Quantum synchronization as a local signature of super and sub-radiance

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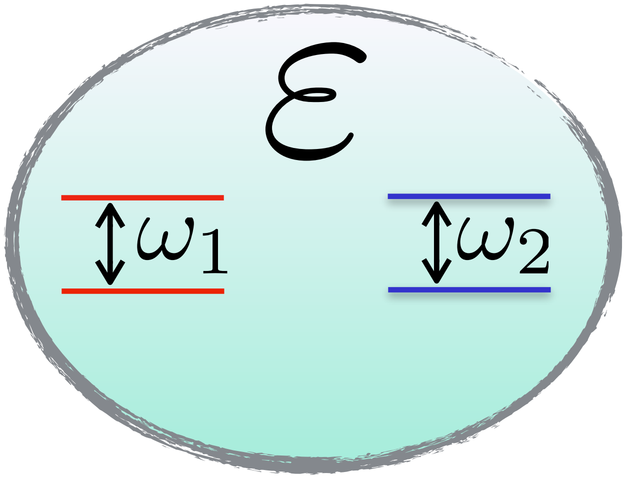
B Bellomo, GL Giorgi, G M Palma, R Zambrini, arXiv:1612.07134 and PRA (in press)

- The simplest case: two atoms in the vacuum
- Liouvillian formalism
- Quantum synchronization induced by a common environment
- Super- and subradiance
- Results

Two qubits in a (partially) common environment



$$H_S = \frac{\omega_1}{2}\sigma_1^z + \frac{\omega_2}{2}\sigma_2^z$$

$$H_I = \sum_i \sigma_i^x \sum_k g_k^i (a_k + a_k^\dagger)$$

Lindblad master equation

$$\dot{\rho} = -i\left[H_S + H_{LS}, \rho\right] + \sum_{ij} \gamma_{ij} \left(\sigma_i^- \rho \sigma_j^+ - \frac{1}{2}\left\{\sigma_j^+ \sigma_i^-, \rho\right\} \right)$$

- Map the density matrix into a 16-entry vector

$$\rho = \sum_{i,j=1}^{4} \rho_{ij} |i\rangle\langle j| \xrightarrow{HS} |\rho\rangle\rangle = \sum_{i,j=1}^{4} \rho_{ij} |ij\rangle\rangle$$

- Equations of motion

$$|\dot{\rho}_t\rangle\rangle = \mathcal{L}|\rho_t\rangle\rangle$$

- Born-Markov limit: the Liouvillian is block-diagonal

$$\mathcal{L} = \bigoplus_{\mu=1}^{5} \mathcal{L}_\mu$$

- This allows for a simple spectral analysis

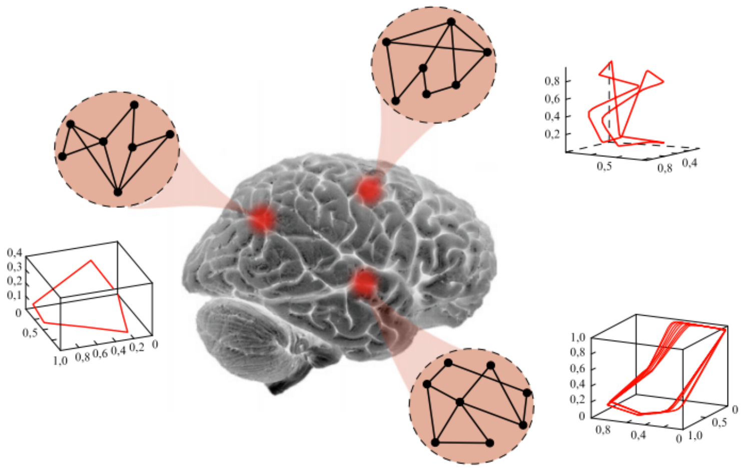
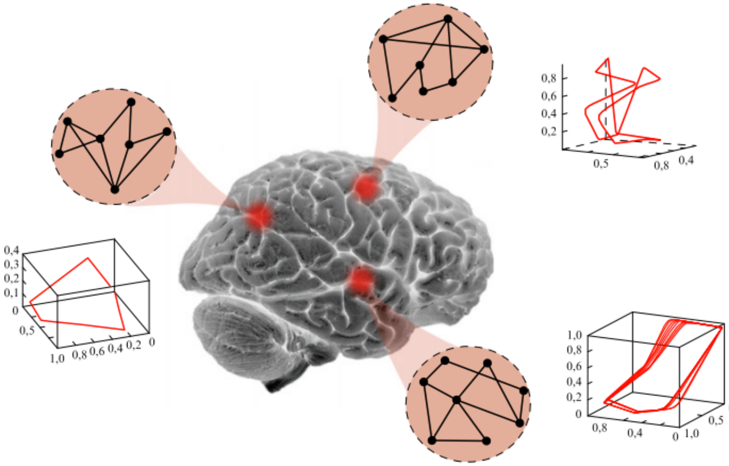


Рис. 2. Каждый отдел (подотдел) мозга, или центр — это сложная нейронная сеть с широким спектром возможного поведения, представляемого в её индивидуальном фазовом пространстве. Дальние возбуждающие и подавляющие связи между центрами объединяют разные центры и формируют коллективные моды, динамика которых представляется в объединённом фазовом пространстве.

Pikovsky, Rosenblum, Kurths, Synchronization: A Universal Concept in Nonlinear Sciences (2001)



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Pikovsky, Rosenblum, Kurths, Synchronization: A Universal Concept in Nonlinear Sciences (2001)

Previous works

- Pair of harmonic oscillators (common-bath sync)

GL Giorgi, F Galve, G Manzano, P Colet, R Zambrini, PRA 85 052101 (2012)

- Network of harmonic oscillators (common-bath sync)

G Manzano, F Galve, GL Giorgi, E Hernández-García, R Zambrini, Sci. Rep. 3, 1439 (2013)

- Pair of spins (common-bath sync, detrimental role of dephasing)

GL Giorgi, F Plastina, G. Francica, R Zambrini, PRA 88 042115 (2013)

- Pair of spins (separate baths, unbalanced dissipation rates, sync as a probe)

GL Giorgi, F Galve, R Zambrini, PRA 94 052121 (2016)

- A review on quantum synchronization measures

F Galve, GL Giorgi, R Zambrini, arXiv:1610.05060

- Synchronization can be identified with the existence of a common phase in the oscillatory dynamics of local observables (“**classical measure**”)

$$C_{A_1(t),A_2(t)}(\Delta t) = \frac{\int_t^{t+\Delta t}[A_1(t) - \bar{A}_1][A_2(t) - \bar{A}_2]dt'}{\sqrt{\prod_{i=1}^{2} \int_t^{t+\Delta t}[A_i(t) - \bar{A}_i]^2 dt'}}$$

- In such systems, synchronization is induced by the presence of a thermal environment

- In the absence of driving (relaxation towards equilibrium), synchronization is observed during a transient (pre-asymptotic limit)

- Fundamental ingredient to observe synchronization: time-scale separation in the spectrum (one eigenvalue much “slower” than anyone else)

Local observable to monitor: $\langle \sigma_i^x \rangle$

The corresponding matrix elements belong to one of the Liouvillian blocks

Eigenvalues

$$-\frac{1}{2}\left[3\gamma_0 \pm V^*\right] - i\omega_0$$

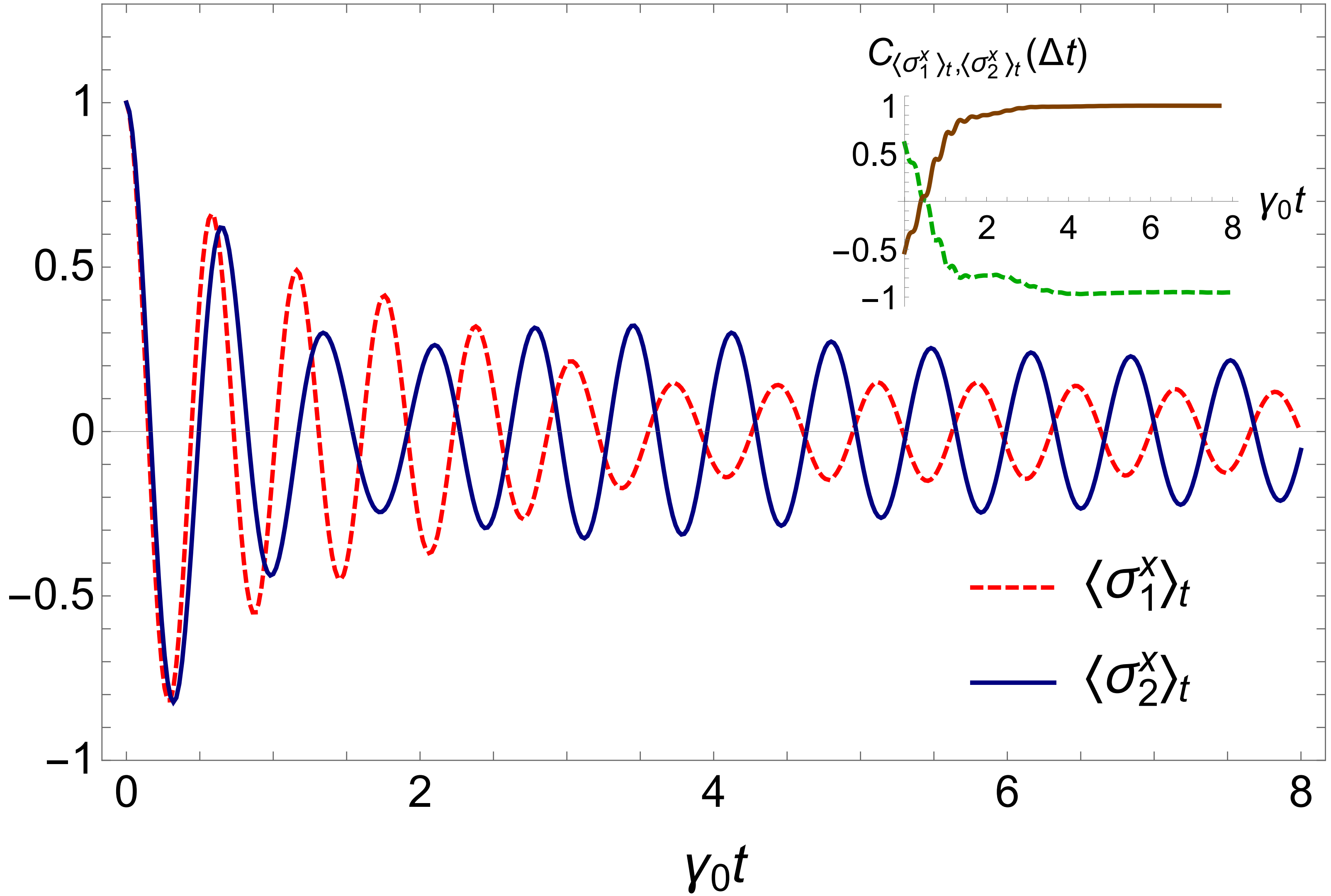
$$-\frac{1}{2}\left[\gamma_0 \pm V\right] - i\omega_0$$

Synchronization measure

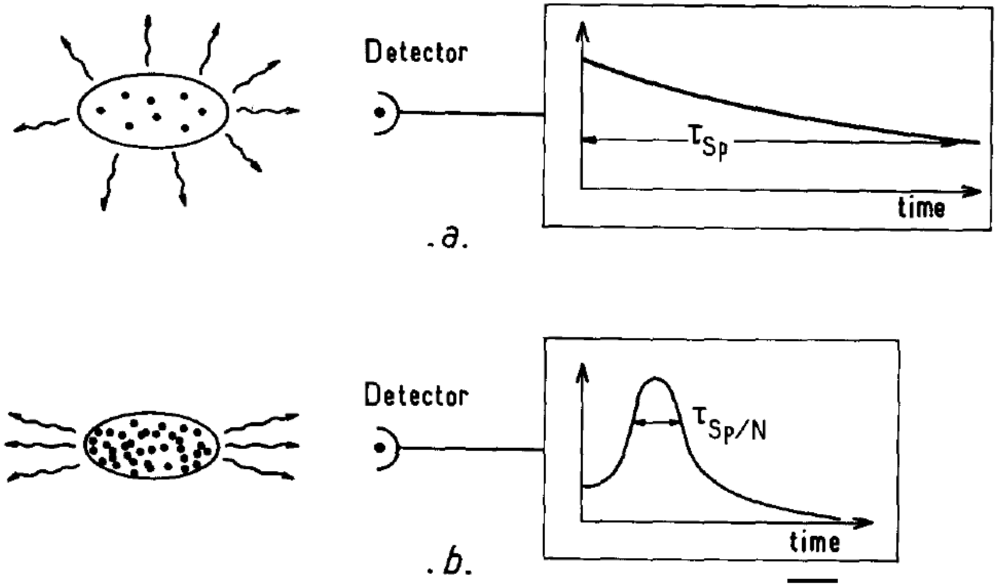
$$V = \sqrt{(\gamma_{12} + i\,2\,s_{12})^2 + \left(\frac{\gamma_1 - \gamma_2}{2} + i\delta\right)^2}$$

$\mathrm{Re}[V]$ measures the difference between the two smaller real parts and can be taken as a measure of the ability of the system to develop monochromatic oscillations

An example


$C_{\langle\sigma_1^x\rangle_t,\langle\sigma_2^x\rangle_t}(\Delta t)$
1
0.5
−0.5
−1
2
4
6
8
$\gamma_0 t$
1
0.5
0
−0.5
−1
0
2
4
6
8
$\gamma_0 t$
$\langle\sigma_1^x\rangle_t$
$\langle\sigma_2^x\rangle_t$

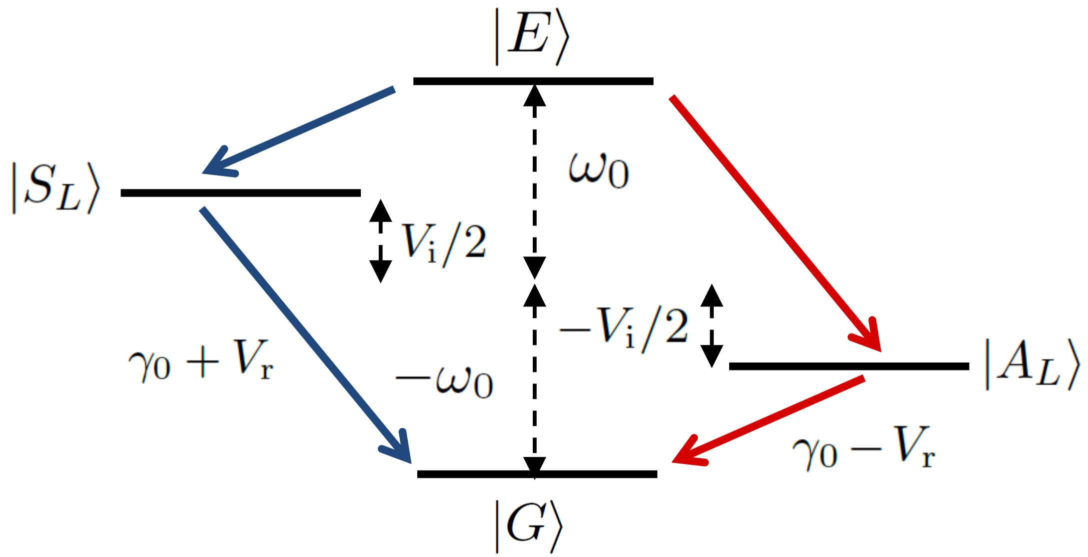
… [Dicke] superradiance is a phenomenon that occurs when a group of N emitters, such as excited atoms, interact with a common light field. If the wavelength of the light is much greater than the separation of the emitters, then the emitters interact with the light in a collective and coherent fashion



Total radiation rate

$$I(t) = \sum_{i,j} \gamma_{ij} \langle \sigma_i^+ \sigma_j^- \rangle_t$$

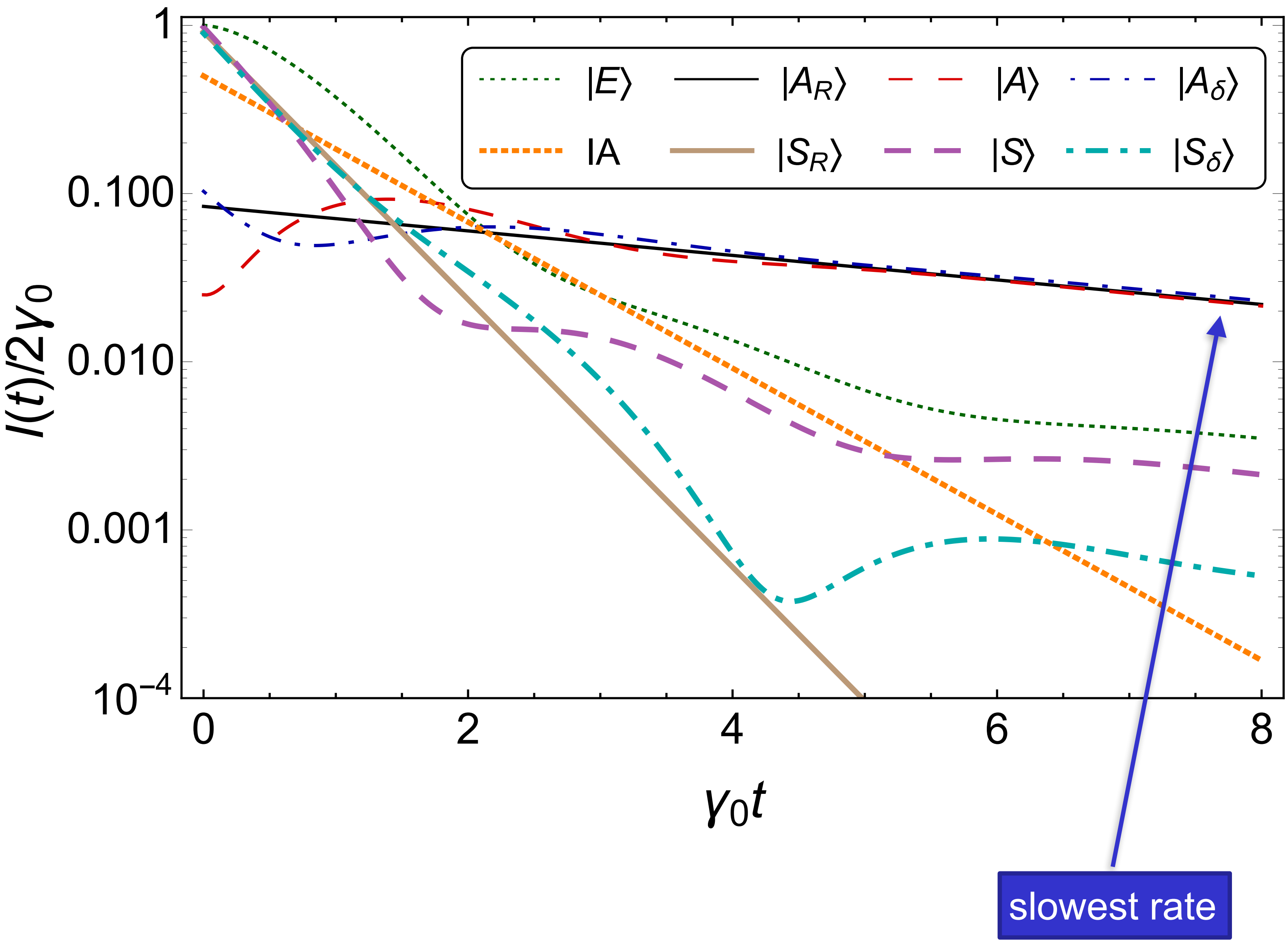
R H Dicke, Phys. Rev. 93, 99 (1954)
M Gross and S Haroche, Phys. Rep. 93, 301 (1982)



Here $|S_L\rangle$, $|A_L\rangle$ are the "evolution" of the symmetric and antisymmetric states dressed both by the interaction (Lamb shift) and by the dissipation rates. A close system of equaitions of motion can be written for these states

Eigenvalues of the Liouvillian block governing the radiation rate

$$0;\ -2\gamma_0;\ -\gamma_0 \pm i\mathrm{Im}[V];\ -\gamma_0 \pm \mathrm{Re}[V]$$



1
0.100
0.010
0.001
10^{-4}
0
2
4
6
8
$I(t)/2\gamma_0$
$\gamma_0 t$
$|E\rangle$
$|A_R\rangle$
$|A\rangle$
$|A_\delta\rangle$
IA
$|S_R\rangle$
$|S\rangle$
$|S_\delta\rangle$
slowest rate

- The slowest decay rate corresponds to the equivalent of the subradiant state (as expected)
- The synchronization-bringing mode is the coherence between such a subradiant state and the ground state
- Synchronization and subradiant are different physical manifestations of the same microscopic term appearing in the dissipative dynamics
- We have used a **classical** indicator to measure synchronization. Nevertheless, the connection to subradiance shows that it is a purely quantum effect

THANK YOU

for your attention