DYNAMICS OF INCOMPATIBILITY OF QUANTUM MEASUREMENTS IN OPEN SYSTEMS

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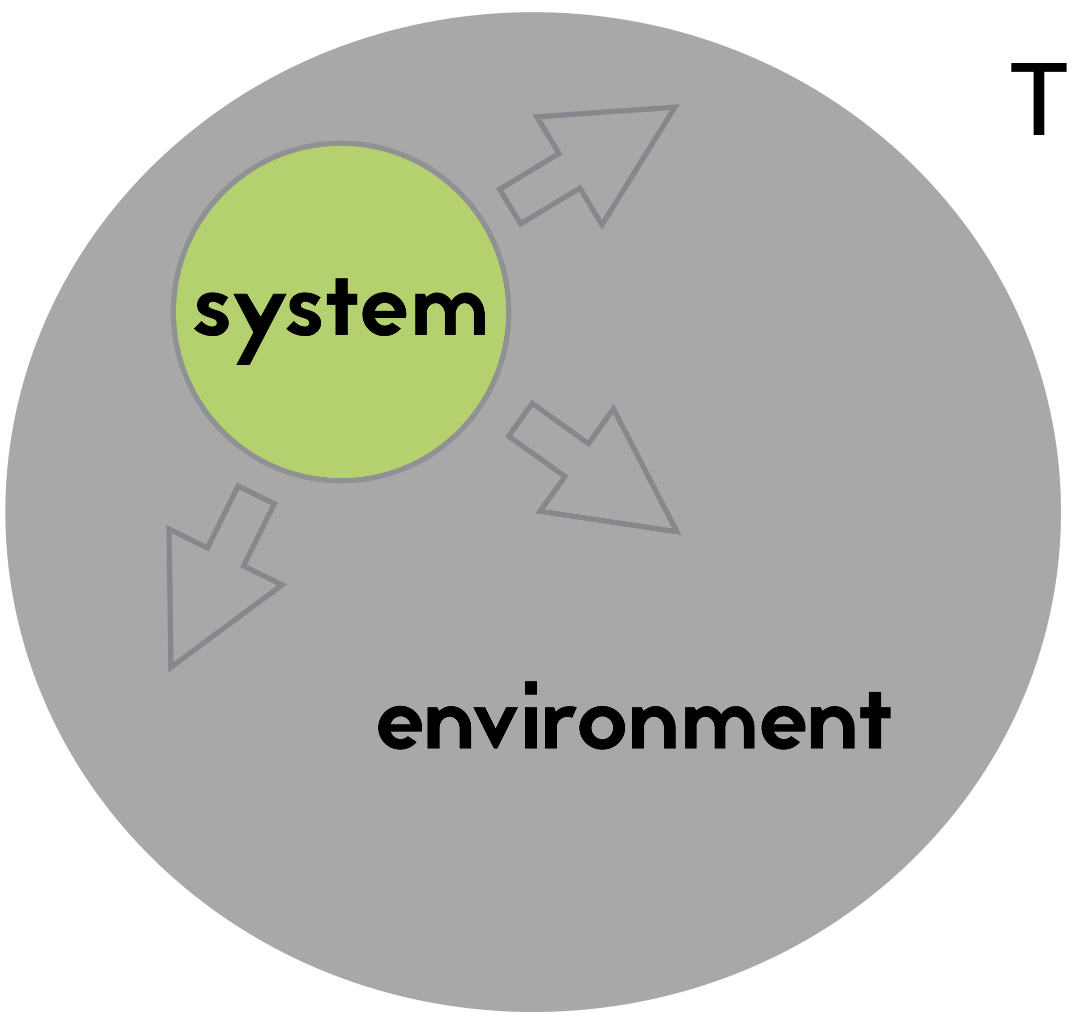
OUTLINE

- MOTIVATION
  - Quantum properties are a resource
  - Effect of noise on quantum properties
- WHAT MAKES MEASUREMENTS QUANTUM?
  - Bell inequalities
  - Compatibility of quantum measurements
- HOW EASILY DO MEASUREMENTS BECOME COMPATIBLE?
  - Inompatibility under Markovian noise
  - Non-Markovian dynamics and inompatibility

THE HOLY GRAIL OF QUANTUM

FRAGILITY OF QUANTUM STATES

Quantum properties are easily lost.



The system can no longer be used as a quantum resource.

Reservoir engineering etc.

FRAGILITY OF QUANTUM STATES

Environment-Induced Sudden Death of Entanglement

M. P. Almeida, F. de Melo, M. Hor-Meyll, A. Salles, S. P. Walborn, P. H. Souto Ribeiro, L. Davidovich*

We demonstrate the difference bet
entangled quantum systems couple
experimental setup, we showed tha
asymptotic, quantum entanglemen
another distinct and counterintuiti

PHYSICAL REVIEW A **79**, 042302 (2009)

Sudden death and sudden birth of entanglement in common structured reservoirs

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Sudden Transition between Classical and Quantum Decoherence

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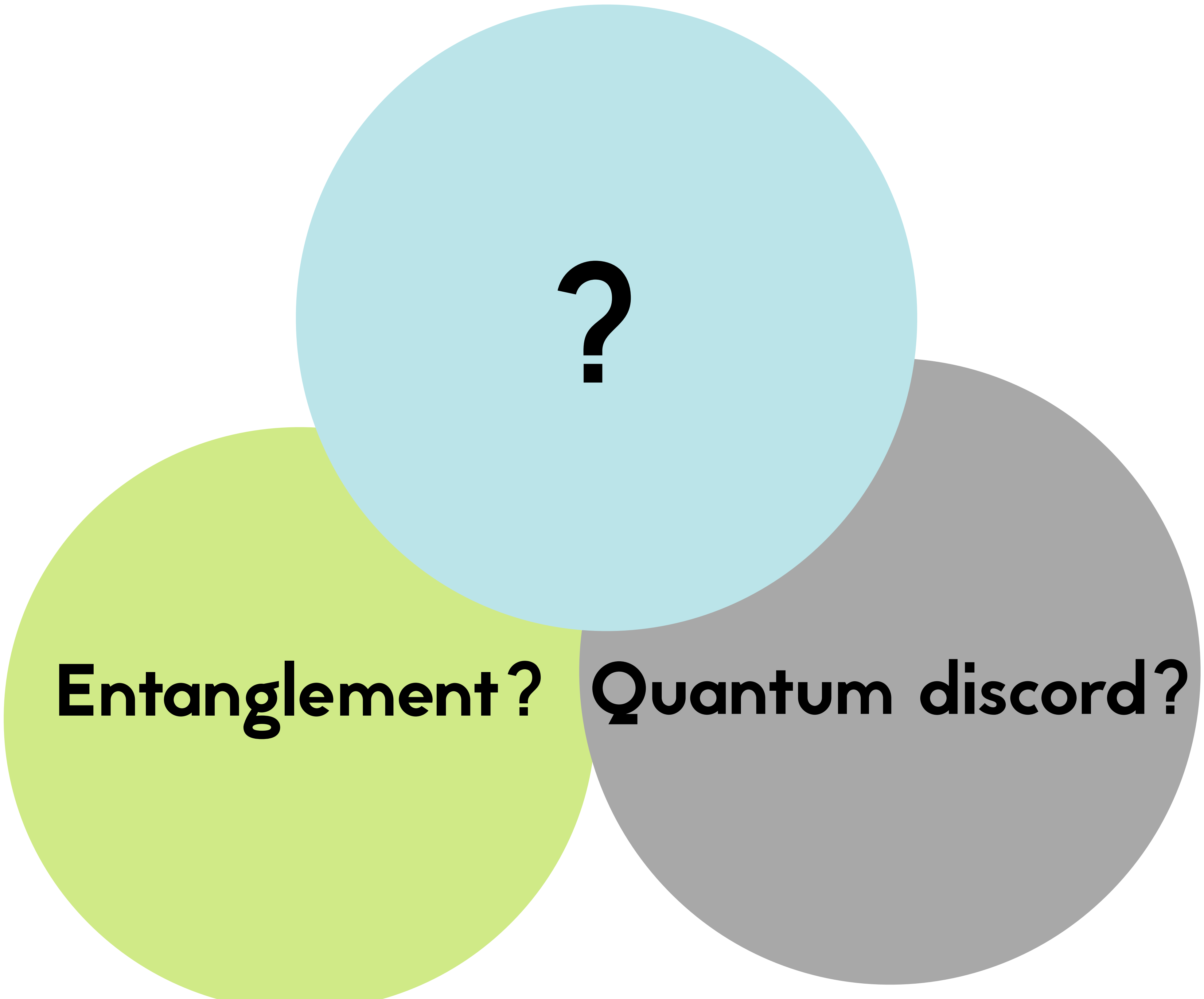
(Received 3 December 2009; revised manuscript received 30 January 2010; published 17 May 2010)

WHAT IS A QUANTUM RESOURCE?



QUANTUM MEASUREMENTS

Sharp measurements (PVM):

Set of orthogonal projections $\{\Pi_m\}$

$$\sum_m \Pi_m = \mathbb{I} \qquad \Pi_m \Pi_n = \delta_{mn} \Pi_n$$

E.g. measurement of spin in x-direction: σ_x

$$|+\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |0\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|1\rangle - |0\rangle)$$

$$S_x(\pm) = |\pm\rangle\langle\pm| = \frac{1}{2}(\mathbb{I} \pm \sigma_x)$$

QUANTUM MEASUREMENTS

Generalised measurements (POVM):

Set of positive operators $\{E_m\}$

$$\sum_m E_m = \mathbb{I} \qquad E_m \geq 0 \quad \forall m$$

E.g. unsharp measurement of spin in x-direction:

$$S_x(\pm) = \frac{1}{2}(\mathbb{I} \pm r\sigma_x)$$

WHAT MAKES QUANTUM QUANTUM?

EPR paradox and Bell inequalities:

The statistics of a set of correlation measurements performed on a maximally entangled state cannot be explained by a classical theory.

Both Alice and Bob measure locally in orthogonal directions.

Also the set of measurements is quantum mechanical.

WAHAT MAKES MEASUREMENTS QUANTUM?

If two observables don't commute, they cannot be measured jointly.

Commutativity?

COMPATIBILITY

Two measurements $\{E_m\}$ and $\{F_n\}$ are compatible, if there exists a measurement $\{G_{mn}\}$ such that

$$E_m = \sum_n G_{mn} \qquad F_n = \sum_m G_{mn}$$

The two measurements can be realised together with one device.

If the measurements are not compatible, they are incompatible.

COMPATIBILITY

E.g. measurements of spin in x- and z-direction:

$$S_x(\pm) = \tfrac{1}{2}(\mathbb{I} \pm \sigma_x) \qquad S_z(\pm) = \tfrac{1}{2}(\mathbb{I} \pm \sigma_z)$$

Incompatible

$$S_x(\pm) = \frac{1}{2}(\mathbb{I} \pm r\sigma_x) \qquad S_z(\pm) = \frac{1}{2}(\mathbb{I} \pm r\sigma_z)$$

Compatible, iff $r \leq \frac{1}{\sqrt{2}}$

MEASURE FOR INCOMPATIBILITY

The minimal amount of classical noise needed to make two measurements compatible.

Measuring a POVM $\{E_m\}$ using a noisy measurement device which sometimes ignores the actual outcome, and instead outputs a random output. The resulting deformed measurement $\{E_m^\lambda\}$ is given as

$$E_m^\lambda = (1-\lambda)E_m + \frac{1}{2}\lambda\mathbb{I}$$

Robustness of the original incompatibility of a pair $\{E_m\}$ $\{F_n\}$:

$$\mathcal{I}_p(E,F) := \inf\{\lambda > 0 \mid (E^\lambda, F^\lambda) \text{ compatible } \}.$$

IS IT A RESOURCE?

Bell inequalities:
CHSH inequality is violated iff the local sharp measurements of Alice are incompatible.

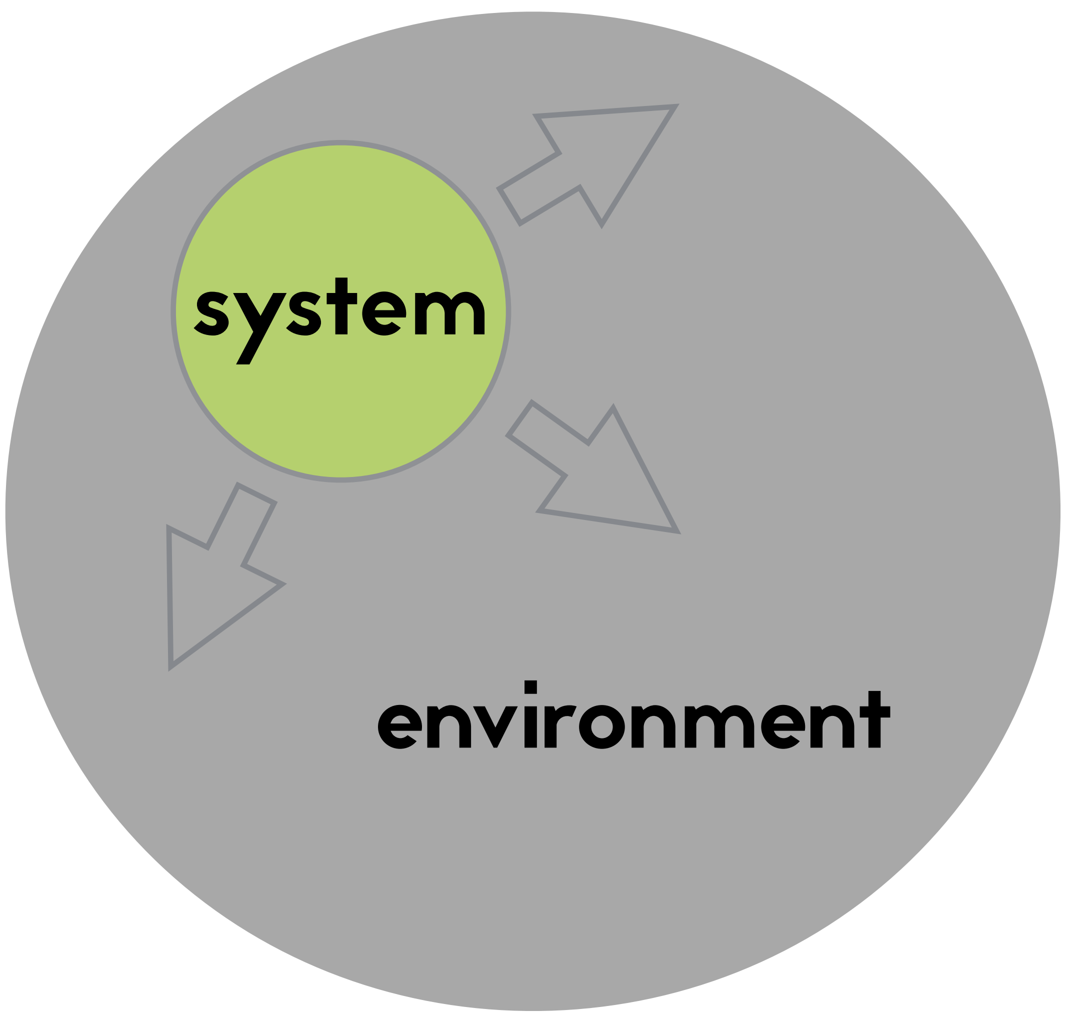
A. Fine, PRL 48, 5 (1982), M. Wolf et al., PRL 103, 230402 (2009).

Quantum steering:
incompatible measurements are nonclassical, as they (and they only) can be used for the task of quantum steering.

R. Uola et al., PRL 113, 160403 (2014).

MEASUREMENTS IN OPEN QUANTUM SYSTEMS

The dynamics an open quantum system can be described with a family of CPT maps



$$\Lambda_t(\rho) = \mathrm{tr}_E[U_t\rho \otimes \rho_E U_t^\dagger]$$

A completely positive unital Heisenberg picture evolution of the measurements of the system.

$$\mathrm{tr}[\rho\Lambda_t^H(F_j)] = \mathrm{tr}[\Lambda_t[\rho]F_j]$$

MEASUREMENTS IN OPEN QUANTUM SYSTEMS

We want to study the incompatibility of measurements under the influence of an environment:

$$\mathcal{I}(\Lambda_t(E), \Lambda_t(F)).$$

$\mathcal{I}$ is a monotone under CPT maps:

$$\mathcal{I}(\Lambda_t(E), \Lambda_t(F)) \leq \mathcal{I}(E, F)$$

For a divisible family of maps the incompatibility can never increase.

QUBIT

Binary POVMs for a qubit can be written in terms of its Bloch four-vector $\mathbf{x} = (x^0, \vec{x})$

$$F_1 = \frac{1}{2}(x^0 \mathbb{I} + \vec{x} \cdot \vec{\sigma}) \qquad F_0 = \mathbb{I} - F_1$$

Two binary measurements are compatible, iff

$$C(\mathbf{x}, \mathbf{y}) \geq 0$$

$$\begin{aligned} C(\mathbf{x}, \mathbf{y}) \quad &= \quad \left[\langle \mathbf{x}|\mathbf{x}\rangle\langle \mathbf{x}^\perp|\mathbf{x}^\perp\rangle\langle \mathbf{y}|\mathbf{y}\rangle\langle \mathbf{y}^\perp|\mathbf{y}^\perp\rangle\right]^{1/2} - \langle \mathbf{x}|\mathbf{x}^\perp\rangle\langle \mathbf{y}|\mathbf{y}^\perp\rangle \\ &\quad + \langle \mathbf{x}|\mathbf{y}^\perp\rangle\langle \mathbf{x}^\perp|\mathbf{y}\rangle + \langle \mathbf{x}|\mathbf{y}\rangle\langle \mathbf{x}^\perp|\mathbf{y}^\perp\rangle \end{aligned}$$

$$\langle \mathbf{x}|\mathbf{y}\rangle := x^0 y^0 - \textstyle\sum_{i=1}^{3} x^i y^i$$

QUBIT

$$\mathcal{I}(x, y) := \inf\{\lambda > 0 \mid \mathsf{C}\,(N_\lambda(\mathbf{x}),\ N_\lambda(\mathbf{y})) \geq 0\}$$

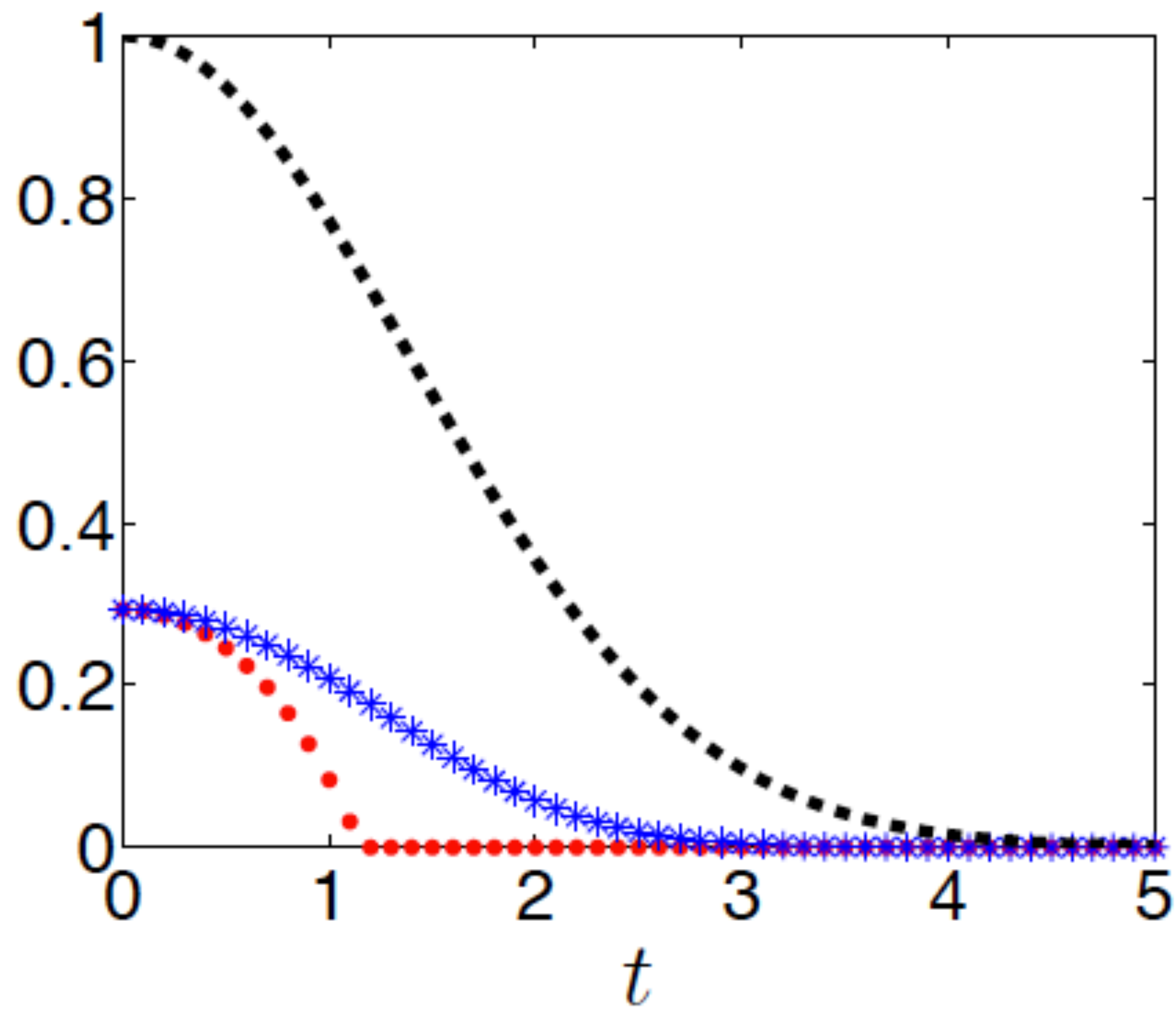
$$N_\lambda(x) := ((1-\lambda)x^0 + \lambda, (1-\lambda)\vec{x})$$

Pure dephasing:

$$\rho_t = \begin{pmatrix} \rho_{HH} & \kappa(t)\rho_{HV} \\ \kappa^*(t)\rho_{VH} & \rho_{VV} \end{pmatrix} \qquad \kappa(t) = \int d\omega |f(\omega)|^2 e^{i\omega\Delta n t}$$

B.-H. Liu et al., Nat Phys. 7, 931 (2011).

SUDDEN DEATH OF INCOMPATIBILITY



concurrence

$x_0 = y_0 = 1,$
$\vec{x} = (1,0,0),$
$\vec{y} = (0,0,1)$

$x_0 = y_0 = 1,$
$\vec{x} = (1,0,0),$
$\vec{y} = (0,1,0)$

SUDDEN DEATH OF INCOMPATIBILITY

Amplitude damping:

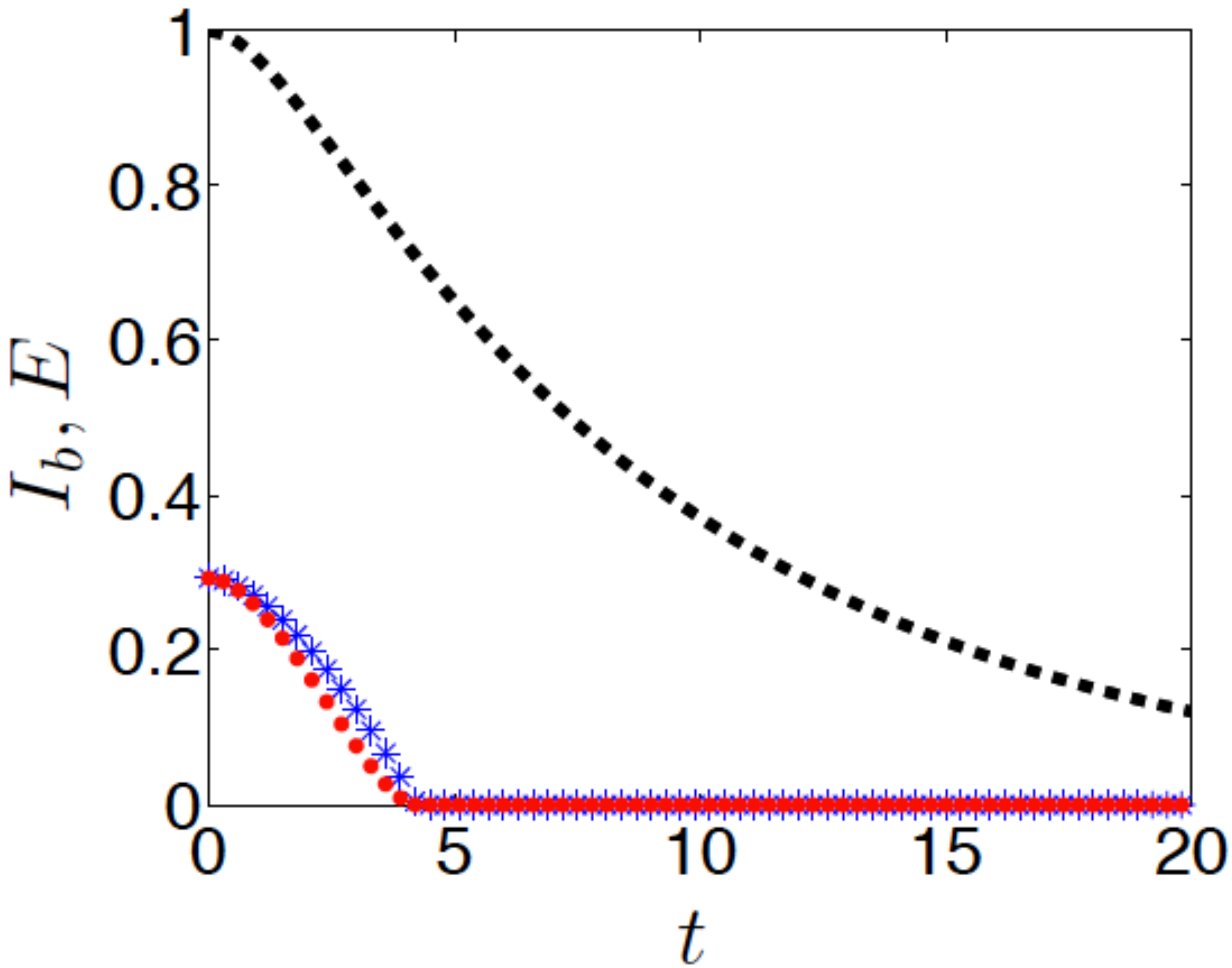
Form given by
the spectral density

$$\rho_t = \begin{pmatrix} |G(t)|^2\rho_{11} & G(t)\rho_{12} \\ G^*(t)\rho_{21} & \rho_{22} + (1 - |G(t)|^2)\rho_{11} \end{pmatrix}$$

A two-state system interacting with a bosonic quantum reservoir at zero temperature.

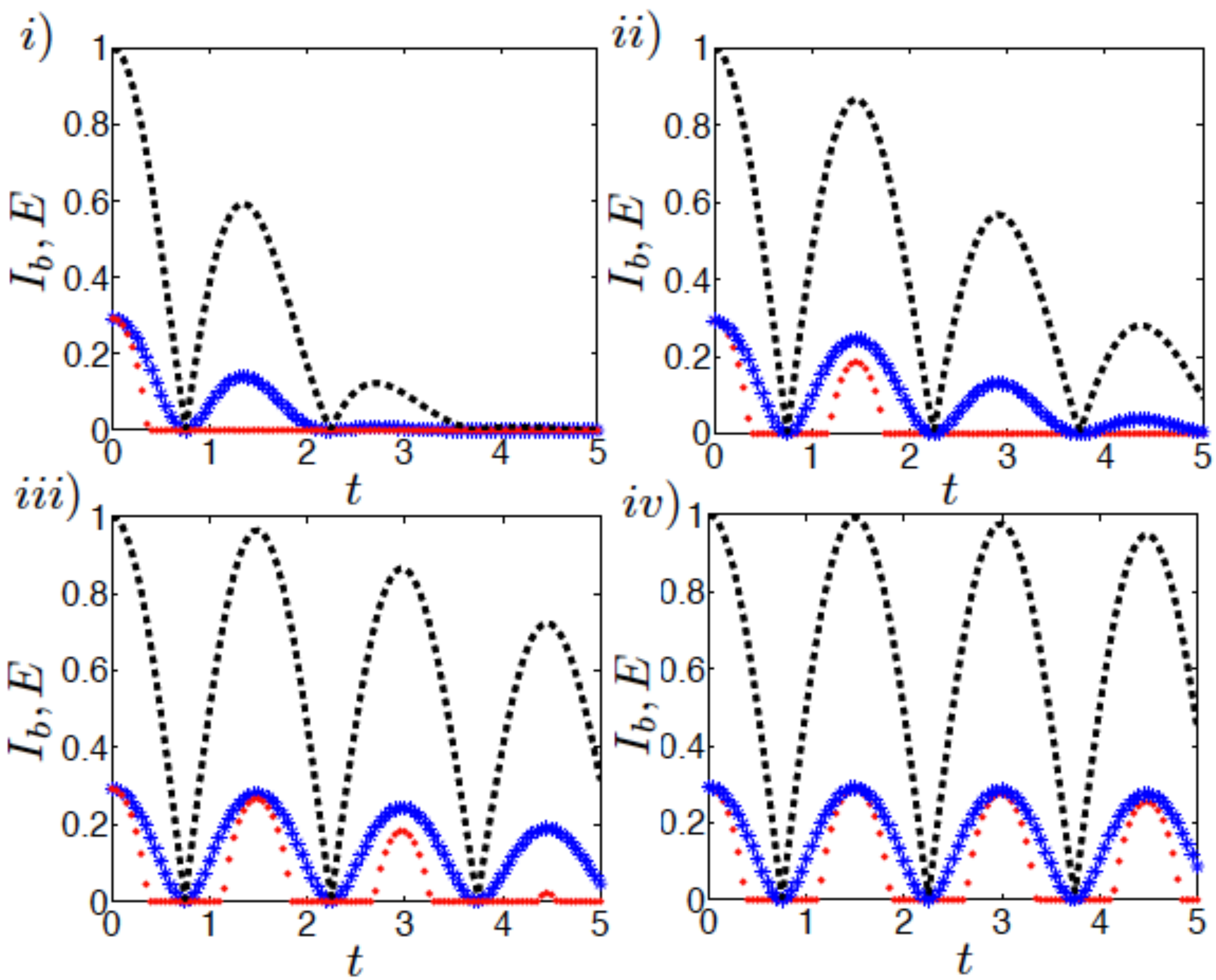
SUDDEN DEATH OF INCOMPATIBILITY

Lorentzian



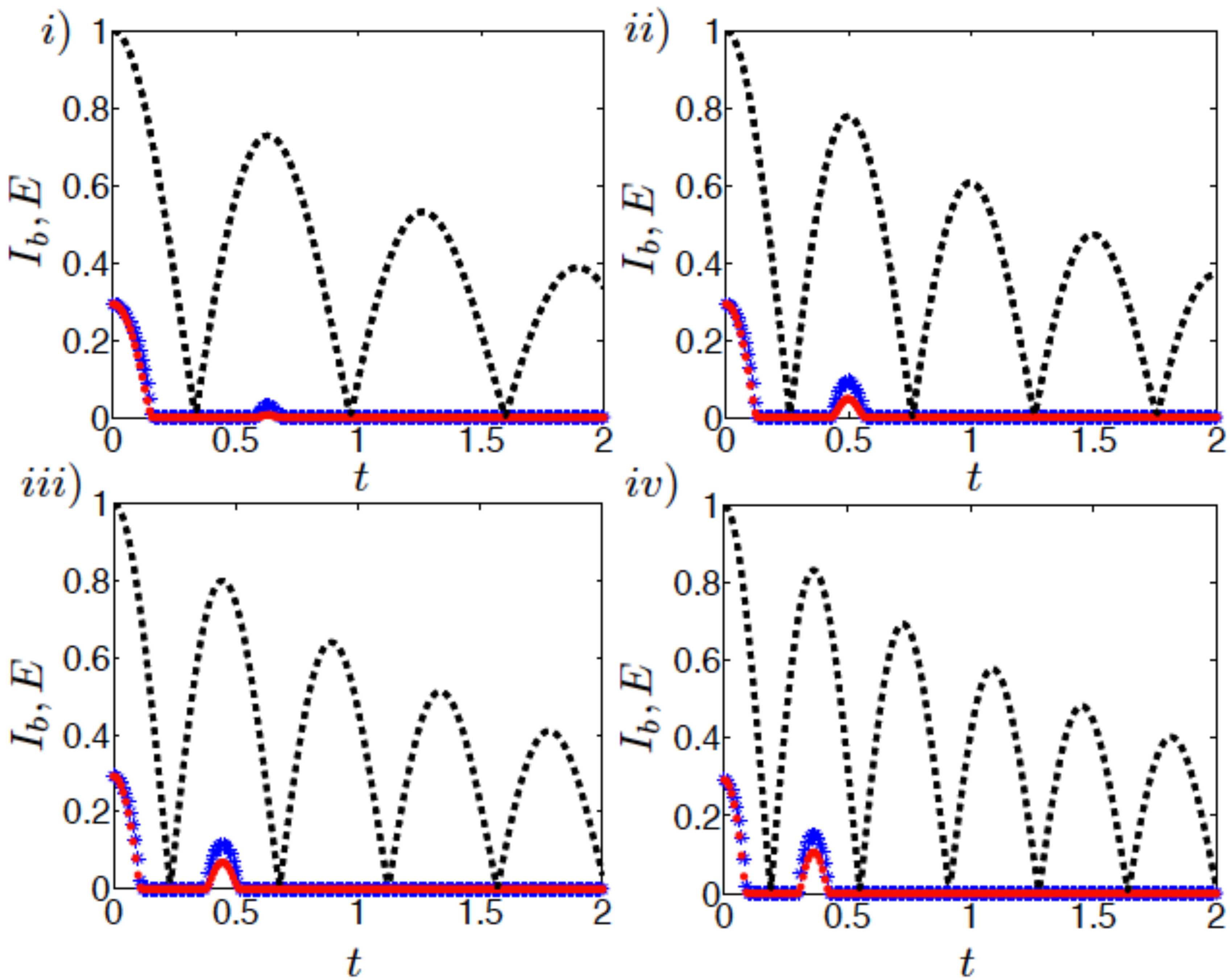
CAN MEMORY EFFECTS IMPROVE THE SITUATION?

Pure dephasing:



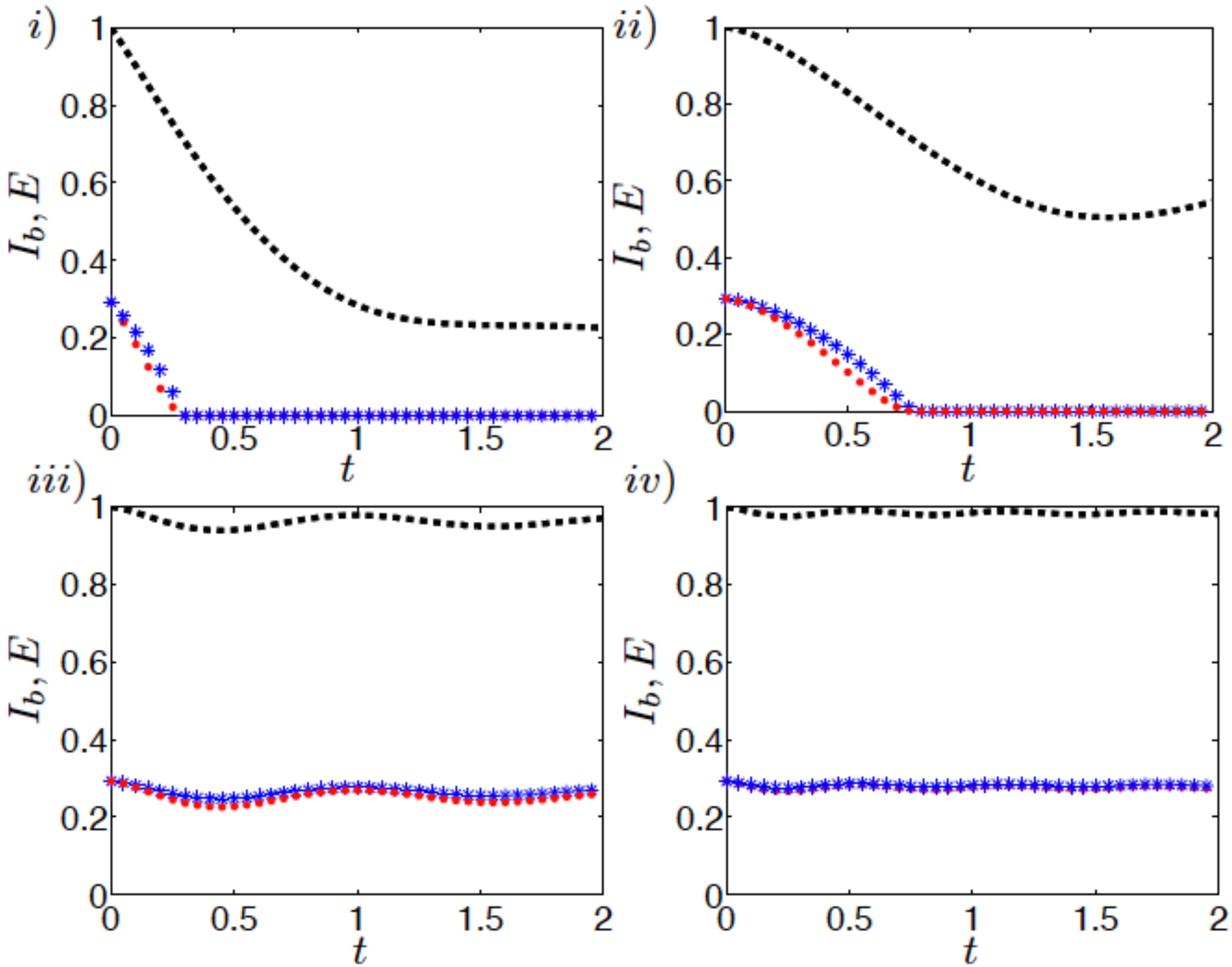
CAN MEMORY EFFECTS IMPROVE THE SITUATION?

Lorentzian:



CAN MEMORY EFFECTS IMPROVE THE SITUATION?

PBG:



CONCLUSIONS

- HOW EASILY DO MEASUREMENTS BECOME COMPATIBLE?
    - Incompatibility is more fragile than entanglement.
    - Quantum protocols requiring incompatible measurements are very sensitive to noise.