Signatures of distinguishability in many-body dynamics

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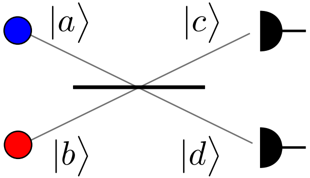
Hong-Ou-Mandel effect

Two photons on a balanced beamsplitter:



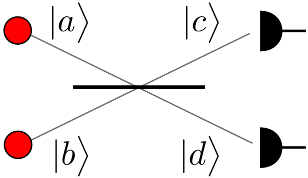
$$|a\rangle \to \frac{1}{\sqrt{2}} \left(|c\rangle + |d\rangle\right)$$
$$|b\rangle \to \frac{1}{\sqrt{2}} \left(|c\rangle - |d\rangle\right)$$

$$|a\rangle\,|b\rangle \in \mathcal{H}^{\otimes 2} \quad \to \quad \frac{1}{2}\left(|c\rangle\,|c\rangle + |d\rangle\,|c\rangle - |c\rangle\,|d\rangle - |d\rangle\,|d\rangle\right)$$

Coincidence probability (both detectors click) $P = \frac{1}{2}$

Hong-Ou-Mandel effect

Two photons on a balanced beamsplitter:



$$|a\rangle \to \frac{1}{\sqrt{2}}(|c\rangle + |d\rangle)$$

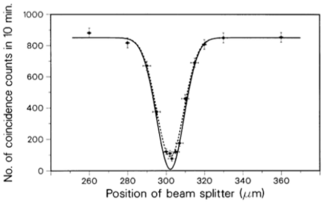
$$|b\rangle \to \frac{1}{\sqrt{2}}(|c\rangle - |d\rangle)$$

$$\frac{1}{\sqrt{2}}(|a\rangle|b\rangle + |b\rangle|a\rangle) \in \mathsf{Sym}(\mathcal{H}^{\otimes 2}) \quad \to \quad \frac{1}{\sqrt{2}}(|c\rangle|c\rangle - |d\rangle|d\rangle)$$

Coincidence probability (both detectors click) $\quad P = 0$

Hong-Ou-Mandel effect

Two photons on a balanced beamsplitter:



$$|a\rangle \to \frac{1}{\sqrt{2}}(|c\rangle + |d\rangle)$$
$$|b\rangle \to \frac{1}{\sqrt{2}}(|c\rangle - |d\rangle)$$

$$\frac{1}{\sqrt{2}}(|a,\alpha\rangle\,|b,\beta\rangle + |b,\beta\rangle\,|a,\alpha\rangle) \in \mathrm{Sym}\left((\mathcal{H}\otimes\mathcal{K})^{\otimes 2}\right)$$

Coincidence probability (both detectors click) $P = \frac{1 - |\langle\alpha|\beta\rangle|^2}{2}$

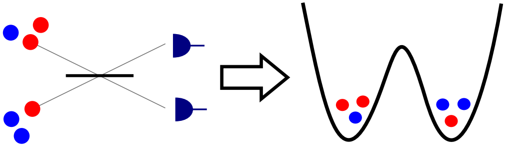
Hong, Ou, Mandel, Phys. Rev. Lett. 59, 2044 (1987)

Motivations

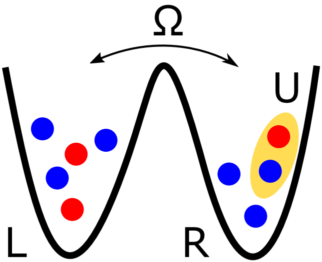
Hong-Ou-Mandel: test indistinguishability of two photons

Similar diagnostic tool for systems of many interacting bosons?

$\Rightarrow$ introduce distinguishability between interacting bosons by considering mixtures of mutually distinguishable species



Two bosonic species in a double well



Bose-Hubbard Hamiltonian with

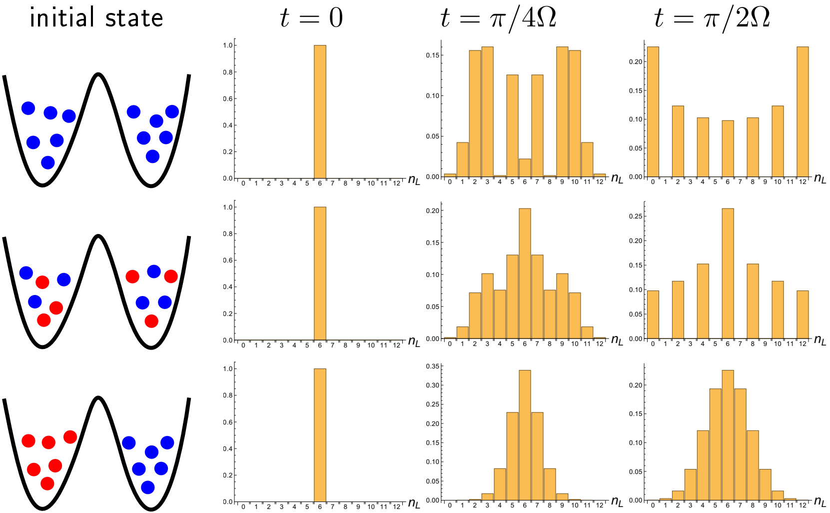
- tunneling rate Ω
- on-site interaction U

Isospecific Hamiltonian: Ω and U independent of species

Protocol:

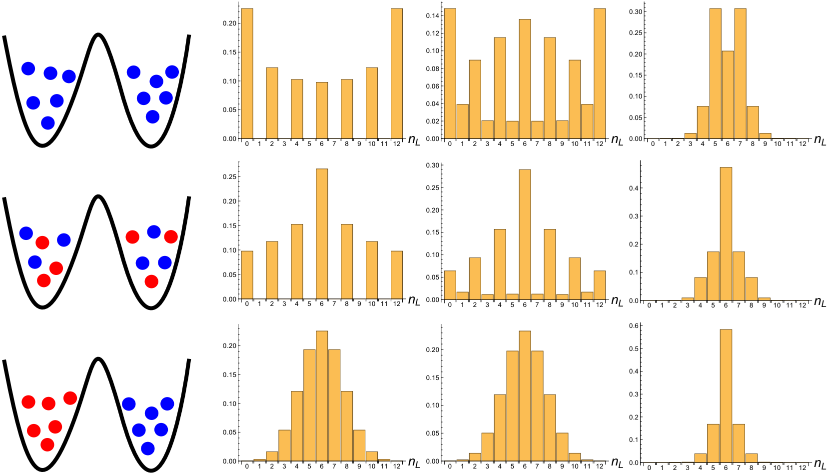
- Prepare a given number of red and blue atoms in each well
- Let the system evolve for a time t
- Measure the number of particles in the left well
- Repeat

Non-interacting case $(U = 0)$



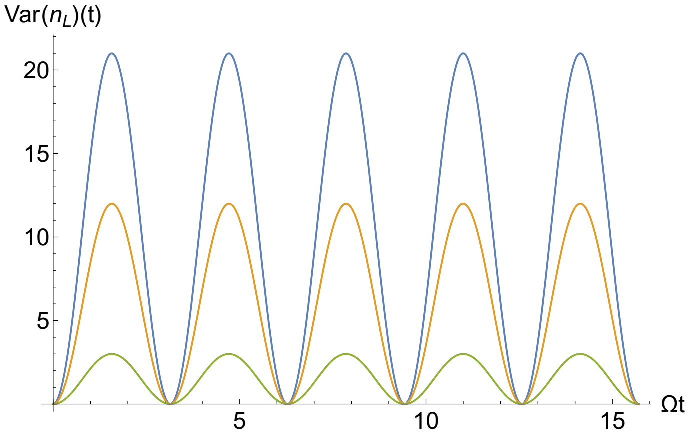
Effect of interactions ($t = \pi/2\Omega$)

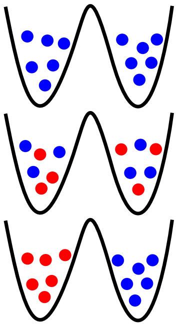
initial state | $U = 0$ | $U = \Omega/8$ | $U = \Omega$



Variance as a measure of distinguishability

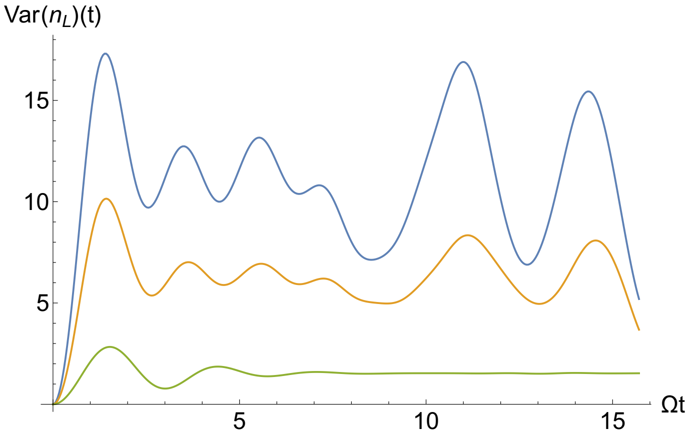
$$\mathbf{U = 0}$$

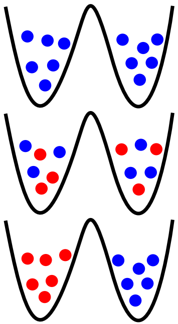




Variance as a measure of distinguishability

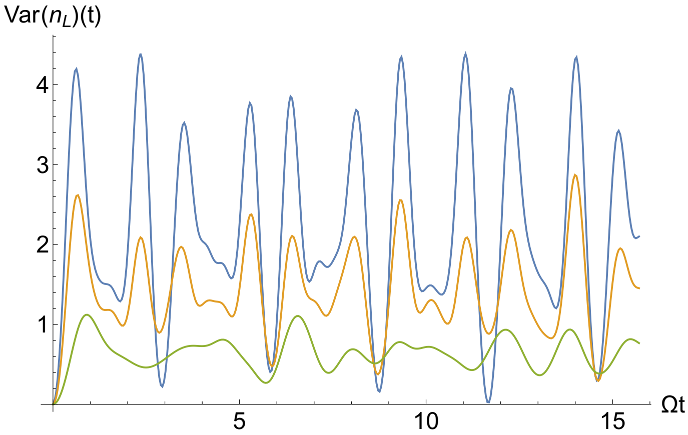
$$\mathbf{U = \Omega/8}$$

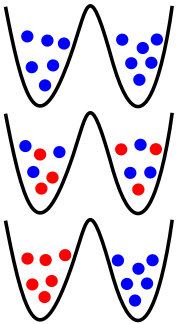




Variance as a measure of distinguishability

$$\mathbf{U} = \mathbf{\Omega}$$





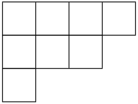
Basis of the Hilbert space

Single particle space $\mathcal{H} \otimes \mathcal{K}$

$\dim(\mathcal{H}) = L$ (number of wells), $\quad \dim(\mathcal{K}) = S$ (number of colors)

The Hilbert space of N bosons admits a basis $\{|\lambda, m, \mu\rangle\}$

- λ = Young diagram with N boxes and $k \leq \min(L, S)$ rows
  common label for irreps of S_N, $U(L)$ and $U(S)$

  e.g. $N = 8, \quad [4, 3, 1] =$ 

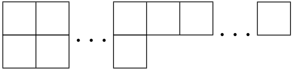
- m indexes basis of λ irrep of $U(L)$ (spatial distribution)
- μ indexes basis of λ irrep of $U(S)$ (species distribution)

λ and μ conserved by isospecific evolution

D.Rowe, M.Carvalho, J.Repka, Rev. Mod. Phys. 84, 711 (2012)

Back to two species in a double well

$$\mathcal{H} = \text{span}(|L\rangle\,, |R\rangle) \qquad \mathcal{K} = \text{span}(|\textcolor{red}{\bullet}\rangle\,, |\textcolor{blue}{\bullet}\rangle)$$

Young diagrams $\lambda = [\lambda_1, \lambda_2] =$ 

characterized by $N = \lambda_1 + \lambda_2$ and $j = (\lambda_1 - \lambda_2)/2$

e.g. $N = 9,\ j = 3/2$

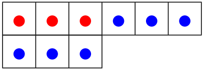
spatial index

L	L	L	L	L	R
R	R	R			

$m = \frac{1}{2}(n_L - n_R) = 1/2$

$-j \leq m \leq j$

species index



$\mu = \frac{1}{2}(n_{\textcolor{red}{\bullet}} - n_{\textcolor{blue}{\bullet}}) = -3/2$

$-j \leq \mu \leq j$

λ describes a spin j irreducible representation of SU(2)

Schwinger representation

Define a spin $\boldsymbol{J}_{\color{gray}\bullet}$ for each species ${\color{gray}\bullet} = {\color{red}\bullet}$ or ${\color{blue}\bullet}$

$$J_{x{\color{gray}\bullet}} = \frac{1}{2}\left(a^\dagger_{L{\color{gray}\bullet}} a_{R{\color{gray}\bullet}} + a^\dagger_{R{\color{gray}\bullet}} a_{L{\color{gray}\bullet}}\right)$$

$$J_{y{\color{gray}\bullet}} = \frac{1}{2i}\left(a^\dagger_{L{\color{gray}\bullet}} a_{R{\color{gray}\bullet}} - a^\dagger_{R{\color{gray}\bullet}} a_{L{\color{gray}\bullet}}\right)$$

$$J_{z{\color{gray}\bullet}} = \frac{1}{2}\left(n_{L{\color{gray}\bullet}} - n_{R{\color{gray}\bullet}}\right)$$

Eigenstates of $J^2_{\color{gray}\bullet}$ and $J_{z{\color{gray}\bullet}}$:

$|j_{\color{red}\bullet}, m_{\color{red}\bullet}, j_{\color{blue}\bullet}, m_{\color{blue}\bullet}\rangle$ = Fock states

$$j_{\color{gray}\bullet} = \frac{1}{2}(n_{L{\color{gray}\bullet}} + n_{R{\color{gray}\bullet}}) = \frac{1}{2} n_{\color{gray}\bullet}$$

$$m_{\color{gray}\bullet} = \frac{1}{2}(n_{L{\color{gray}\bullet}} - n_{R{\color{gray}\bullet}})$$

Isospecific Bose-Hubbard Hamiltonian:

$$H = -\Omega(J_{x{\color{red}\bullet}} + J_{x{\color{blue}\bullet}}) + U(J_{z{\color{red}\bullet}} + J_{z{\color{blue}\bullet}})^2 \qquad [H, (\boldsymbol{J}_{\color{red}\bullet} + \boldsymbol{J}_{\color{blue}\bullet})^2] = 0$$

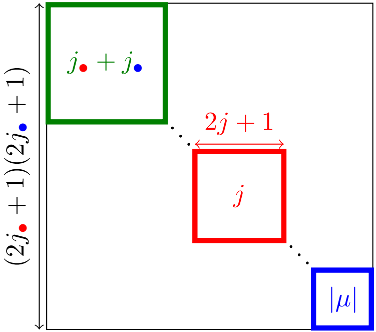
We can identify $|\lambda, m, \mu\rangle$ with the eigenstates of the total spin

$$|j, m\rangle = \sum_{m_{\color{red}\bullet} + m_{\color{blue}\bullet} = m} C^{j_{\color{red}\bullet}, j_{\color{blue}\bullet}, j}_{m_{\color{red}\bullet}, m_{\color{blue}\bullet}, m} \, |j_{\color{red}\bullet}, m_{\color{red}\bullet}, j_{\color{blue}\bullet}, m_{\color{blue}\bullet}\rangle$$

J.Schwinger, Report NYO-3071 to the AEC (1952)

Structure of the Hilbert space

Fix number of particles in each species $\Rightarrow \mu = j_{\color{red}\bullet} - j_{\color{blue}\bullet}$ fixed
$[H, (\boldsymbol{J}_{\color{red}\bullet} + \boldsymbol{J}_{\color{blue}\bullet})^2] \Rightarrow$ Hamiltonian block diagonal in the coupled basis



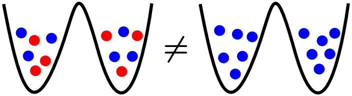
$$\exp(-iHt)\,|j,m\rangle = \sum_{m'=-j}^{j} U^{(j)}_{m,m'}(t)\,|j,m'\rangle$$

$U^{(j)}(t)$ is the evolution operator of $2j$ indistinguishable bosons

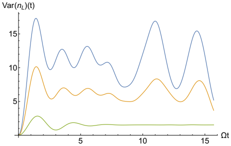
$\Rightarrow$ The state of a mixture of $N = n_{\color{red}\bullet} + n_{\color{blue}\bullet}$ particles has components behaving like $N,\ N-2,\ N-4,\ \ldots,\ |n_{\color{red}\bullet} - n_{\color{blue}\bullet}|$ indistinguishable bosons

Conclusion

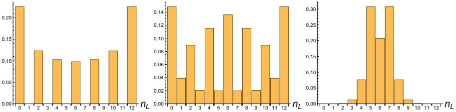
- Adding an extra “label” to particles changes their dynamics



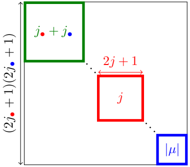
- The variance of populations reflects the degree of distinguishability



- Interactions tend to suppress the effects of indistinguishability



- Distinguishability induces a rich structure in the many-boson Hilbert space



The end

Thank you for your attention