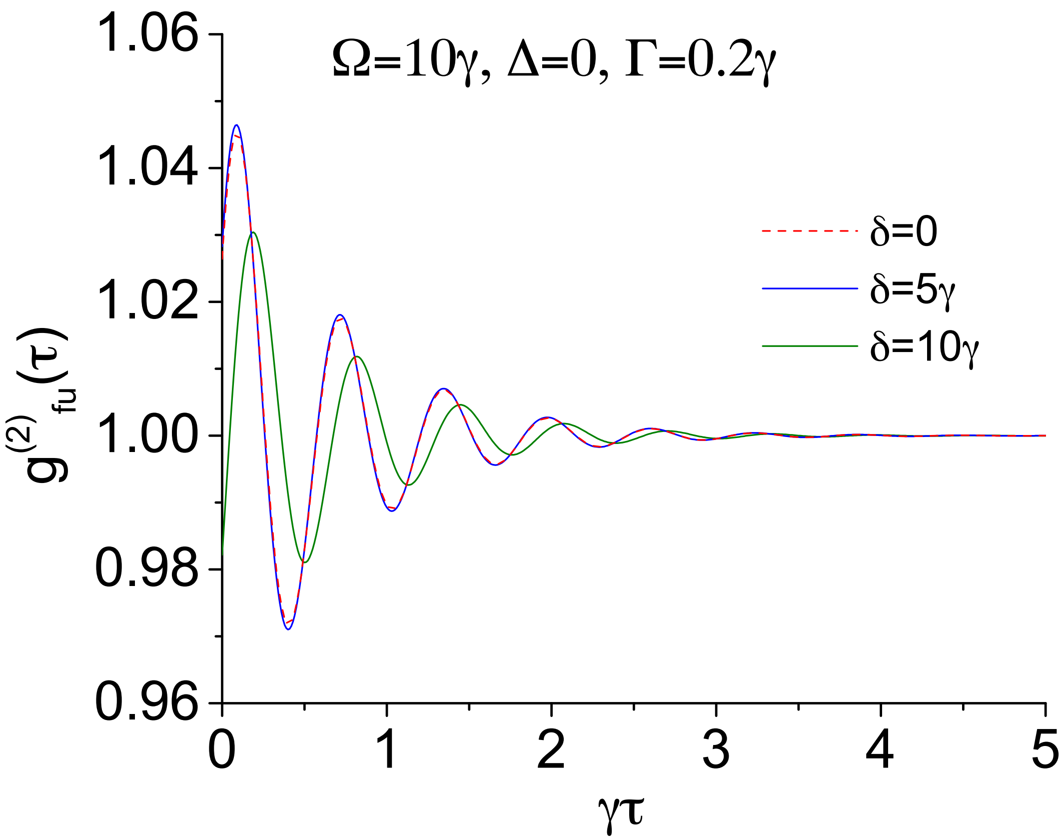

$$g_{fu}^{(2)}(\tau) = \frac{\rho_{22}(\tau)|_{\rho(0)=\rho^c(\infty)}}{\rho_{22}(\infty)}$$

Non-Markovianity?

Conditioned state: narrowband detection

Time-dependent conditioned atomic state

Non-stationary cross-correlation function

$$\rho^c(t) = \frac{\chi(t)}{\mathrm{Tr_A}[\chi(\mathrm{t})]} \longleftarrow g^{(2)}_{fu}(t;t+\tau)$$

$$\chi(t) = \mathrm{Tr_B}[\mathrm{U(t)}\sigma^{\mathrm{f}}_{-}(\mathrm{t})\rho_{\mathrm{AB}}(0)\sigma^{\mathrm{f}}_{+}(\mathrm{t})\mathrm{U}^{\dagger}(\mathrm{t})]$$

$$U(t) = \exp\left(-\frac{iHt}{\hbar}\right)$$

$$\sigma^{\mathrm{f}}_{-}(t) = \int_0^t dt' T(t-t')\sigma_{-}(t')$$

Conditioned state: narrowband detection

Time-dependent
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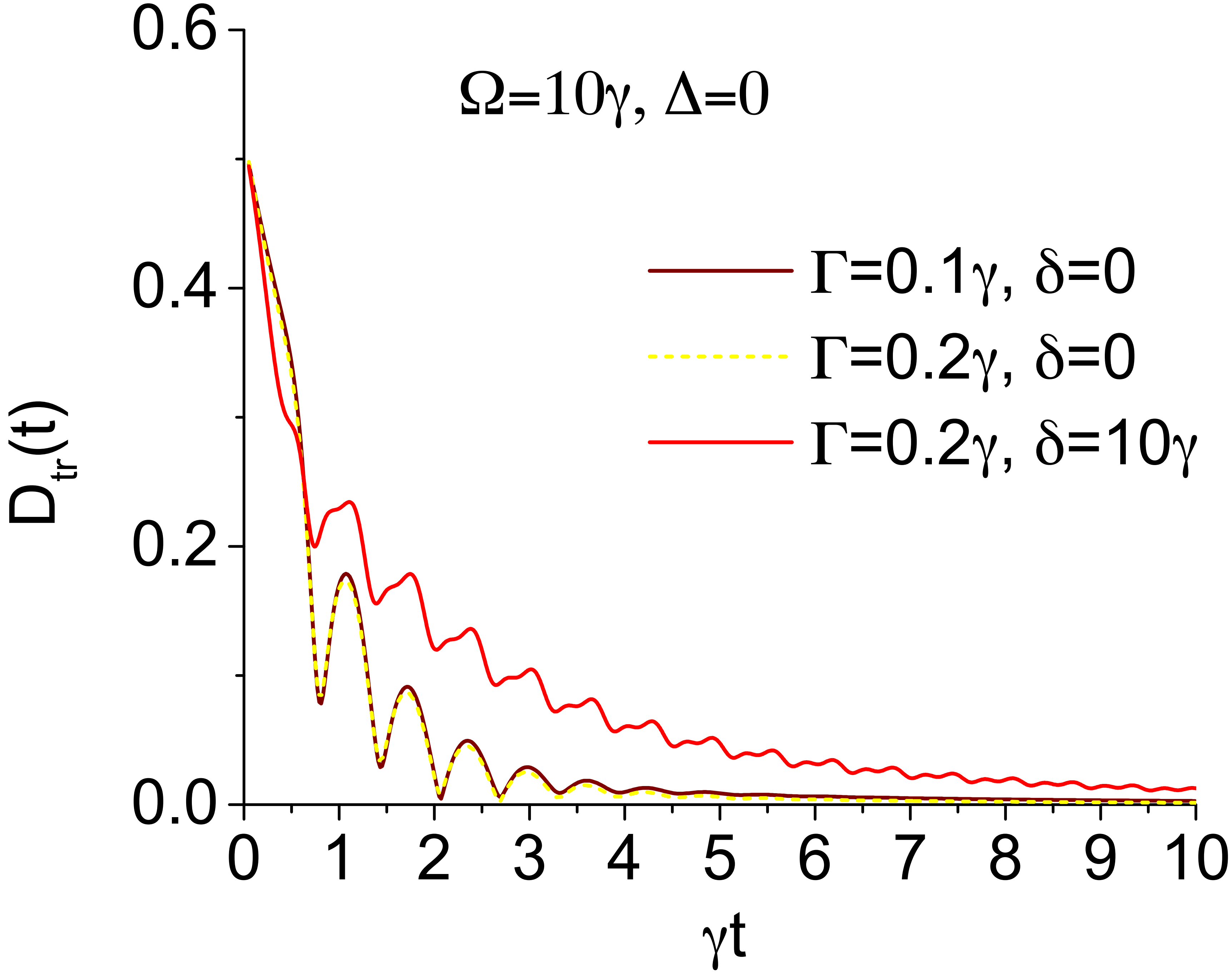
Trace distance [H.-P. Breuer, E.-M. Laine, J. Piilo, PRL (2009)]

$$D_{tr}(t) = \frac{1}{2}||\rho^c(t) - \rho^c(\infty)||$$

$||A|| = \mathrm{Tr}\sqrt{A^{\dagger}A}$ - trace norm

Trace distance of the conditioned state



Conclusion

- Spectral detection is an indispensable tool in resonance fluorescence
- Spectral detection introduces memory effects
- Memory effects can be observed in measurements of temporal cross correlations between filtered and unfiltered photons