Thermodynamic meaning and power of non-Markovianity

arXiv:1504.06533

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Outline

- Non Markovian open quantum dynamics

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- Landauer principle in open systems

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- Memory effects and Work

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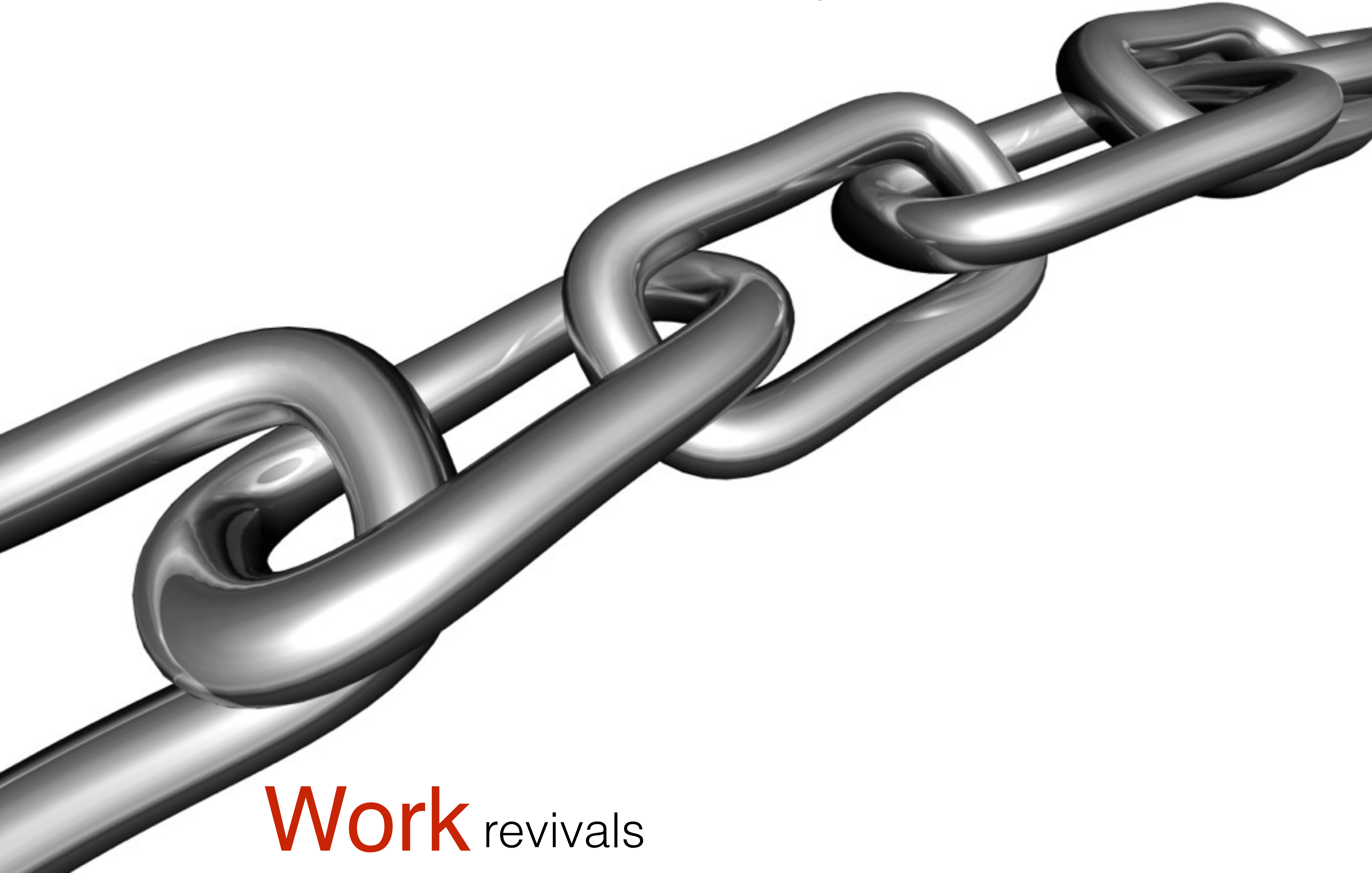
- The power of memory

Non-Markovianity

memory effects



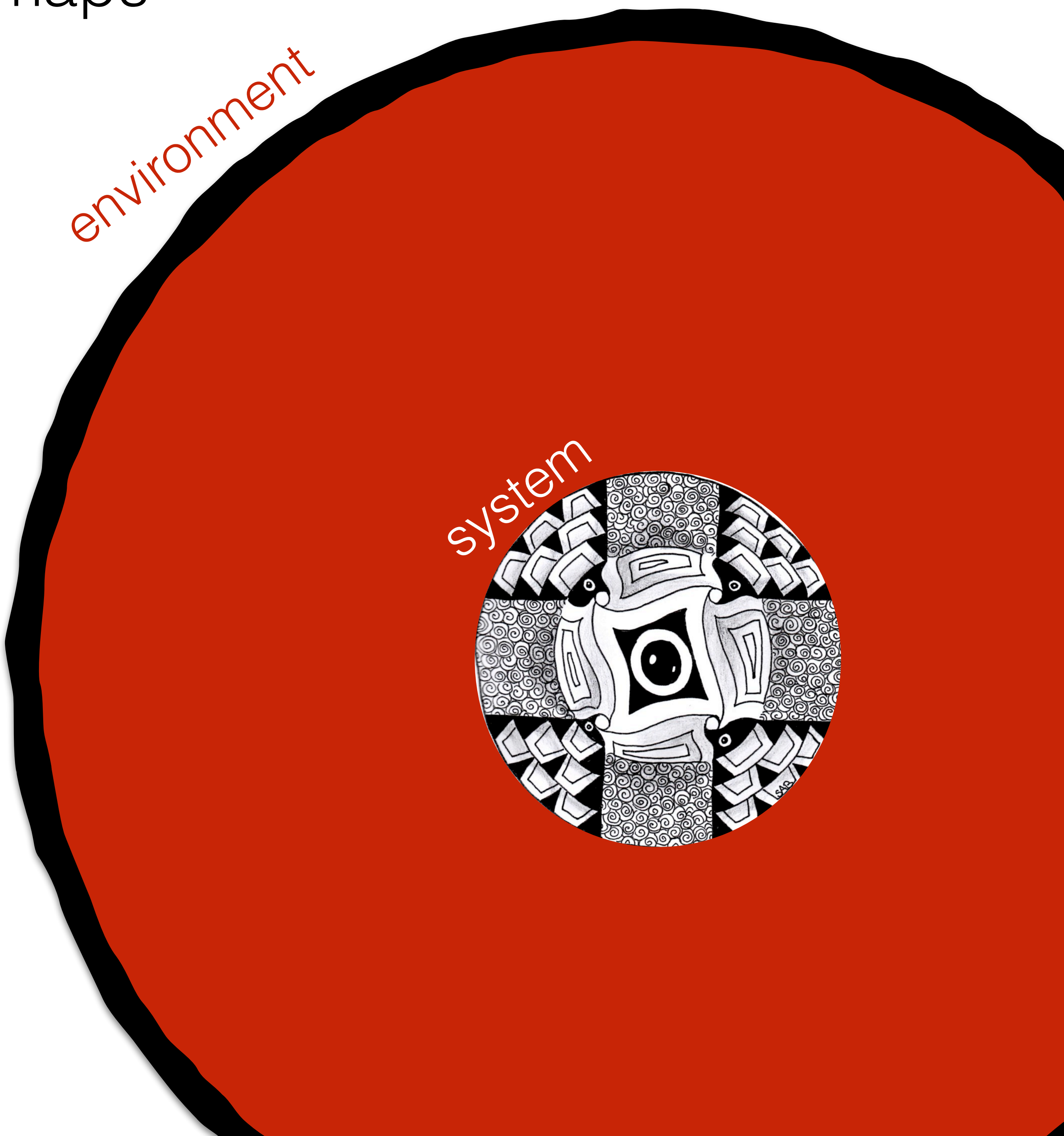
Work revivals

Outline

- Non Markovian open quantum dynamics
- Landauer principle in open systems
- Memory effects and Work
- The power of memory

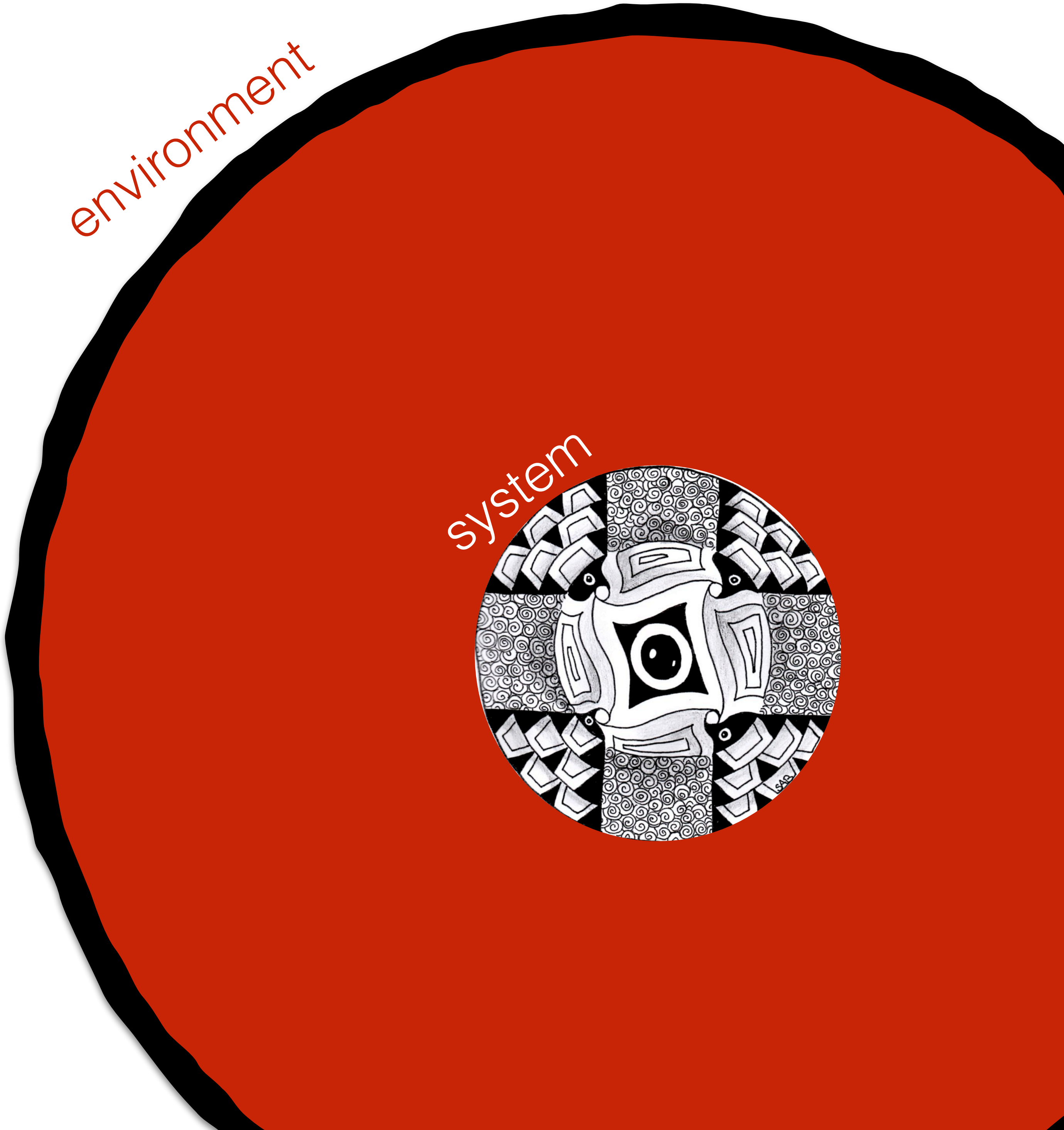
$$\rho(t) = \Phi_t \rho(0)$$

t-parametrised family of CPTP maps



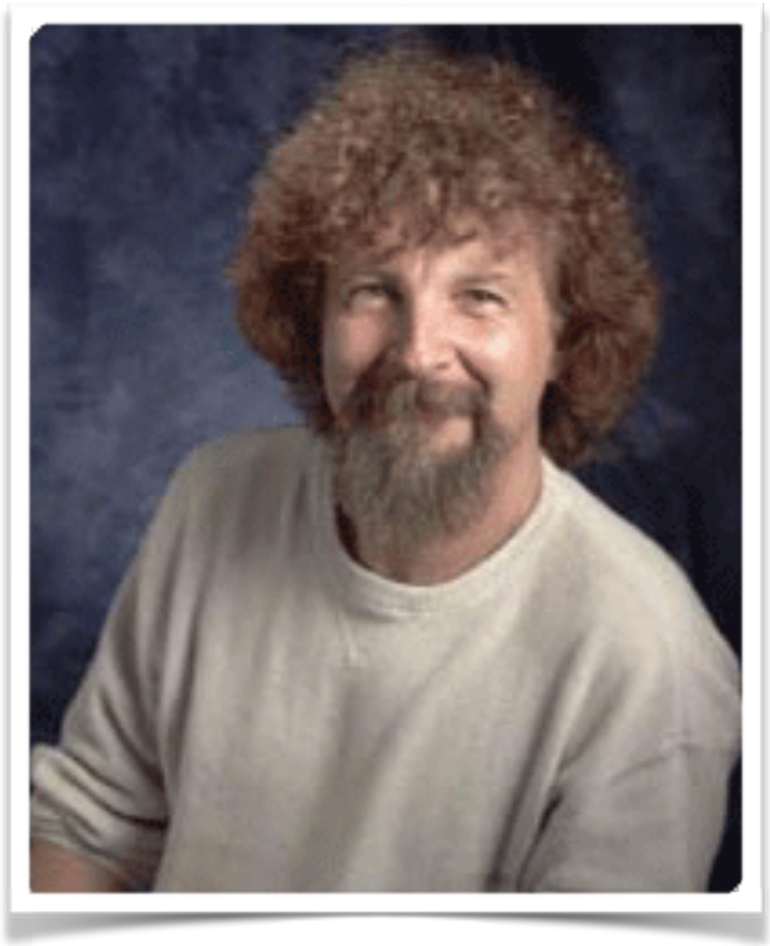
Environment induced decoherence

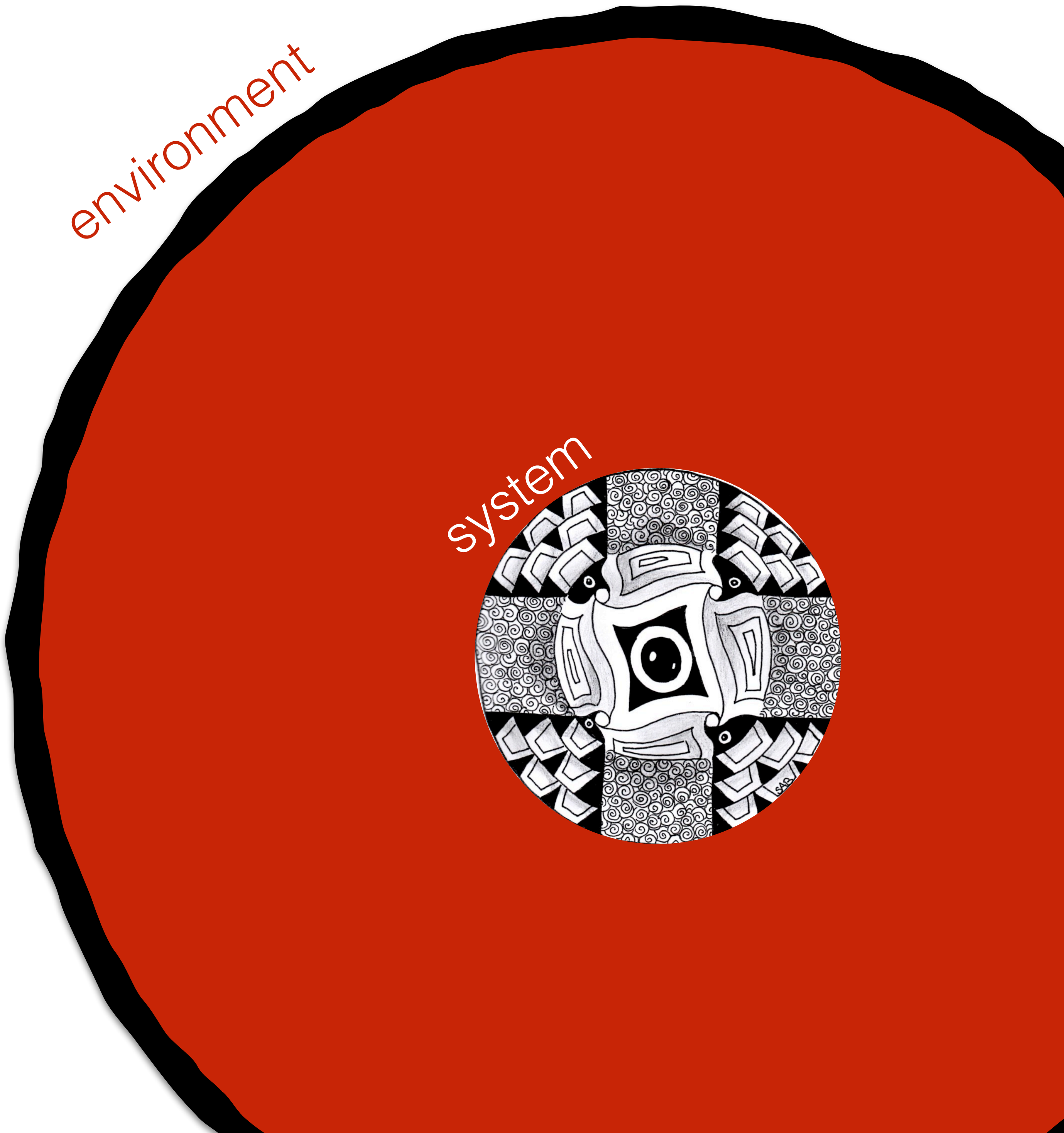
$$\rho(t) = \Phi_t \rho(0)$$



Environment induced decoherence

Information loss due to the action of the environment continuously monitoring the system

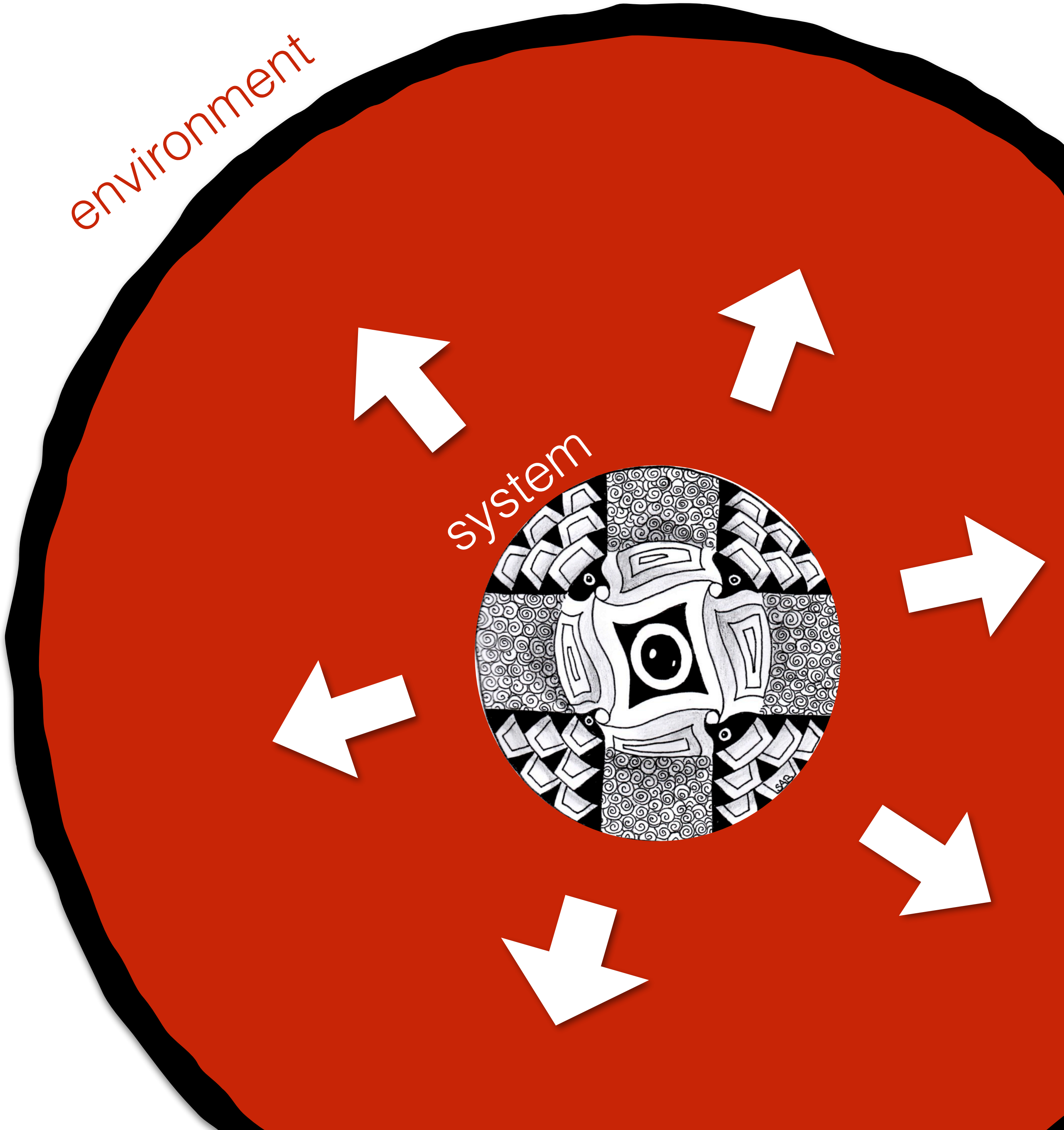




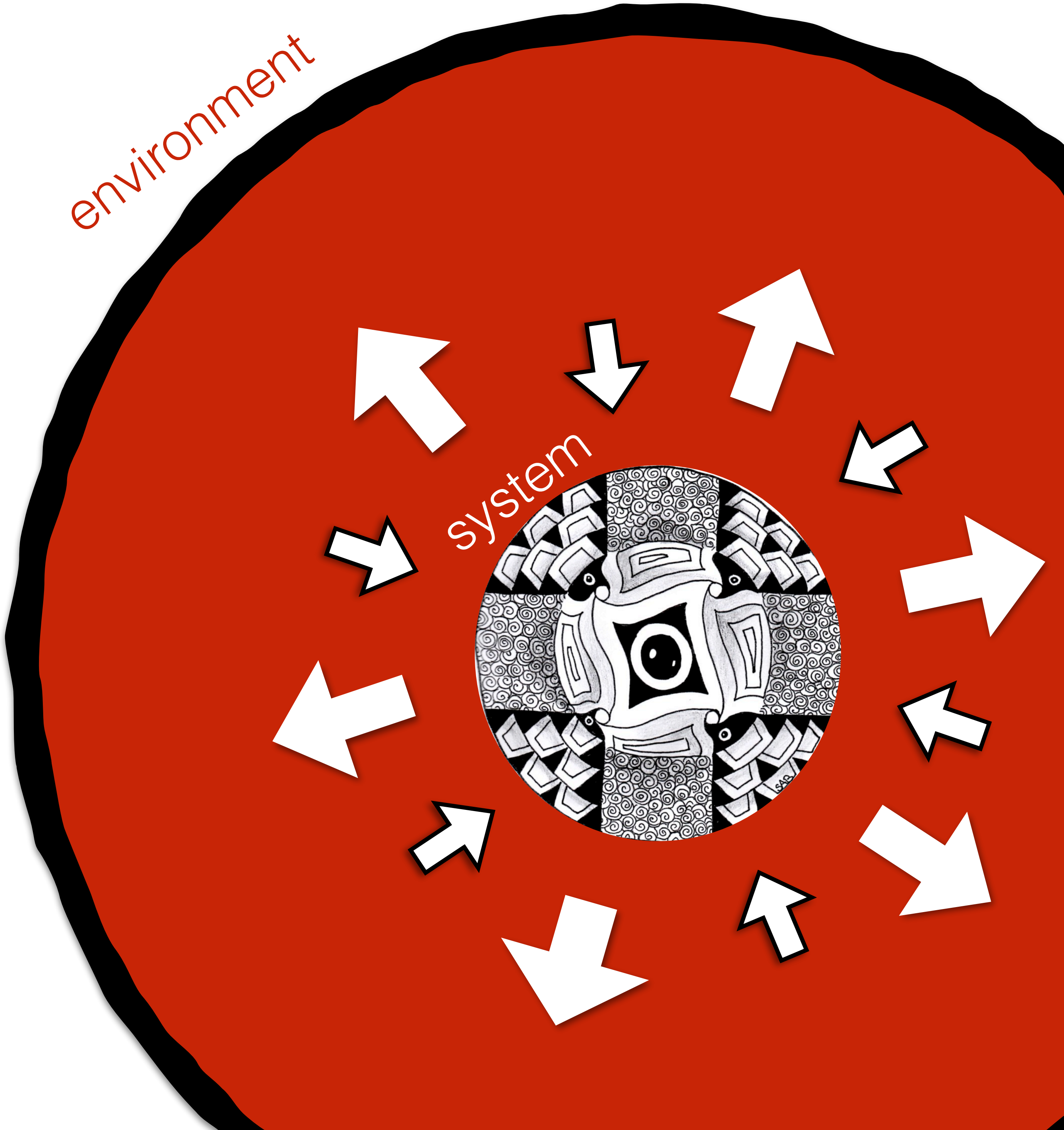
What do we exactly mean with

information loss ?

Markovian dynamics

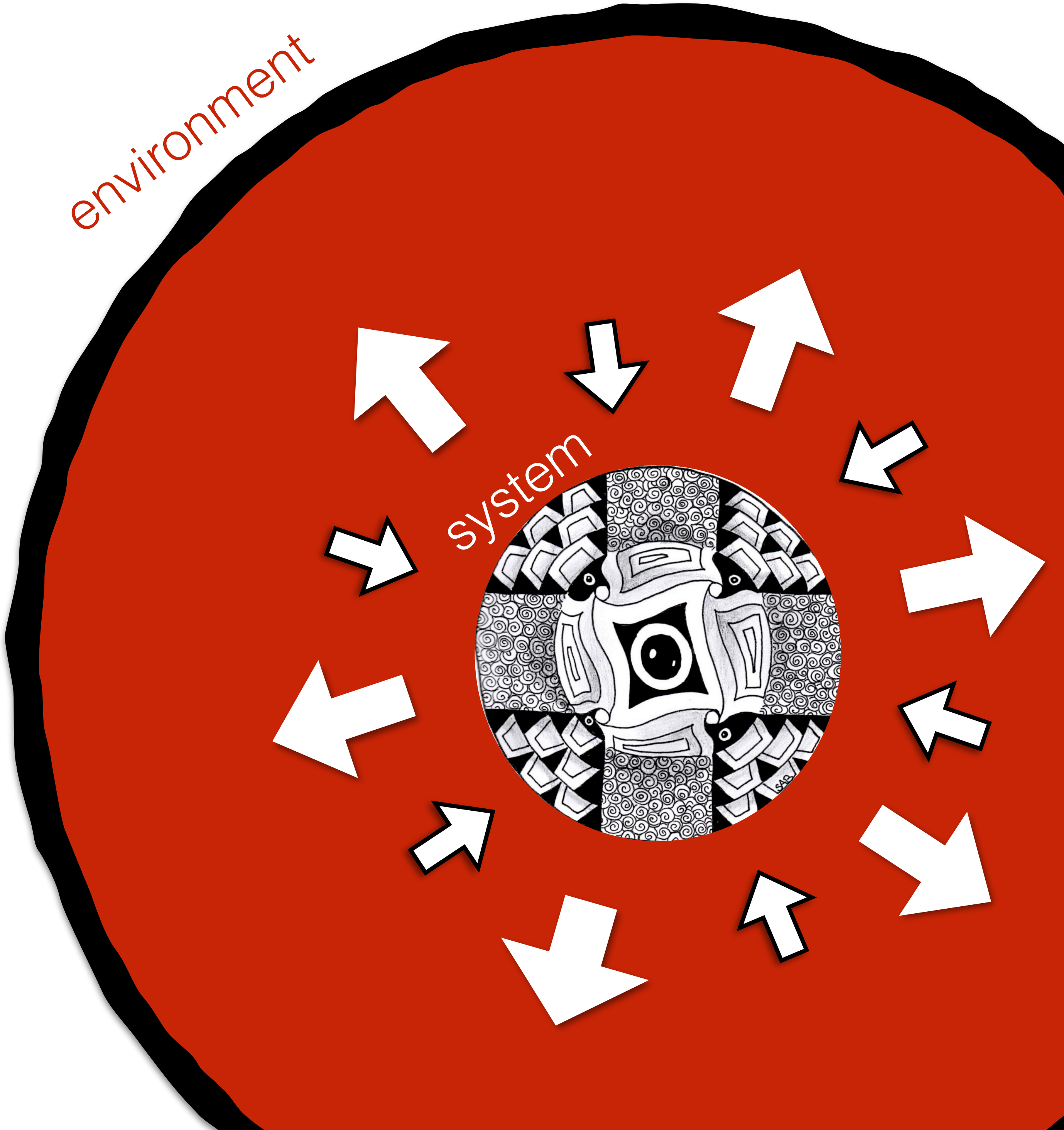


Non-Markovian dynamics



Non-Markovian dynamics

memory effects manifest themselves
as information back flow

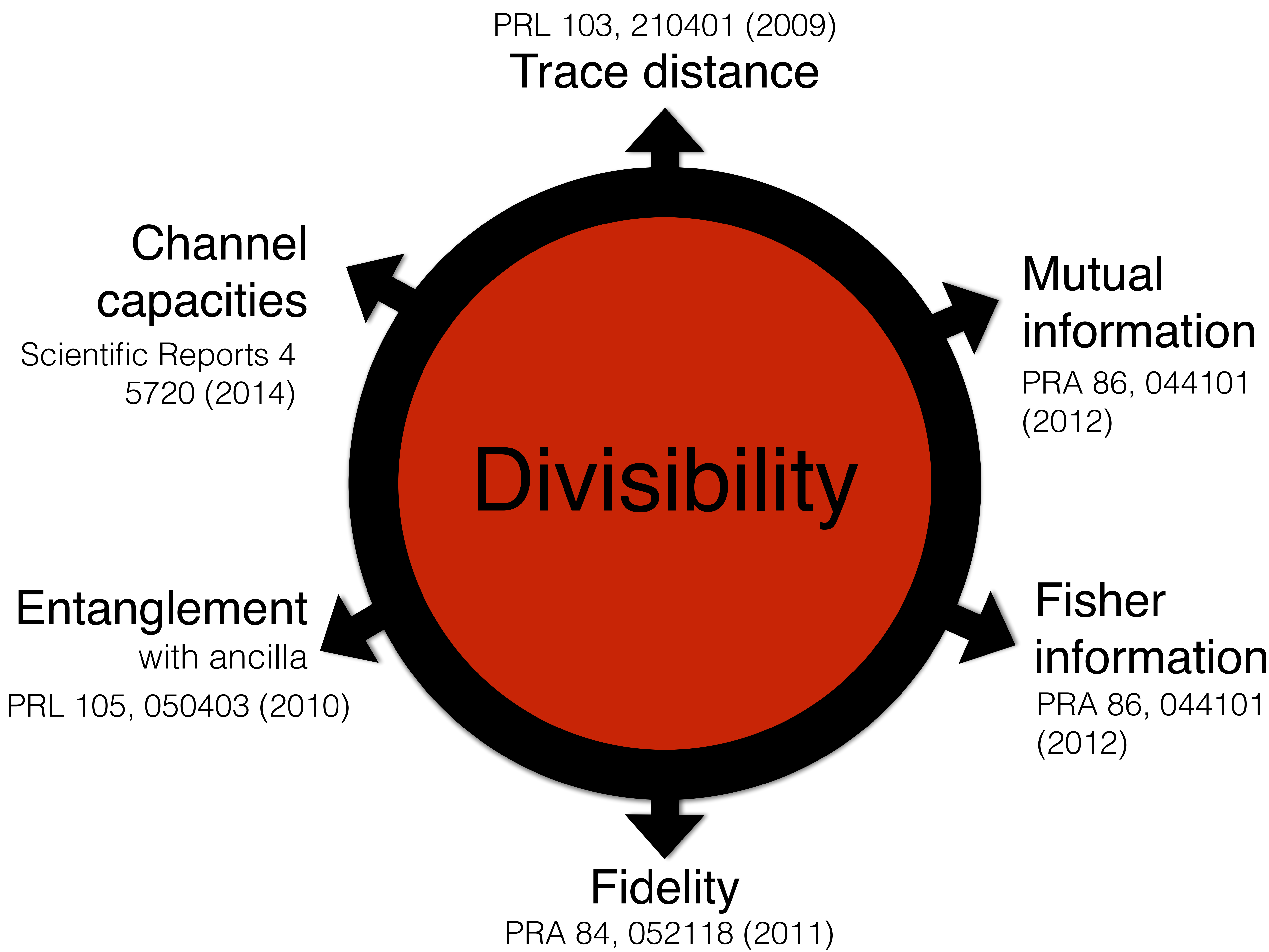


Information back-flow?

Divisibility

The “traditional” mathematical definition of Markovian dynamics

$$\Phi_t = \Phi_{t,s}\Phi_s$$


PRL 103, 210401 (2009)
Trace distance
Channel capacities
Scientific Reports 4 5720 (2014)
Mutual information
PRA 86, 044101 (2012)
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Entanglement with ancilla
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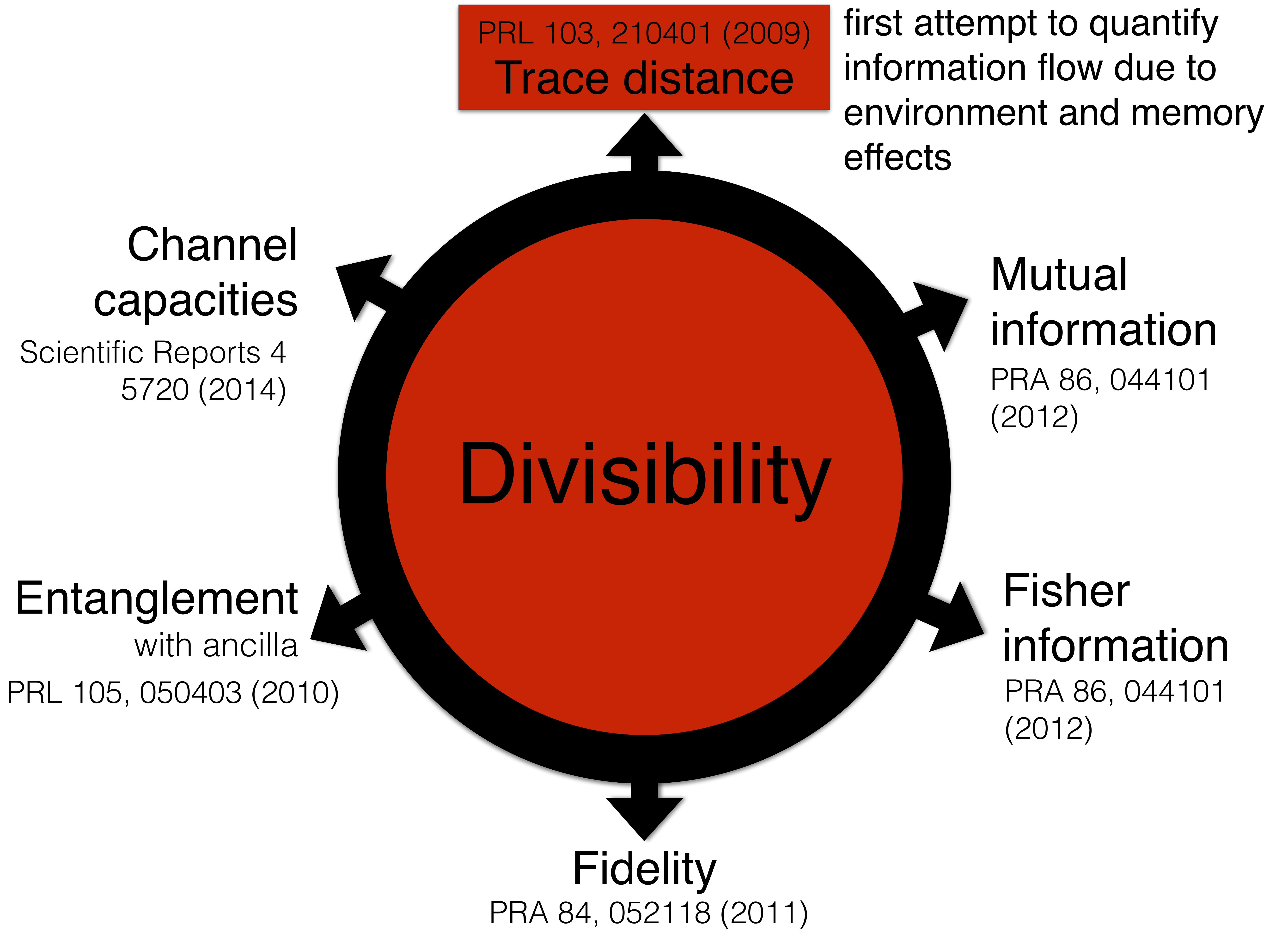
PRL 103, 210401 (2009)
Trace distance
first attempt to quantify information flow due to environment and memory effects
Channel capacities
Scientific Reports 4 5720 (2014)
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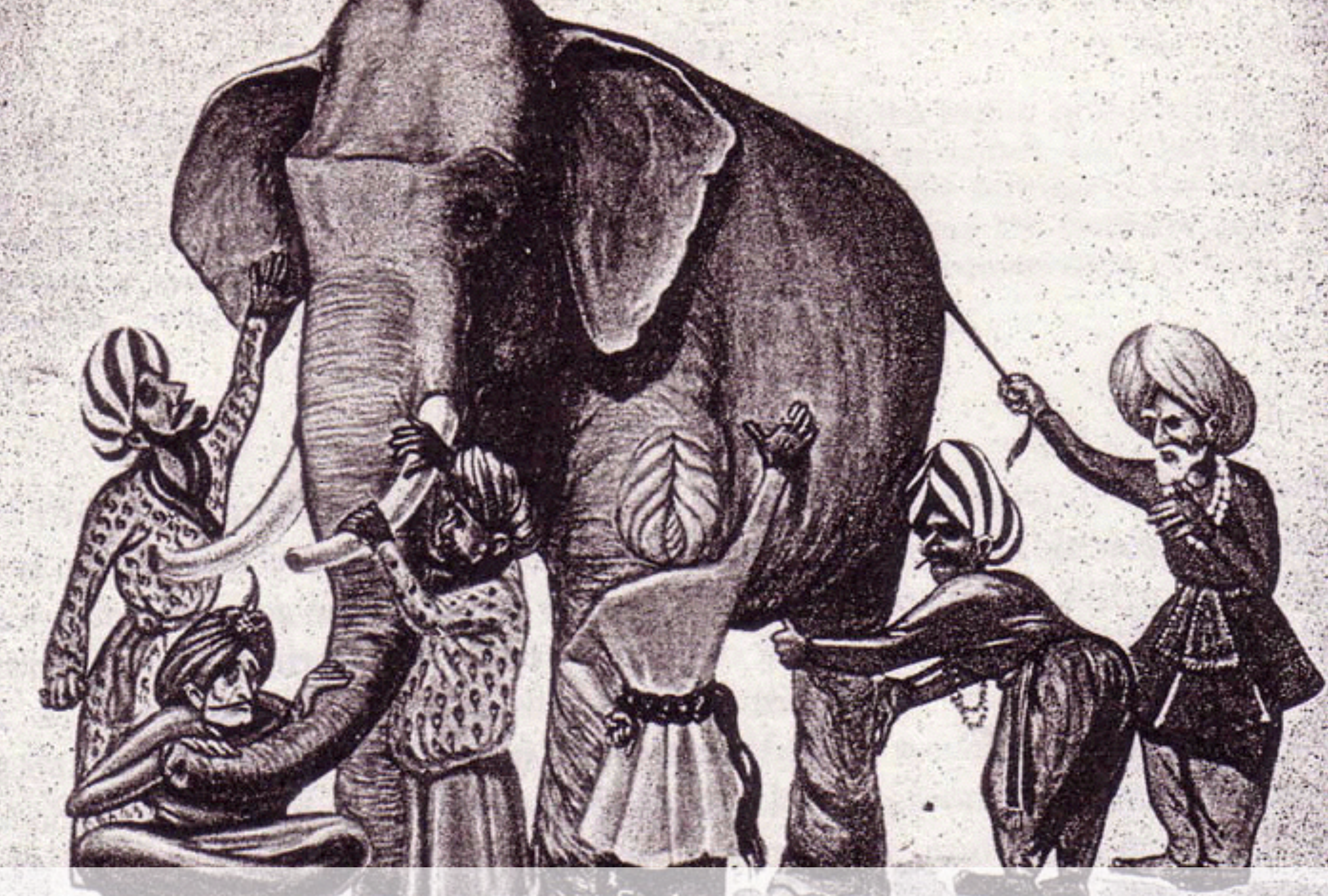
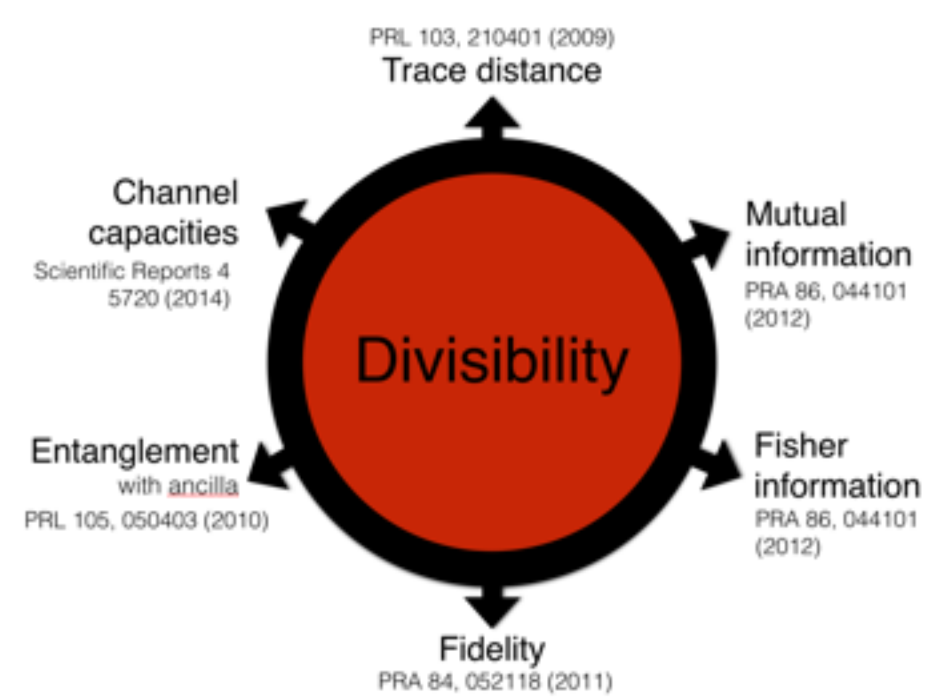


C. Addis, B. Bylicka, D. Chruściński and S. Maniscalco, *“What we talk about when we talk about non-Markovianity”*, arXiv 1402.4975 (published in PRA with a much more boring title)

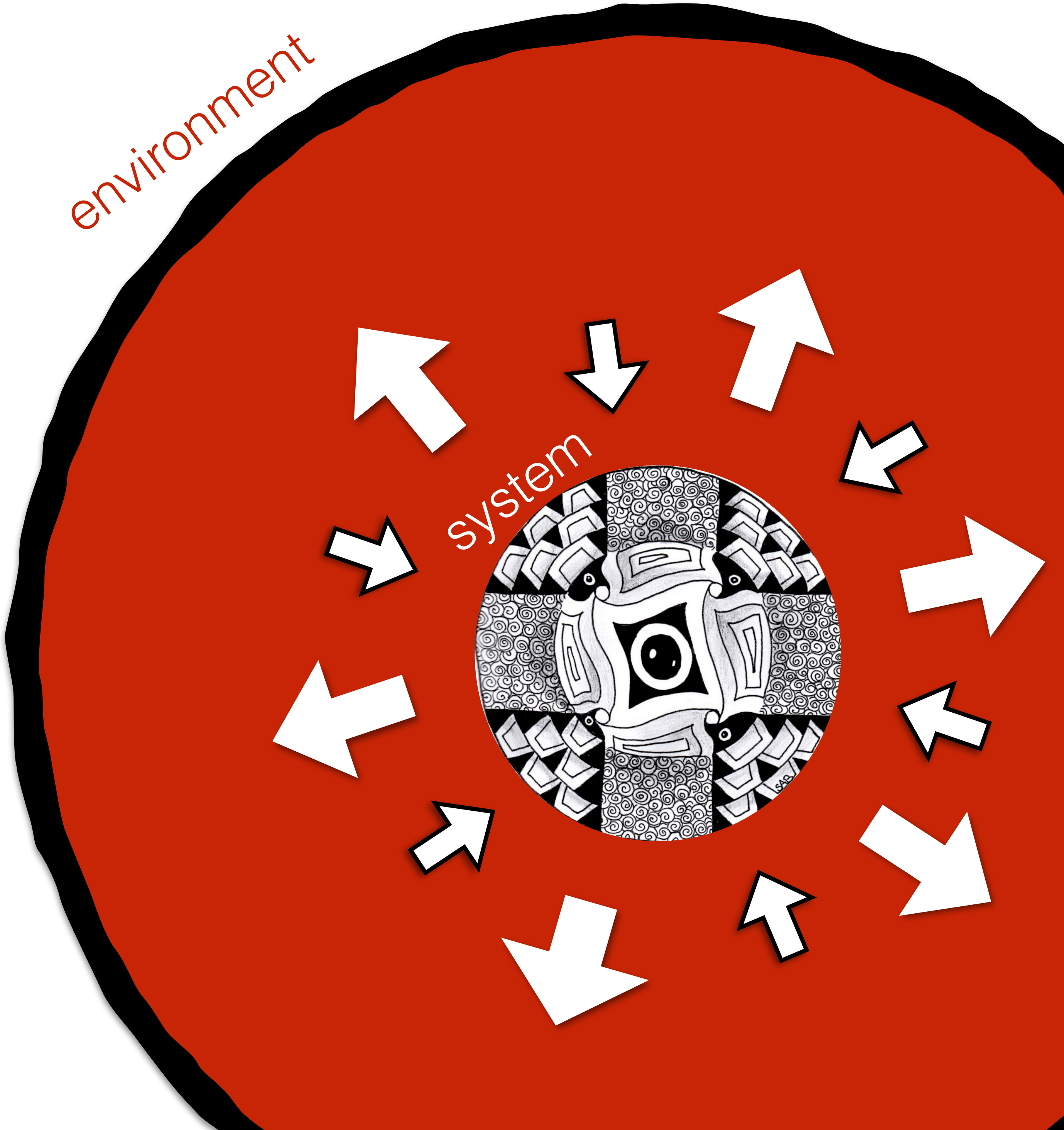
Non-Markovianity

memory effects

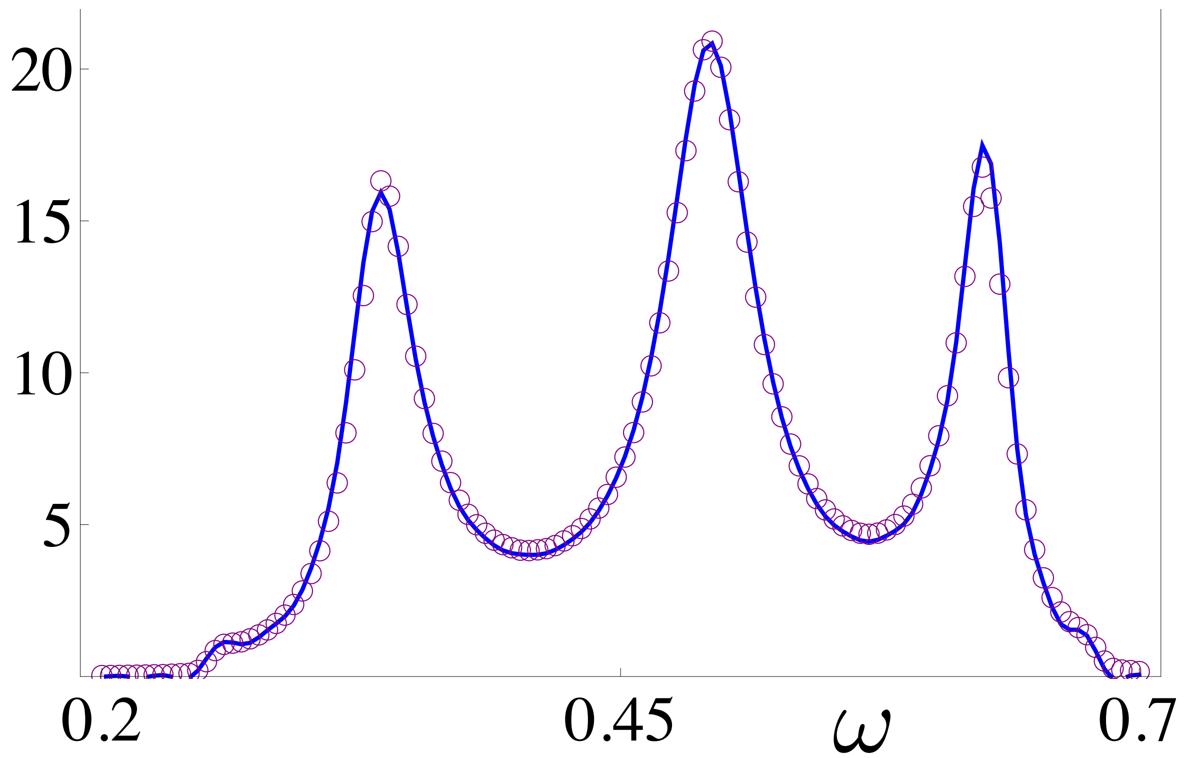


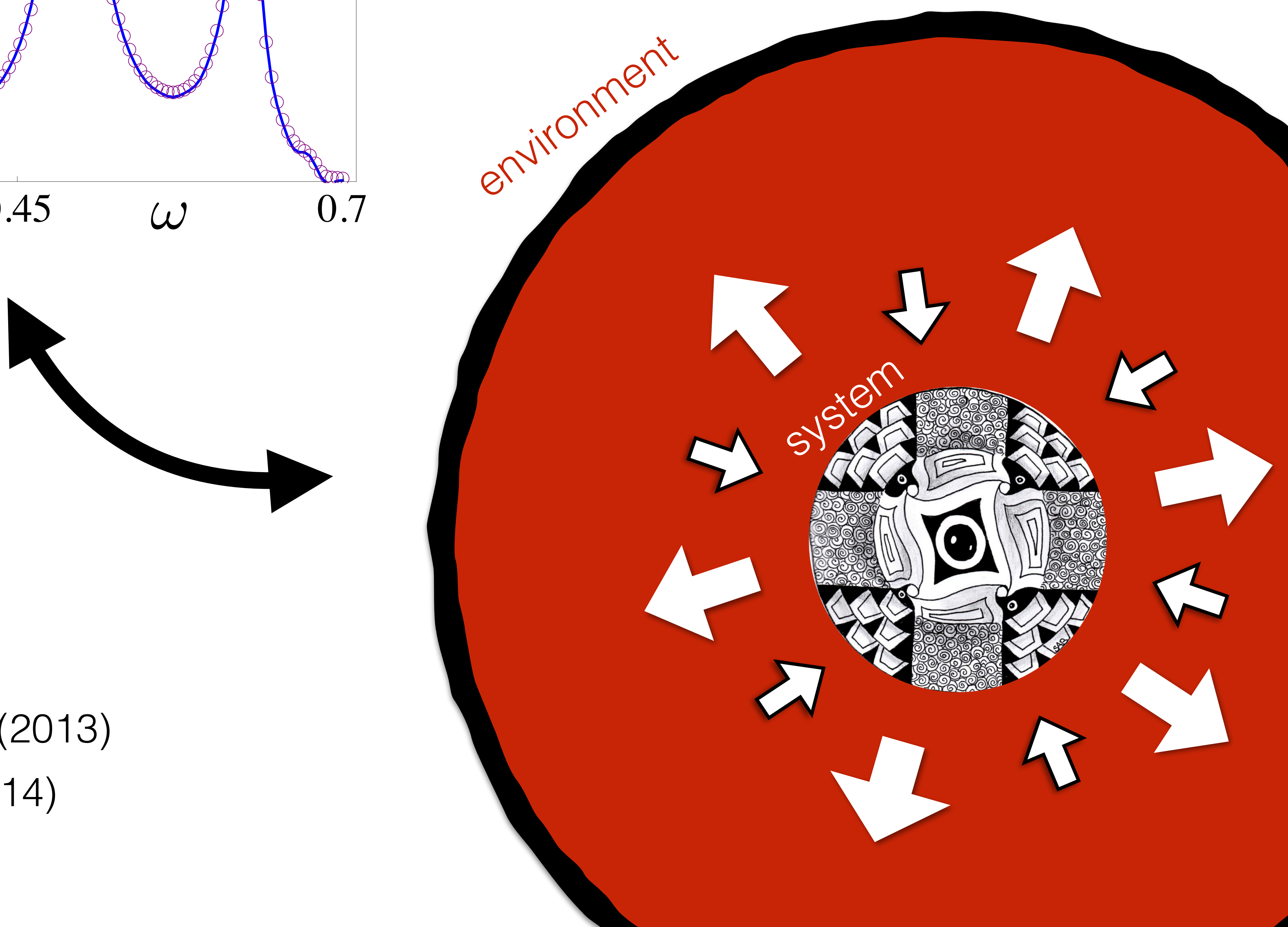
Work revivals

Reservoir engineering



Reservoir spectral density





PRA 87, 010103(R) (2013)

PRA 89, 022109 (2014)

arXiv:1503.04635

What for?

- **Quantum metrology**
  PRL 109, 233601 (2012)
- **Teleportation**
  Scientific Reports 4, 4620 (2013)
- **Quantum communication**
  Scientific Reports 5720 (2014)
- **Superdense coding**
  arXiv:1504.07572 (2015)
- **Quantum key distribution**
  Phys. Rev. A 83, 042321 (2011)

Work

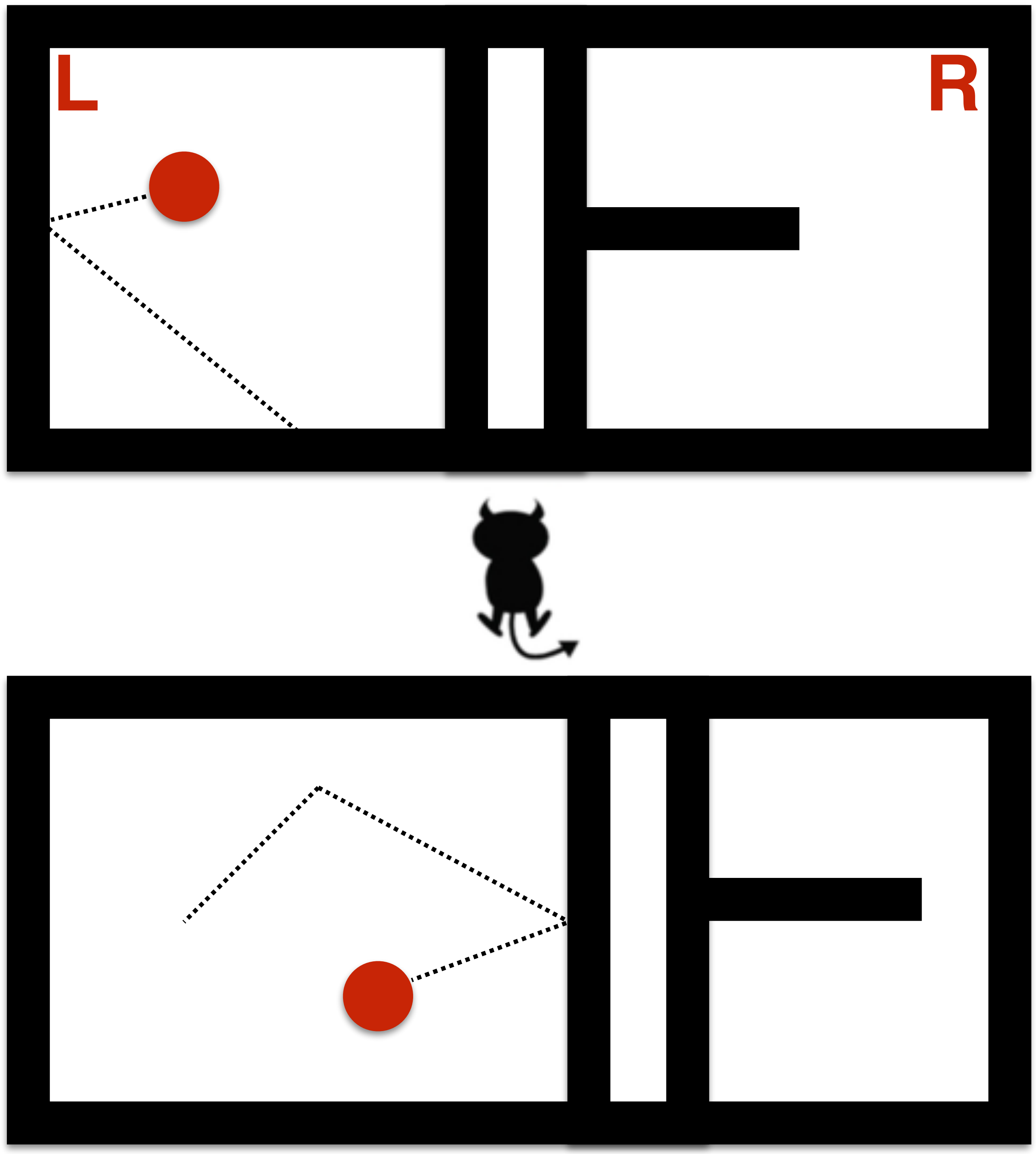
Memory effects

Outline

- Non Markovian open quantum dynamics
- Landauer principle
- Memory effects and Work
- The power of memory

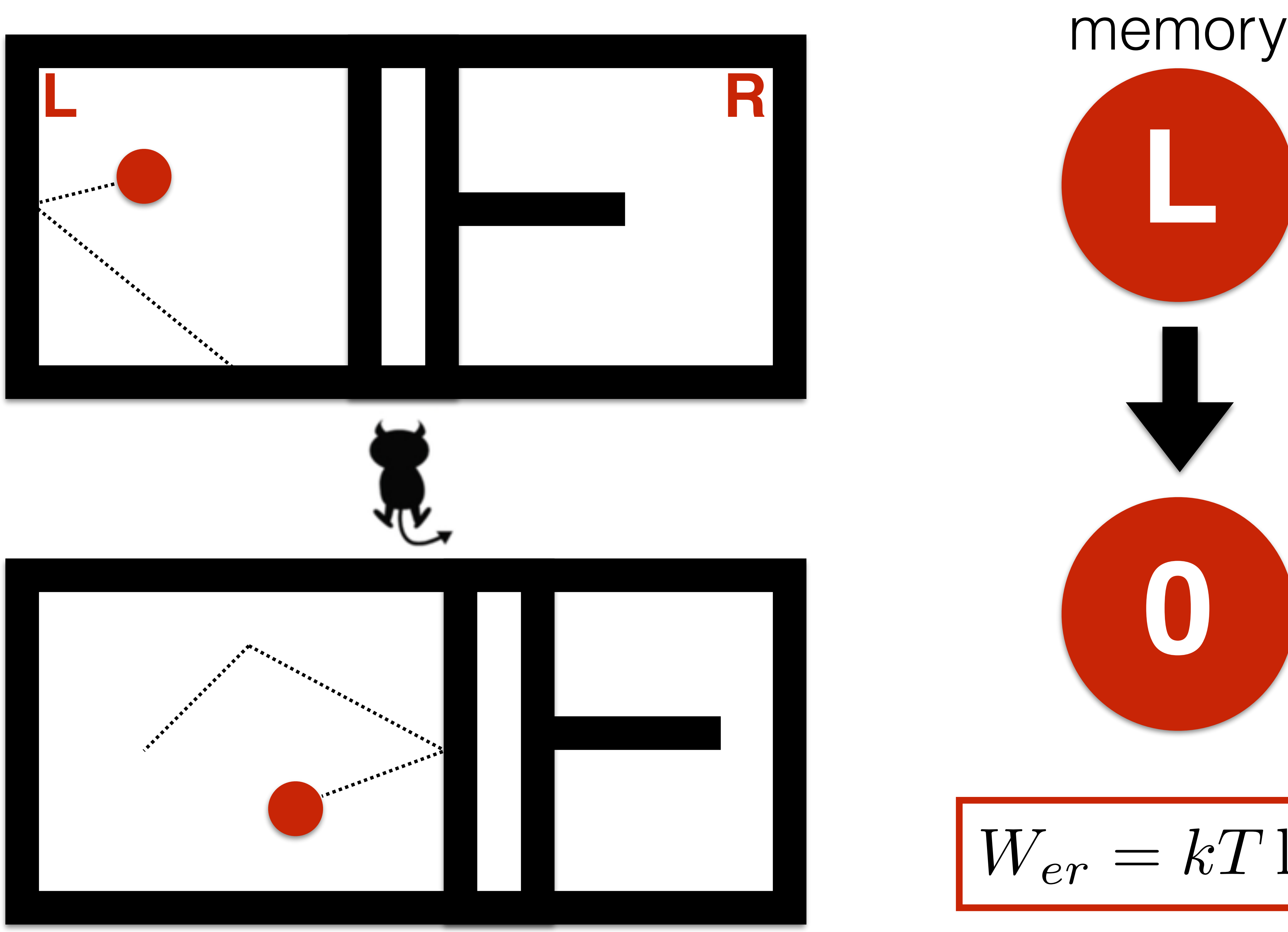
Landauer principle

Exorcising Maxwell's demon



Landauer principle

Exorcising Maxwell's demon



$$\boxed{W_{er} = kT \ln 2}$$

Landauer principle

$$\overset{\text{work of erasure}}{W(S)} = \overset{\text{von-Neuman entropy}}{H(S)}\, kT \ln 2$$

Landauer principle

Subjectivity of information

“Information constraints may result in observers having considerable different knowledge about physical reality”

Landauer principle

Subjectivity of information

von-Neuman entropy

$$W(S) = \boxed{H(S)}kT\ln 2$$

conditional entropy

$$W(S|O) = \boxed{H(S|O)}kT\ln 2$$

Entropy of a system given all information available to an observer

Landauer principle

Subjectivity of information

von-Neuman entropy

$$W(S) = \boxed{H(S)}kT\ln 2$$

conditional entropy

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Entropy of a system given all information available to an observer

classically…

Quantum
system

Quantum
memory

Nature 474, 61–63 (2011)

Work cost of erasure

The amount of work that an observer with quantum memory Q needs to perform to erase S

$$W_{er}(S|Q) = H(S|Q)kT\ln 2$$

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Maximally entangled S and Q

$\Downarrow$

Work extraction by erasure

Nature 474, 61–63 (2011)

Work cost of erasure

The amount of work that an observer with quantum memory Q needs to perform to erase S

$$W_{er}(S|Q) = H(S|Q)kT\ln 2$$

Extractable work

The amount of work that an observer can extract from a n-qubit system

$$W_{ex} = [n - H(S|Q)]kT\ln 2$$

Nature 474, 61–63 (2011)

Work cost of erasure

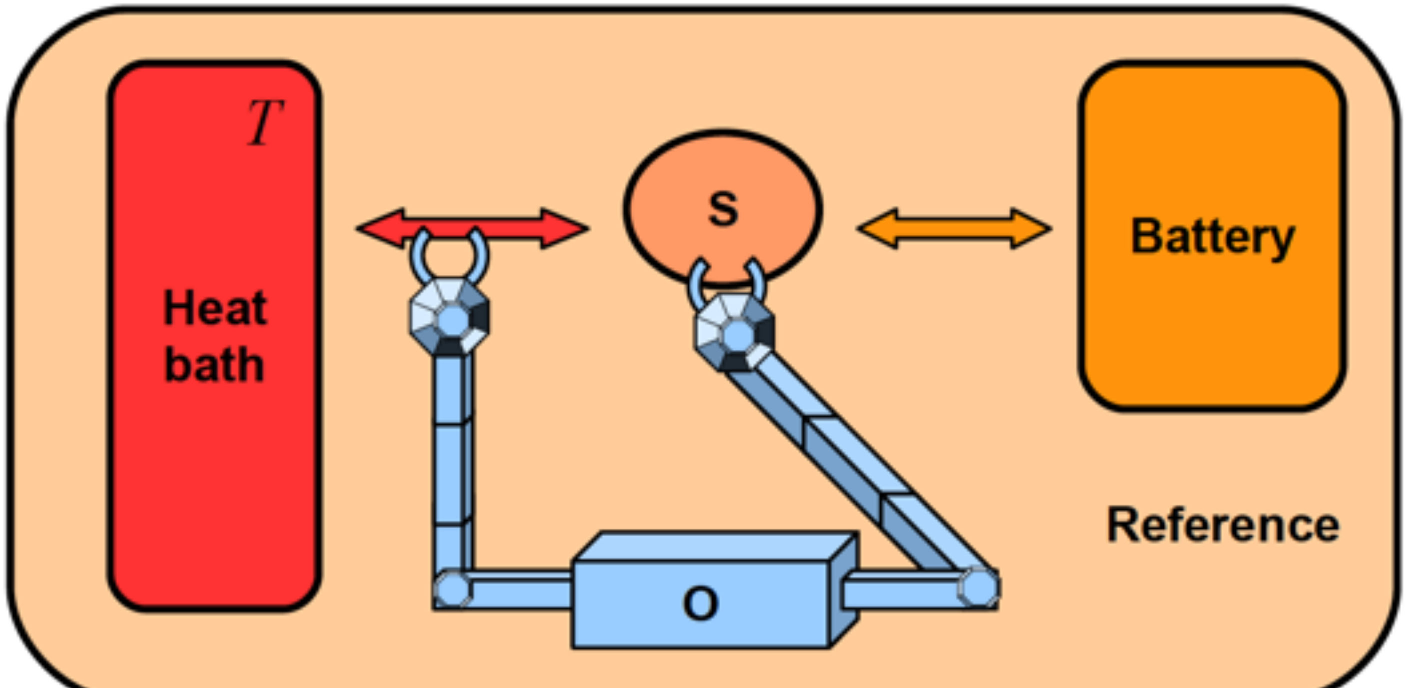
$$W_{er}(S|Q) = H(S|Q)kT\ln 2$$

Nature 474, 61–63 (2011)

Existence of a work erasure protocol which attains this bound

(in the thermodynamic limit)

Ingredients:



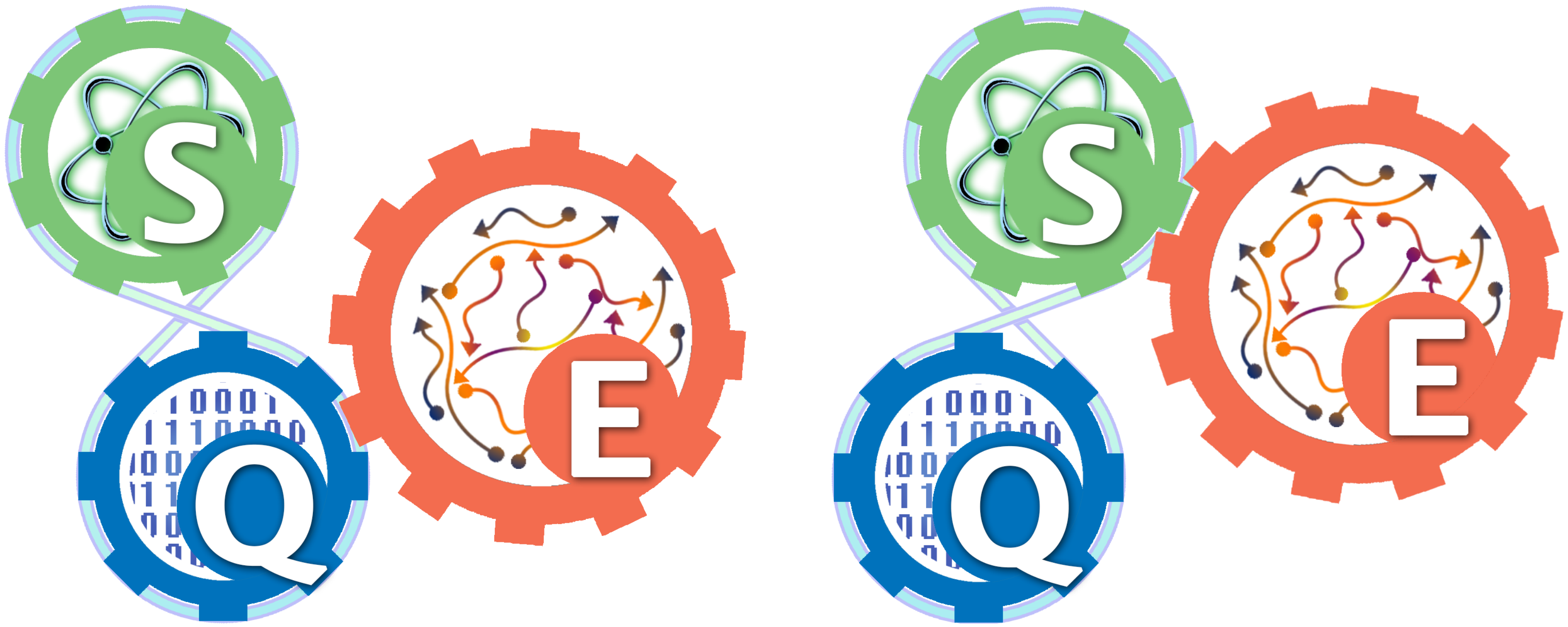
Entanglement

can “do physical work”

Open Quantum Dynamics?

Open Quantum Memory

Open Quantum System



Work cost of erasure

$$W_{er}(S|Q) = H(S|Q)kT\ln 2$$

Extractable work

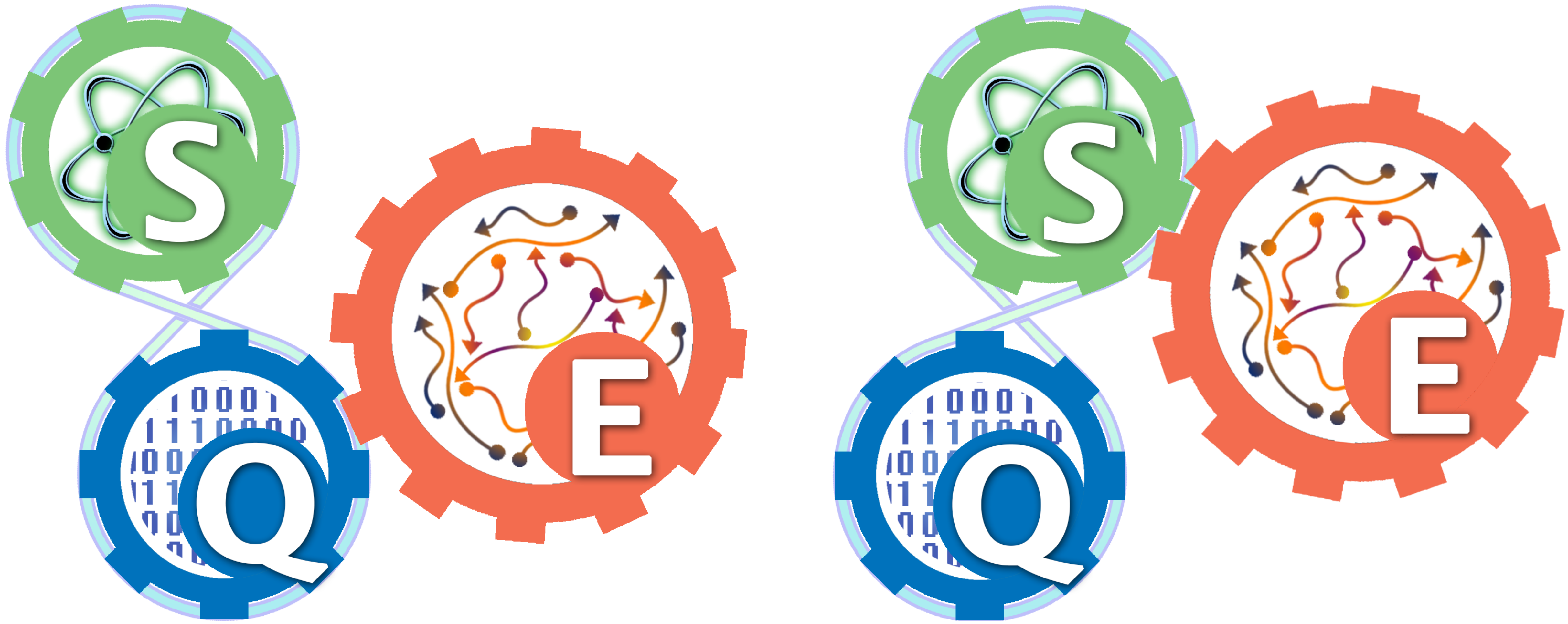
$$W_{ex} = [n - H(S|Q)]kT\ln 2$$

are now time dependent

Extractable work

$$\Delta W_{ex}(t_1, t_2) = W_{ex}(t_2) - W_{ex}(t_1) \qquad t_2 \geq t_1$$

$$= [H(S|Q)_{t_1} - H(S|Q)_{t_2}]kT \ln 2$$



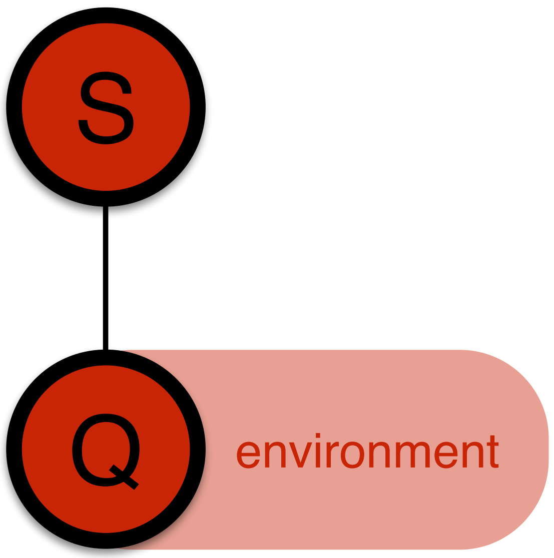
Extractable work

$$\Delta W_{ex}(t_1, t_2) = W_{ex}(t_2) - W_{ex}(t_1) \qquad t_2 \geq t_1$$

$$= [H(S|Q)_{t_1} - H(S|Q)_{t_2}]kT \ln 2$$

Work of erasure $\Delta W_{er}(t_1, t_2) = -\Delta W_{ex}(t_1, t_2)$

Non Markovianity and correlations

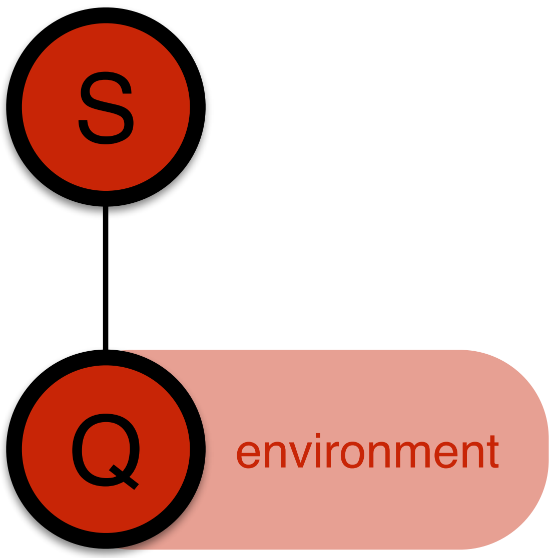


Dynamics of coherent information

$$I_c = -H(S|Q)$$

- correlations between S and Q

Non Markovianity and correlations



Dynamics of coherent information

$$I_c = -H(S|Q)$$

Physical Meaning

I_c measures the extent to which we know less about a part of a system than we do about its whole

- correlations between S and Q

Open Quantum Memory

$$SQ \xrightarrow{\text{time}} \mathbb{I}_S \otimes \Phi_t(Q)$$

$$\Delta W_{ex}(t_1, t_2) = [I_C(Q, \Phi_{t_2}) - I_C(Q, \Phi_{t_1})]kT \ln 2$$

coherent information

Open Quantum Memory

$$SQ \xrightarrow{\text{time}} \mathbb{I}_S \otimes \Phi_t(Q)$$

$$\Delta W_{ex}(t_1, t_2) = [I_C(Q, \Phi_{t_2}) - I_C(Q, \Phi_{t_1})]kT \ln 2$$

coherent information

mutual information

$$I(S : Q_t) = I_C(Q, \Phi_t) - H(S)$$

Open Quantum Memory

$$SQ \xrightarrow{\text{time}} \mathbb{I}_S \otimes \Phi_t(Q)$$

$$\Delta W_{ex}(t_1, t_2) = [I_C(Q, \Phi_{t_2}) - I_C(Q, \Phi_{t_1})]kT \ln 2$$

coherent information

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Divisibility

$\Downarrow$ Data processing inequality

$$I_C(Q, \Phi_{t_2}) \leq I_C(Q, \Phi_{t_1}) \qquad t_2 \geq t_1$$

Divisibility

Data processing inequality

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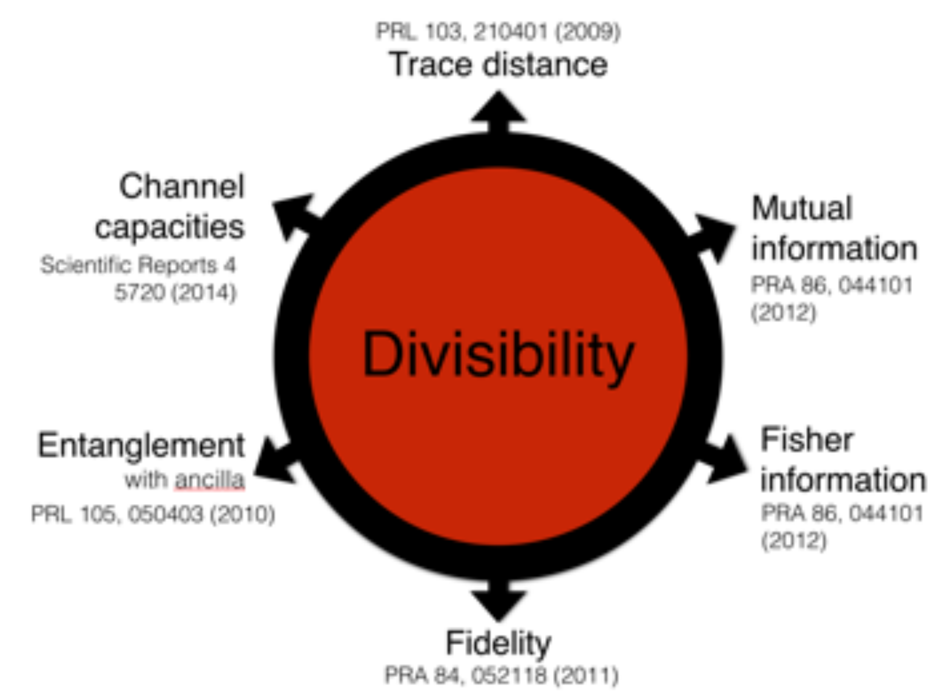
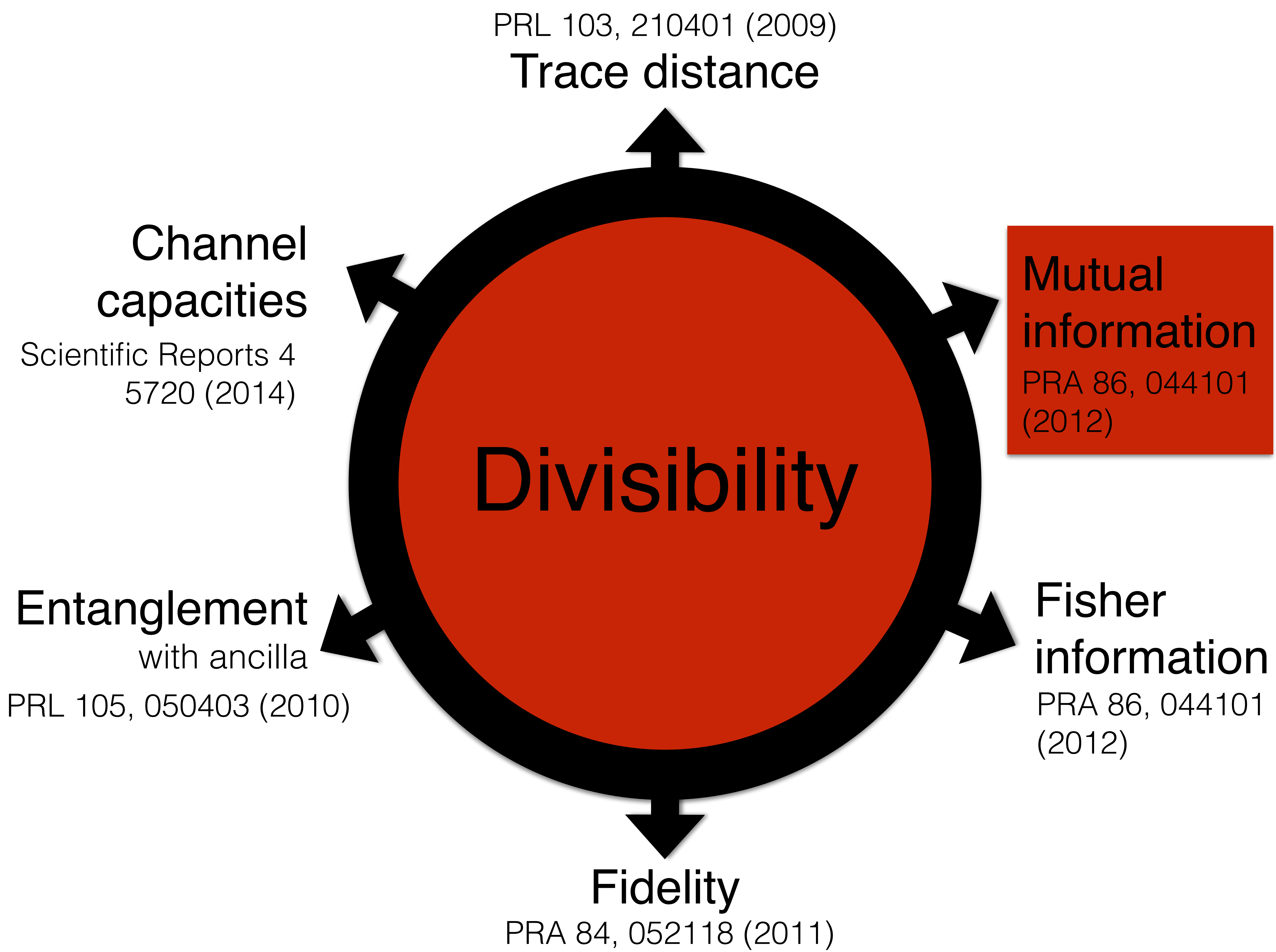
Non-Markovianity

memory effects

Work revivals

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Fidelity
PRA 84, 052118 (2011)

Non Markovianity and correlations

Memory effects (non-Markovianity) as the ability to “retain or regain” coherent information/mutual information between system and ancilla in presence of an environment

oscillations

revivals

trapping

Non-Markovian memory effects

are

revivals of extractable work

Outline

- Non Markovian open quantum dynamics
- Landauer principle in open systems
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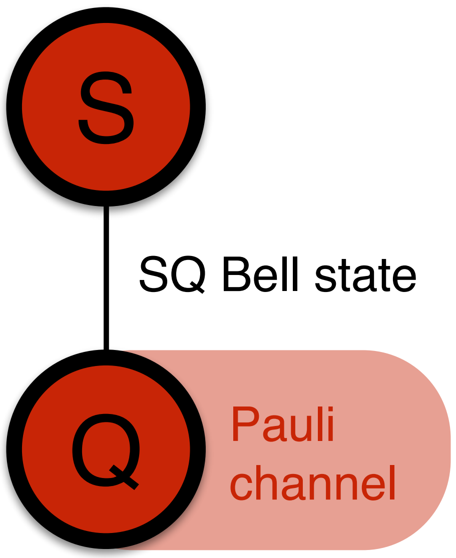
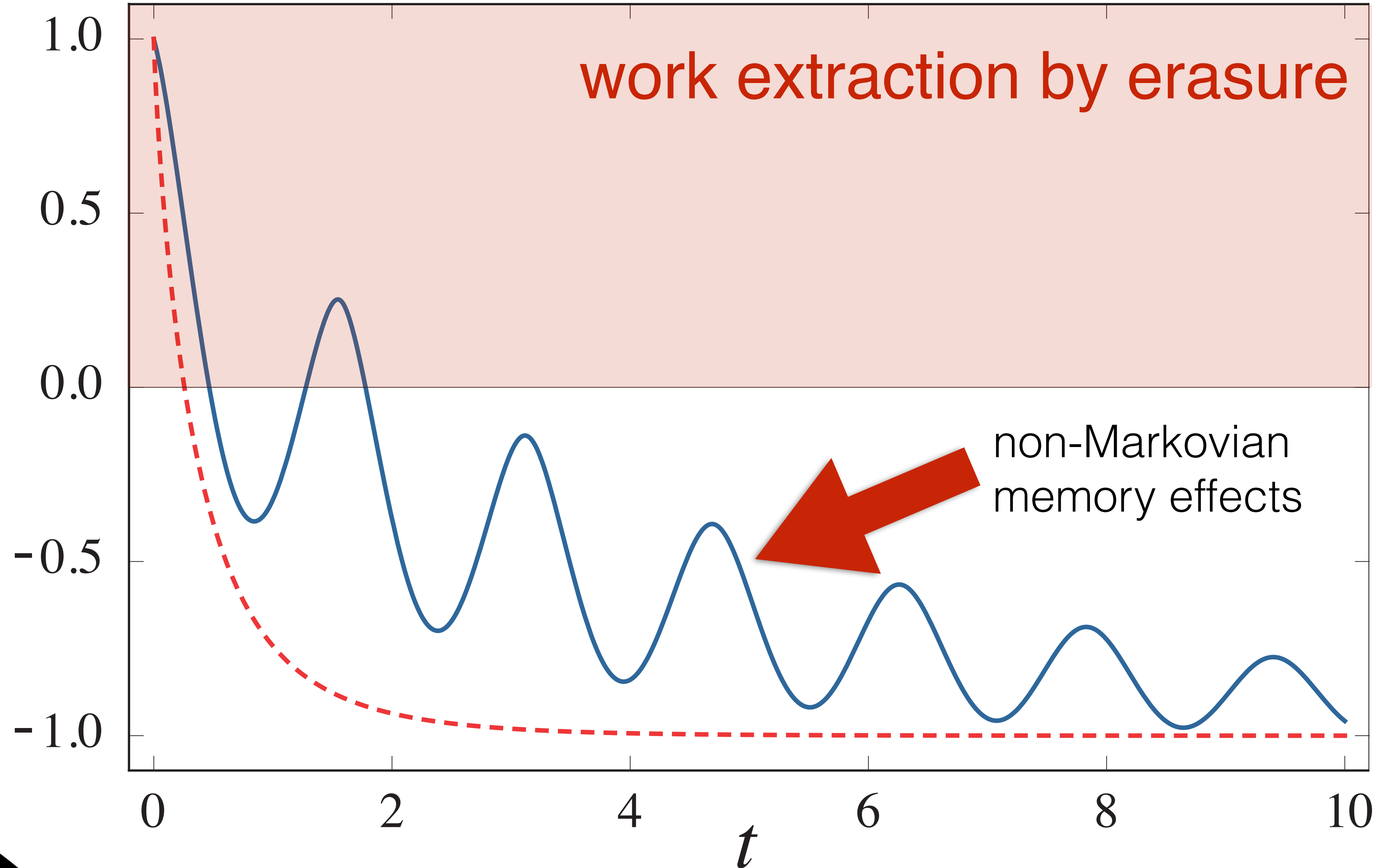
- The power of memory



coherent information



$$I_c = -H(S|Q)$$

$$W_{er}(S|Q) = H(S|Q)kT\ln 2$$

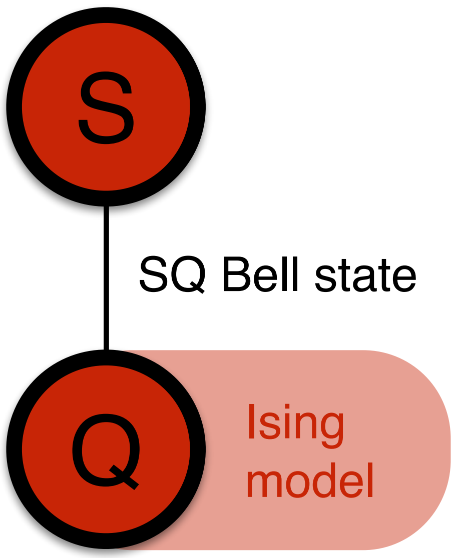
Non-Markovian memory effects

are

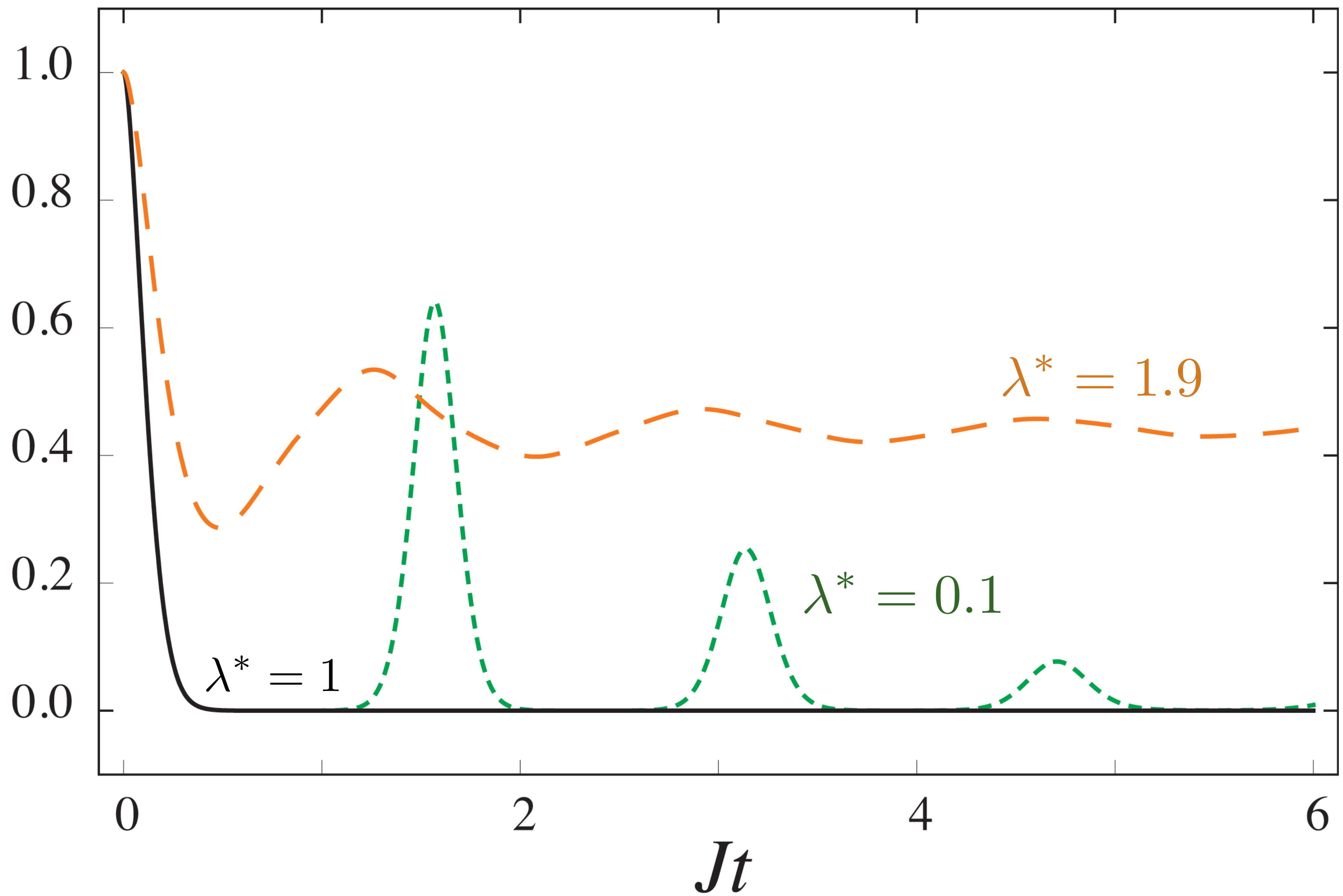
revivals of extractable work

→ reservoir engineering

- time matters (in a non-trivial way)
- optimal time can be controlled



coherent information



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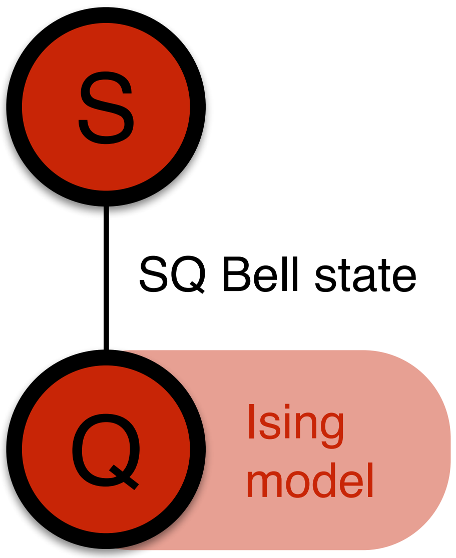
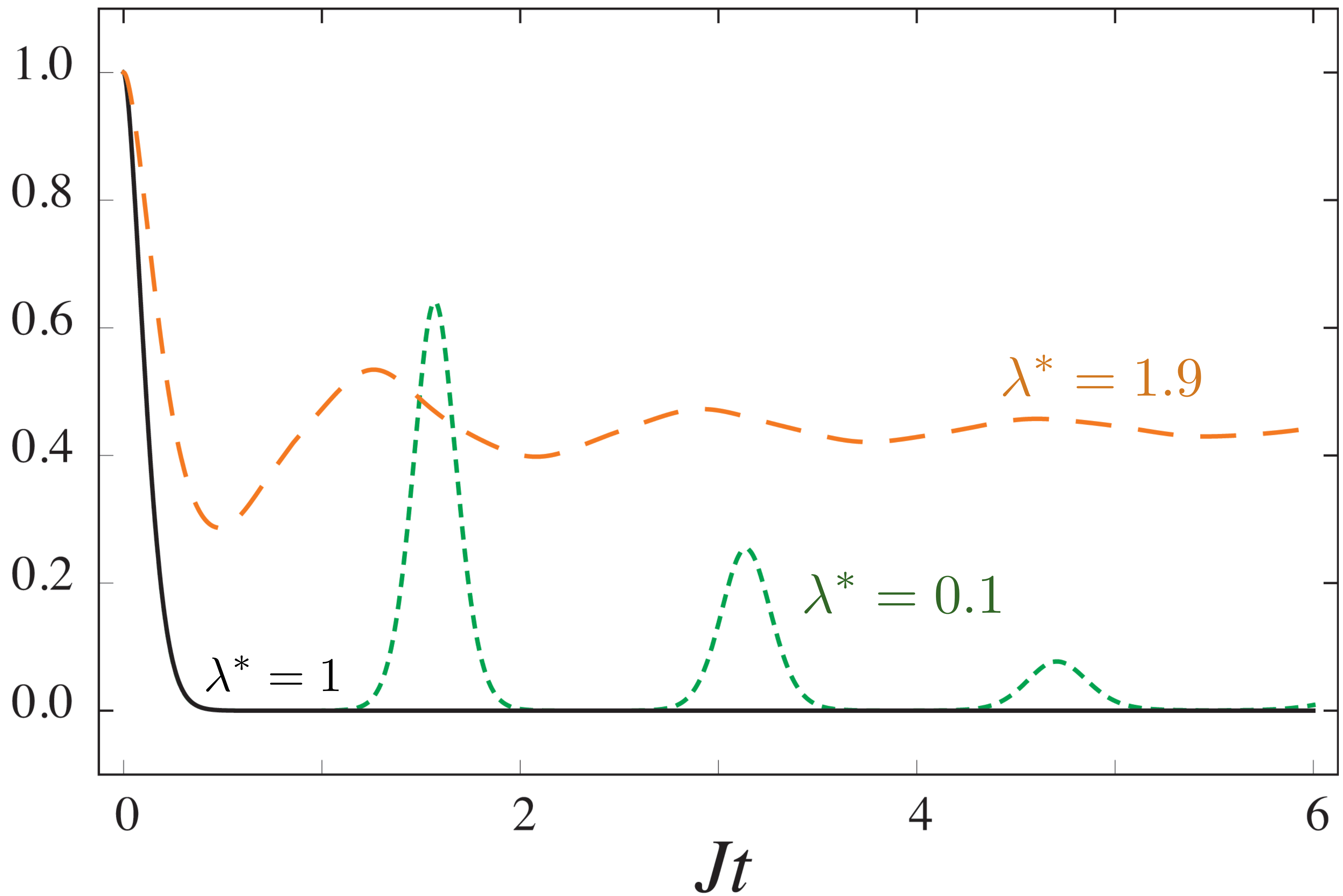
coherent information





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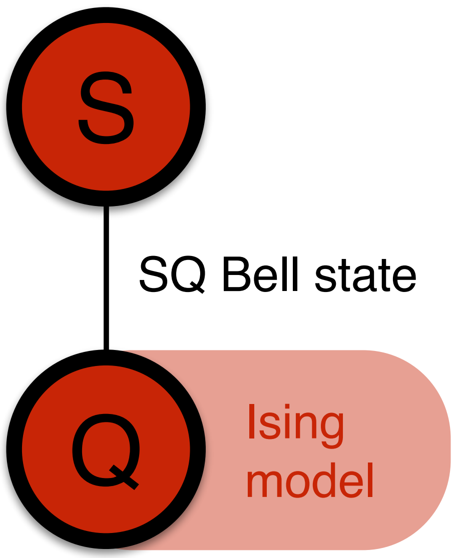
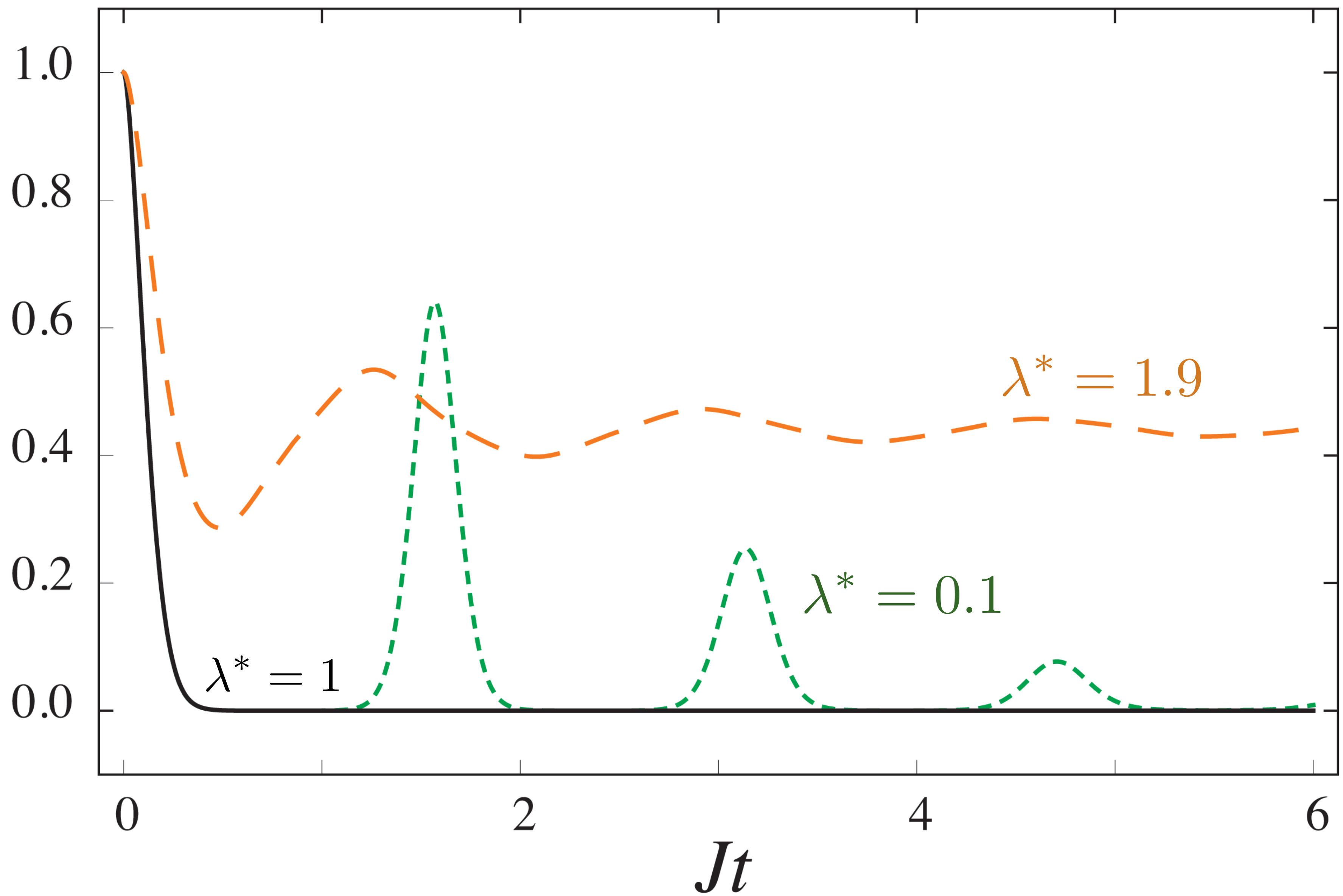
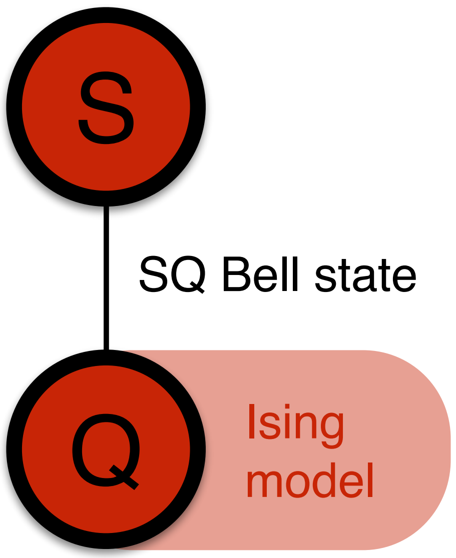
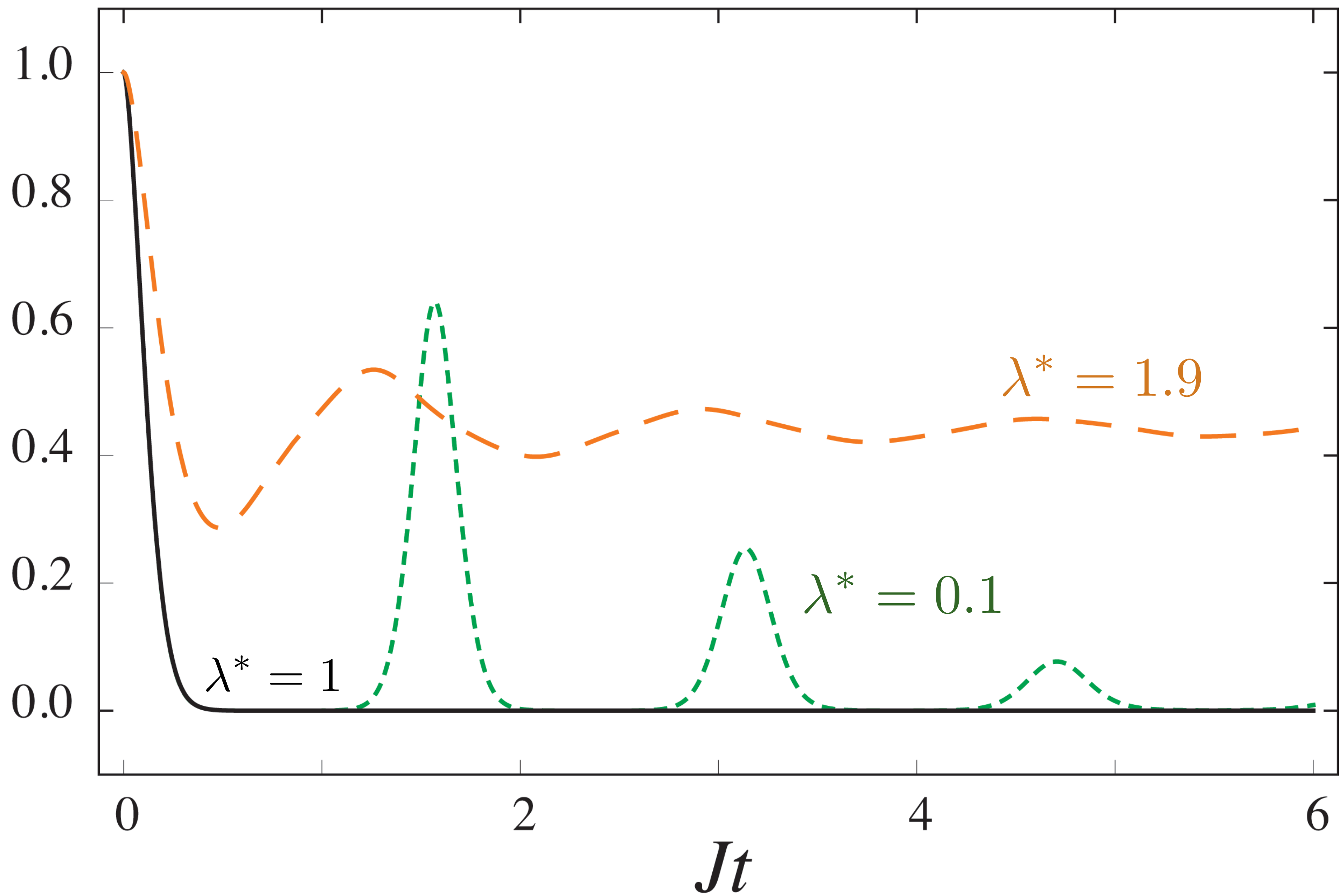




Trapping of work extraction (by erasure)



coherent information



- Trapping of work extraction (by erasure)
- Revivals of work extraction (by erasure)

Non-Markovian enhancement

of

extractable work

Outline

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Open Quantum System

$$SQ \xrightarrow{\text{time}} \Phi_t(S) \otimes \mathbb{I}_Q$$

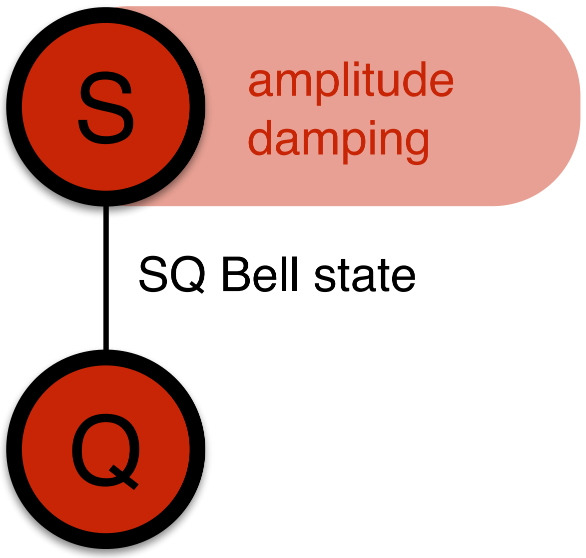
$$\Delta W_{ex}(t_1, t_2) = \left[-\Delta H(S_t) + \Delta I(S_t : Q)\right] kT \ln 2$$

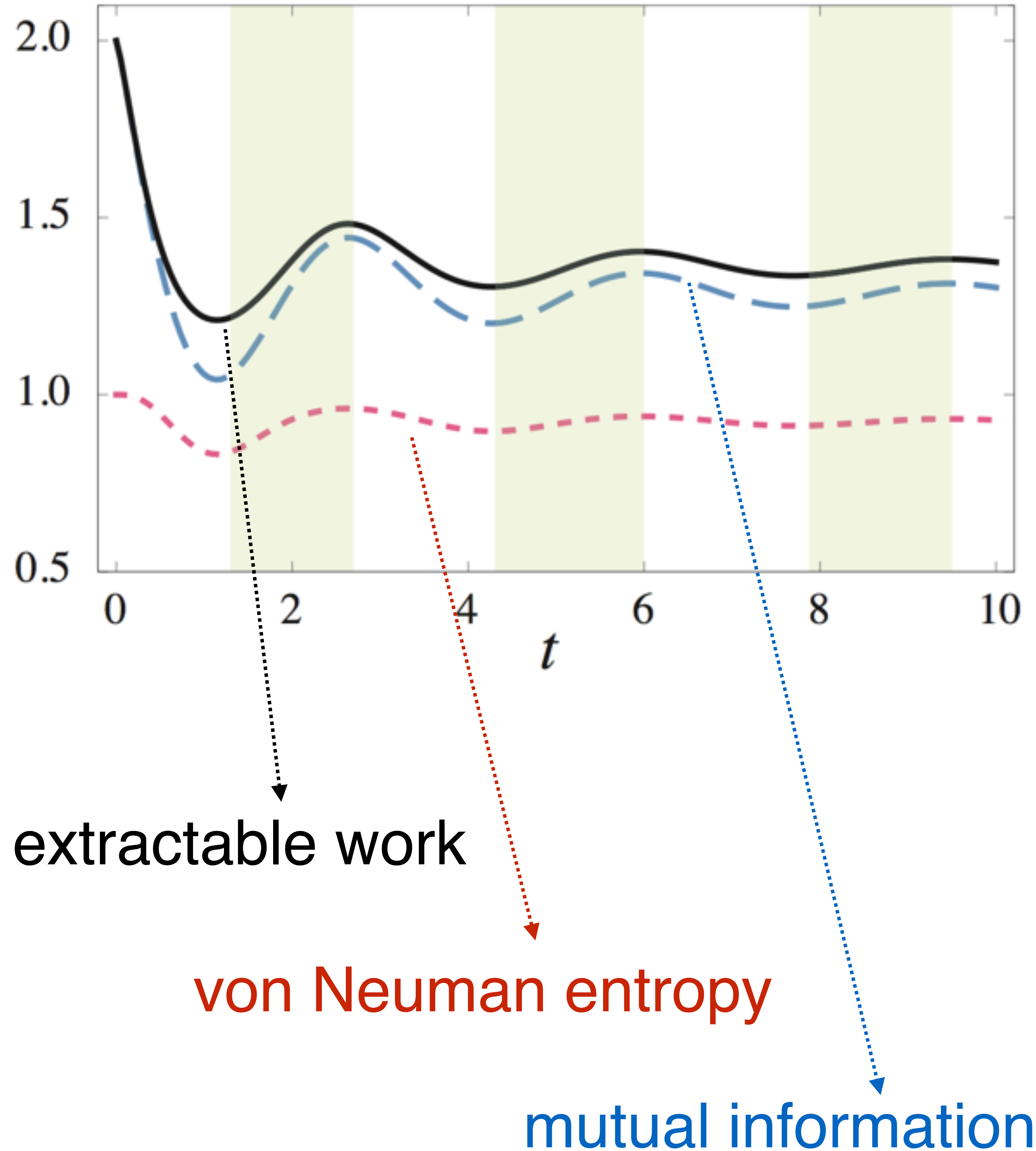
change of
mutual information

change of
von Neuman entropy

can we obtain increase in extractable Work

when the entropy of the system is increasing?


S
amplitude
damping
SQ Bell state
Q


2.0
1.5
1.0
0.5
0
2
4
6
8
10
t
extractable work
von Neuman entropy
mutual information

Revivals of extractable work

due to

quantum correlations

Revival of extractable work

due to

quantum correlations

memory effects

revivals of mutual information

dominant with respect to entropy increase

Outlook

- Resource theory of non-Markovianity
- Work in non-Markovian open quantum systems
- Memory effects and entropic uncertainty relations

  arXiv:1504.02391

Funding:

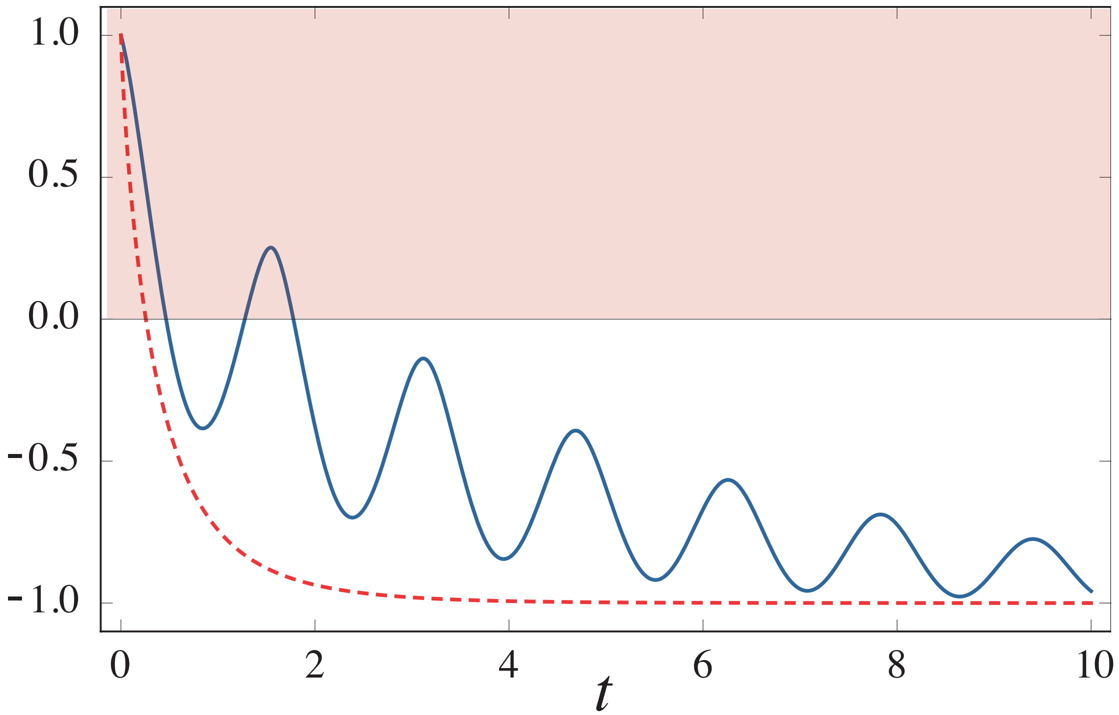
European Commission

QuProCS

Magnus Ehrnrooth
foundation

ACADEMY
OF FINLAND

Extractable work

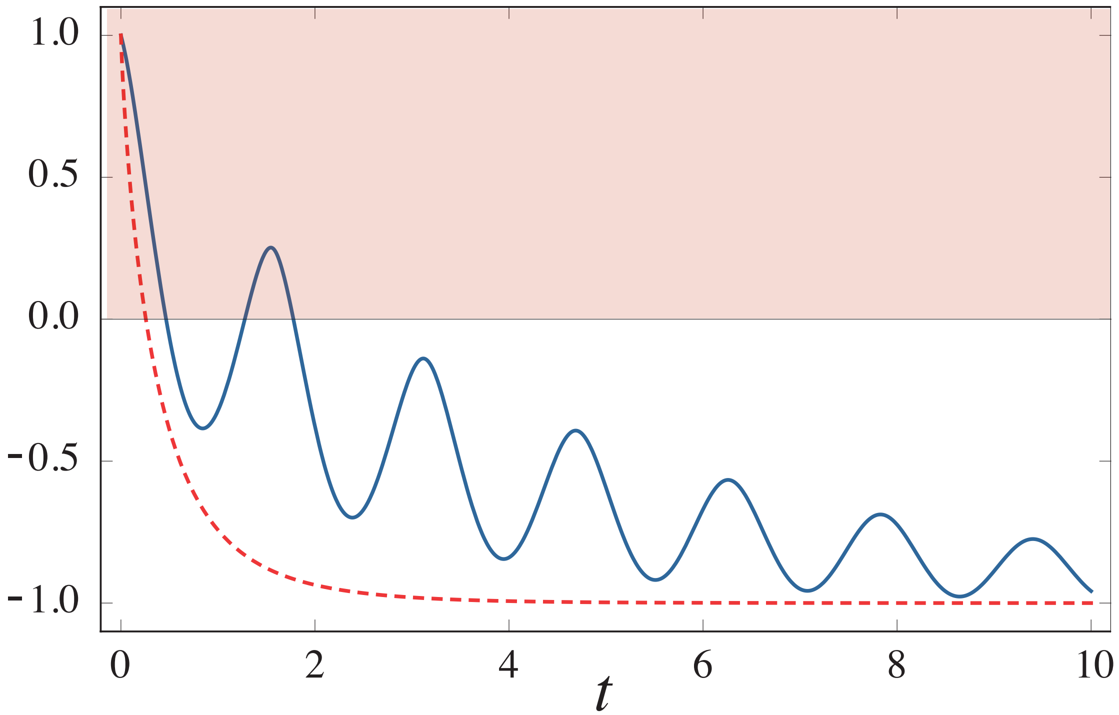


Pauli channel

$$L_t\rho = \sum_{i=1}^{3} \gamma_i(t)\sigma_i\rho\sigma_i - \rho$$

$$\gamma_1(t) = \frac{\lambda}{2} = \gamma_2(t)$$

Extractable work



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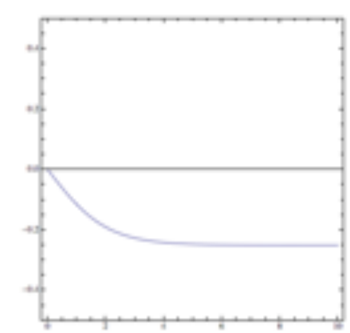
- $\gamma_3(t) = -\frac{\omega}{2} \tanh(\omega t)$



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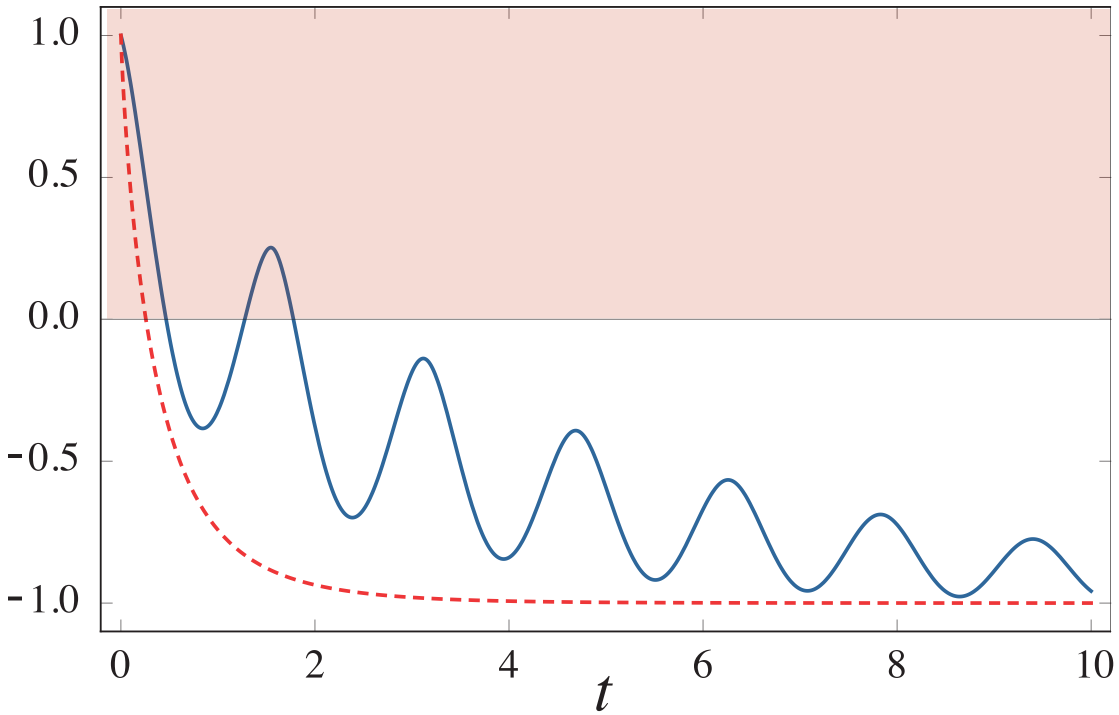

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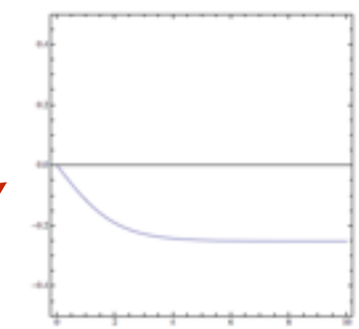
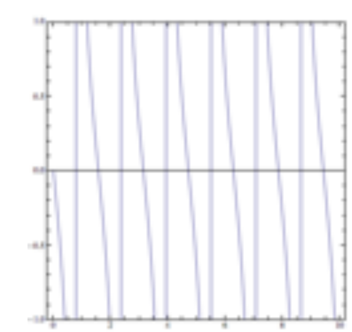
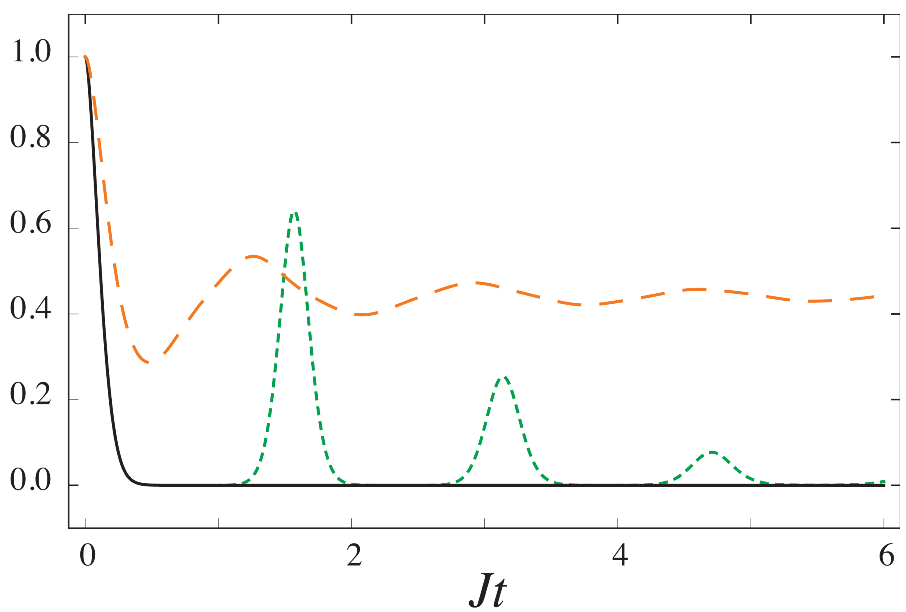


- $\gamma_3(t) = \frac{\omega}{2} \tan(\omega t)$



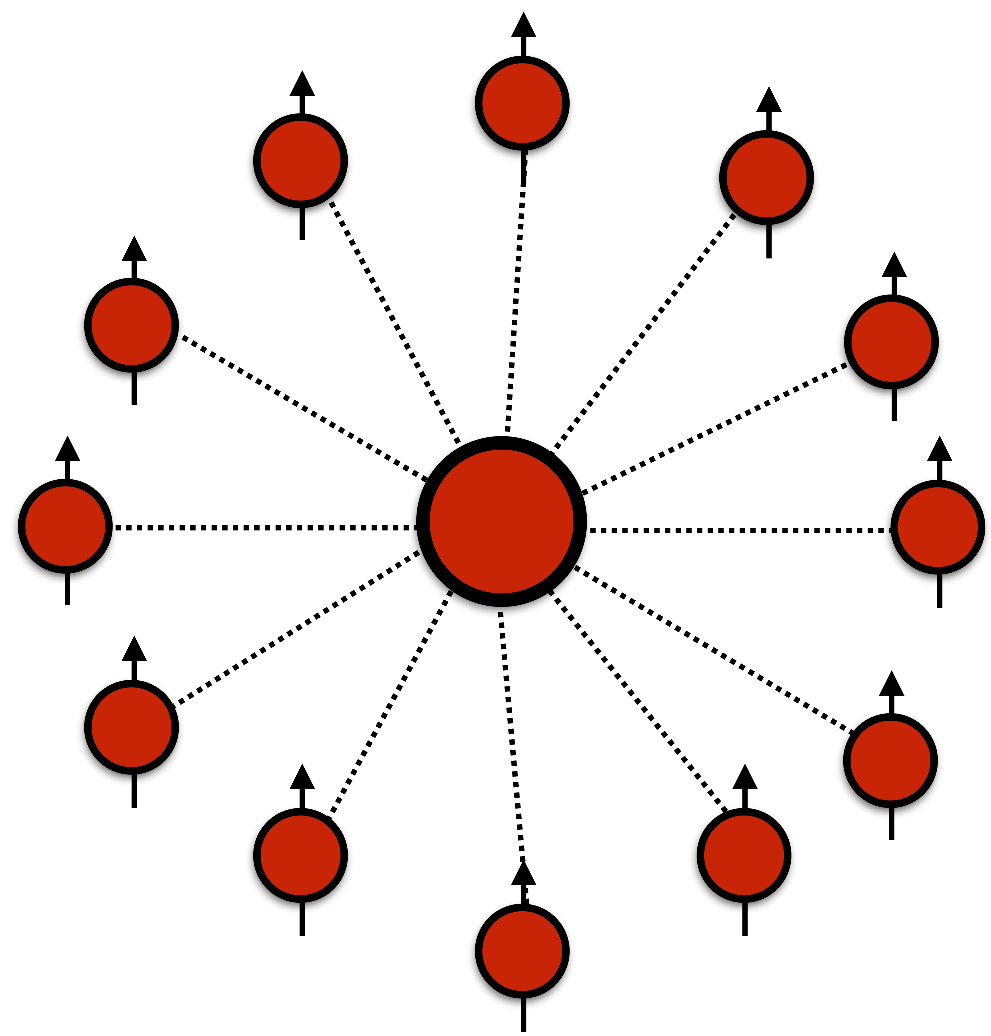
Extractable work



$$H_{env}(\lambda) = -J \sum_j \sigma_j^z \sigma_{j+1}^z + \lambda \sigma_j^x$$

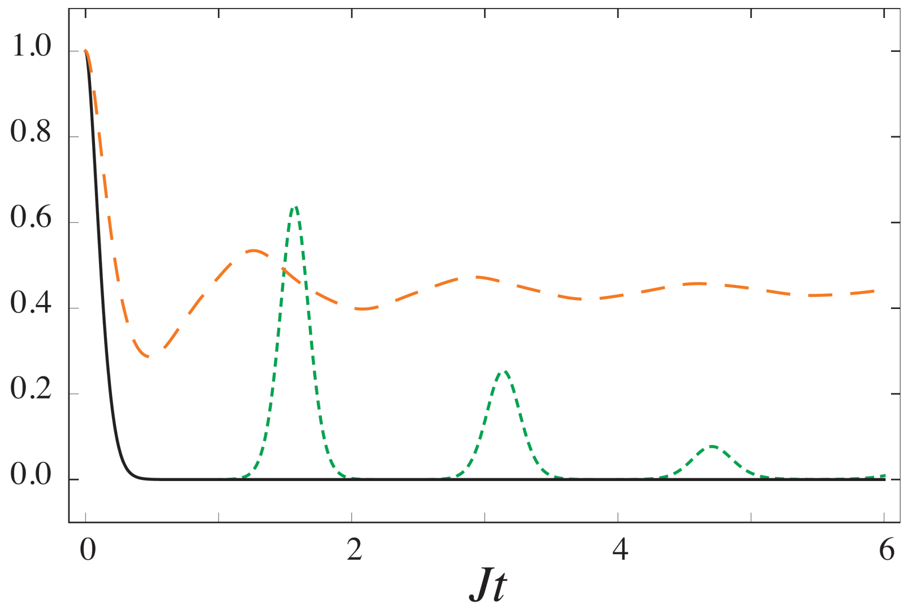
$$H_{int}(\delta) = \delta |e\rangle\langle e| \sum_j \sigma_j^x$$

$$\lambda^* = \frac{\lambda + \delta}{J}$$



PRL 96, 140604 (2006)

Extractable work



$$I_c(Q_t) = 1 + \frac{1}{2}(1 - \sqrt{L(t)})\log_2(\frac{1}{2}(1 - \sqrt{L(t)}))$$
$$+\frac{1}{2}(1 + \sqrt{L(t)})\log_2(\frac{1}{2}(1 + \sqrt{L(t)}))$$

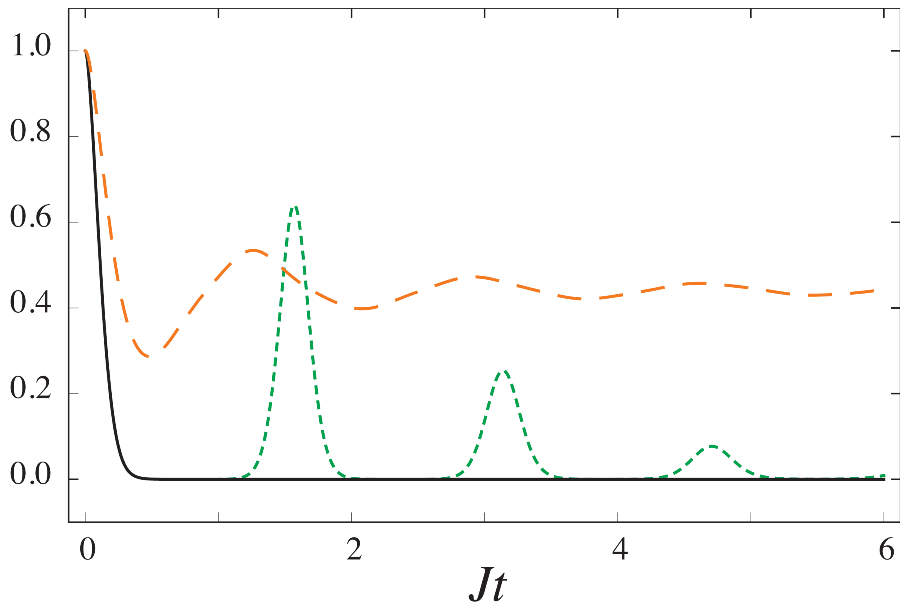
Loschmidt echo

$$L(\lambda, \delta, t) = \Pi_{k>0}\left[1 - \sin^2(2\beta_k)\sin^2(\epsilon_e^k t)\right]$$

$$\lambda^* = \frac{\lambda + \delta}{J}$$

PRA 85, 060101(R) (2012)

Extractable work



$$I_c(Q_t) = 1 + \frac{1}{2}(1 - \sqrt{L(t)})\log_2(\frac{1}{2}(1-\sqrt{L(t)}))$$
$$+\frac{1}{2}(1 + \sqrt{L(t)})\log_2(\frac{1}{2}(1+\sqrt{L(t)}))$$

Loschmidt echo

$$L(\lambda,\delta,t) = \Pi_{k>0}\left[1 - \sin^2(2\beta_k)\sin^2(\epsilon_e^k t)\right]$$

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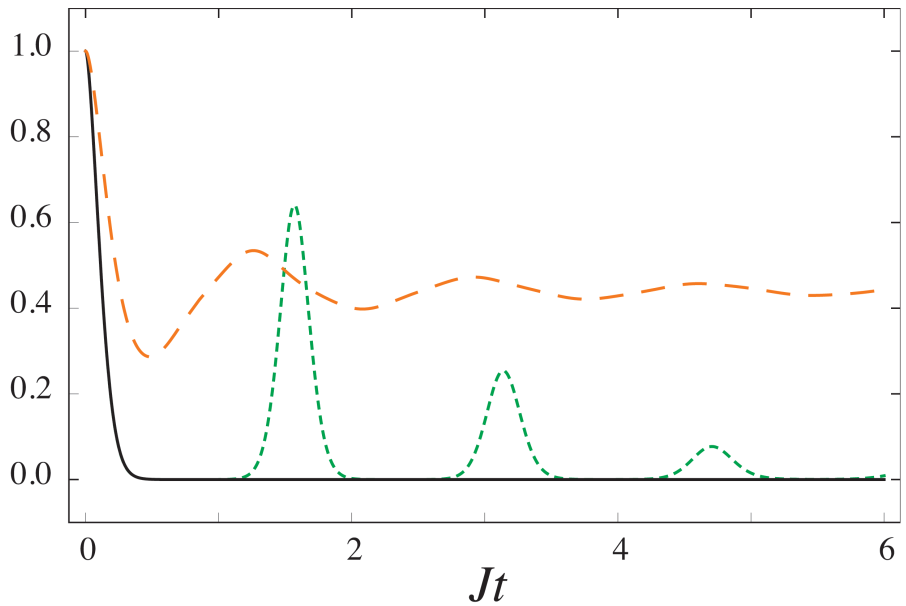
- Trapping of work extraction (by erasure)

PRA 85, 060101(R) (2012)

Extractable work



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Loschmidt echo

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$$\lambda^* = \frac{\lambda + \delta}{J}$$

- Trapping of work extraction (by erasure)
- Revivals of work extraction (by erasure)

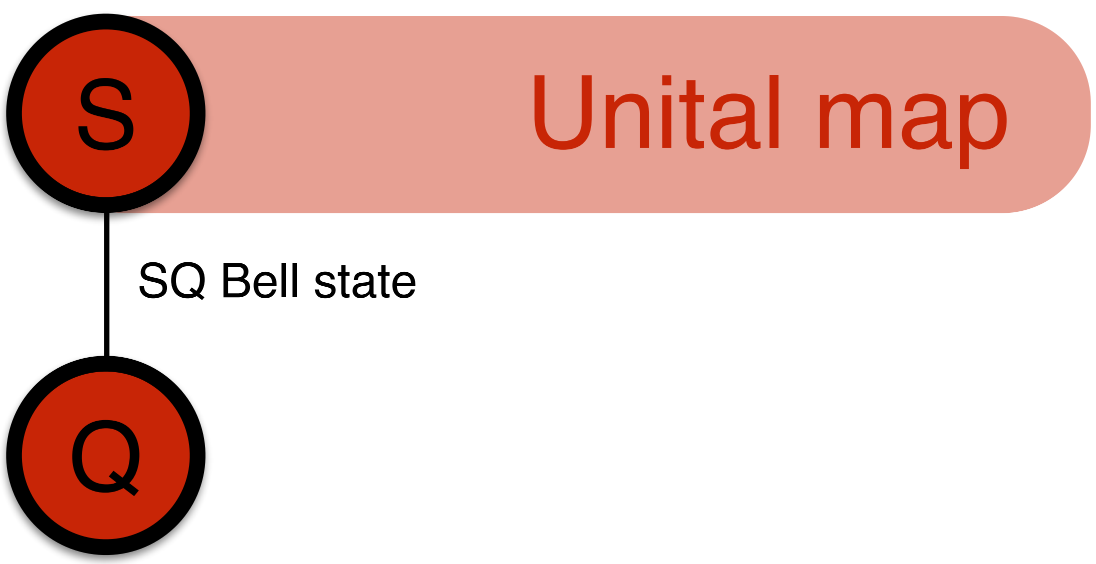
PRA 85, 060101(R) (2012)

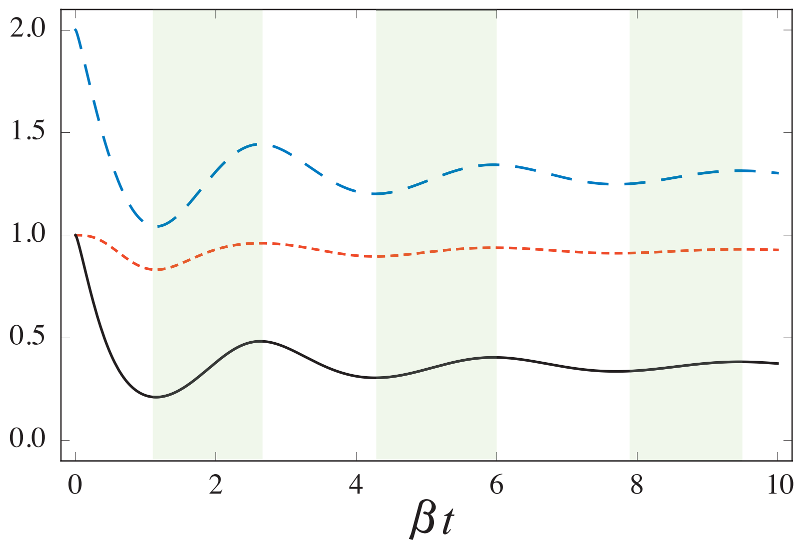
Open Quantum System

$$SQ \xrightarrow{\text{time}} \Phi_t(S) \otimes \mathbb{I}_Q$$

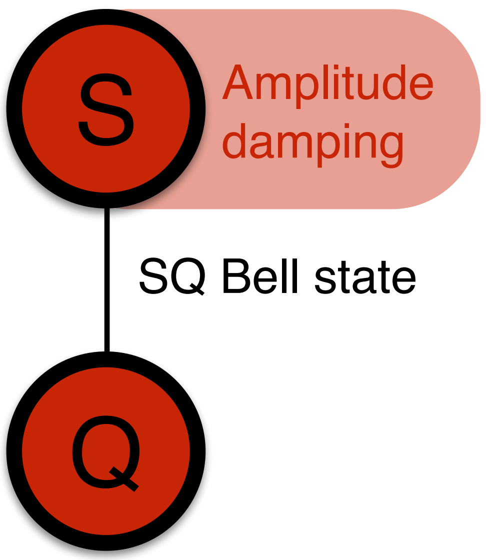
$$\Delta W_{ex}(t_1, t_2) = [-\Delta H(\Phi_t(S)) + \Delta I_C(S, \Phi_t)]kT \ln 2$$

$$= 0$$





Amplitude damping: band gap model

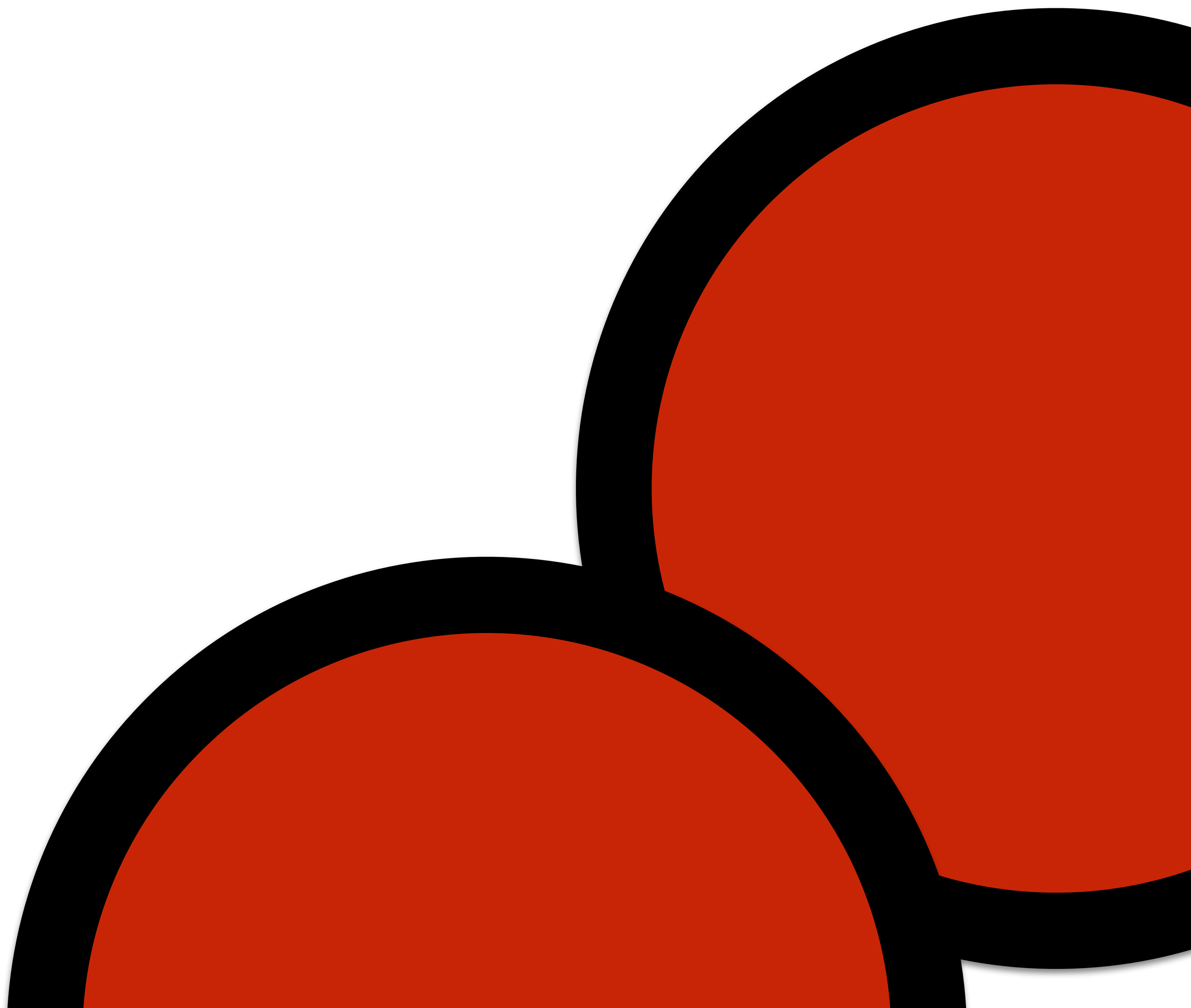


$$L_t \rho = -\frac{is(t)}{2}[\sigma_+\sigma_-, \rho] + \gamma(t)\left(\sigma_-\rho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho\}\right)$$

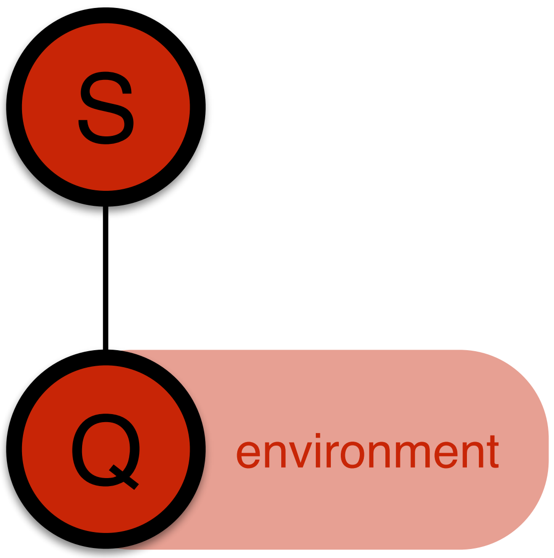
$$s(t) = -2\Im\{\dot{G}(t)/G(t)\} \qquad \gamma(t) = -2\Re\{\dot{G}(t)/G(t)\}$$

depend on the spectral properties

$$\Delta W_{ex}(t_1, t_2) = [-\Delta H(\Phi_t(S)) + \Delta I_C(S, \Phi_t)]kT \ln 2$$



Non Markovianity and correlations



Dynamics of quantum mutual information

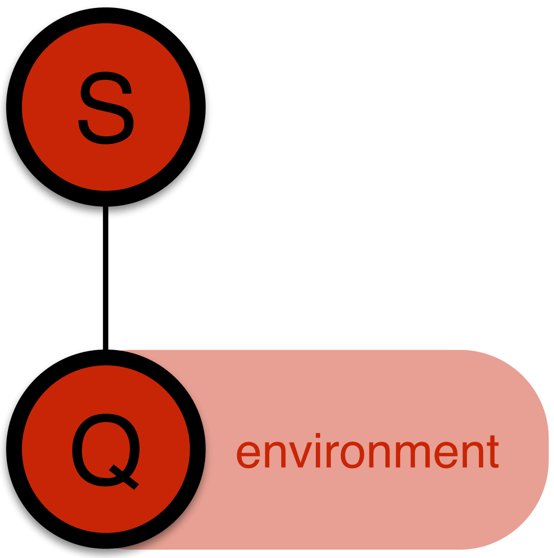
$$I(S:Q) = H(S) + H(Q) - H(SQ)$$

$$= H(S) + H(\Phi_t Q) - \underbrace{H(Q, \Phi_t Q)}_{\text{exchange entropy}}$$

Dynamics of coherent information

$$I_c(Q, \Phi_t Q) = I(S:Q) - H(S)$$

Non Markovianity and correlations



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Dynamics of coherent information

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$$\frac{dI_c}{dt} = \frac{dI}{dt}$$