Local quench and probing in the Ising chain

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Outline:

1. Ising chain + defect : phase diagram

   zero modes and localized states

2. Surface Magnetization: singular behavior

   finite size scaling

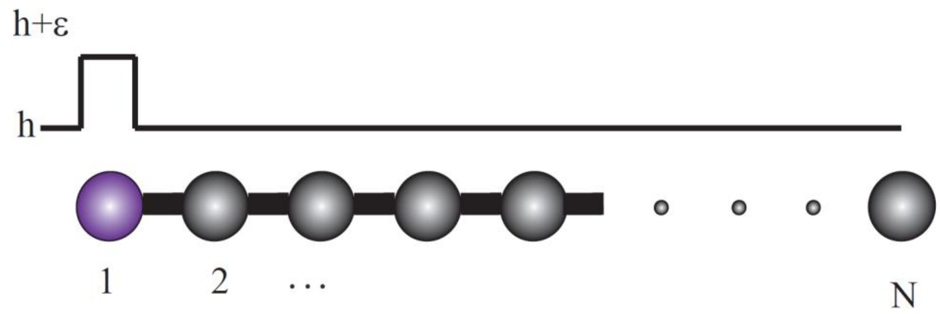
3. Local Quench: propagation of a “magnetization wave”

   the role of the Majorana mode

Ising chain + (surface) defect

Hamiltonian:

$$H = -\epsilon\hat{\sigma}_1^z - h\sum_{n=1}^{N}\hat{\sigma}_n^z - \sum_{n=1}^{N-1}\hat{\sigma}_n^x\hat{\sigma}_{n+1}^x$$



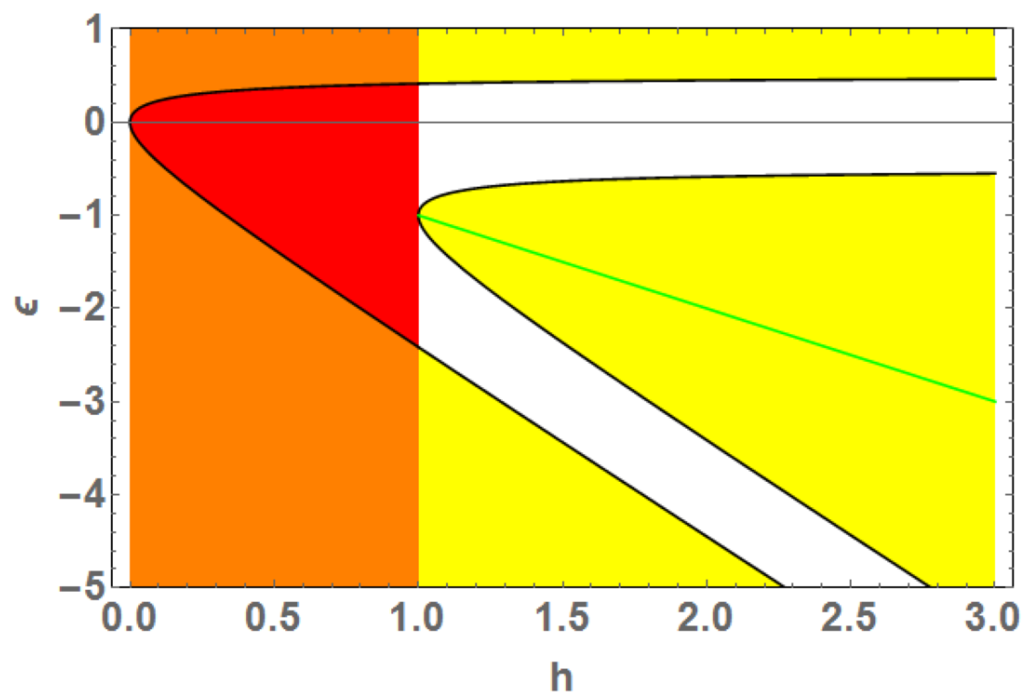
JW + Bogoliubov

$$\hat{\sigma}_n^\alpha \Rightarrow \hat{c}_n$$

$$\hat{\eta}_k = \sum_i g_{k\,i}\hat{c}_i + h_{k\,i}\hat{c}_i^\dagger$$

$$\Downarrow$$

$$H = \sum \Lambda_k \hat{\eta}_k^\dagger \hat{\eta}_k$$



some details……

Nambu-vector formalism:

$$\vec{\hat{c}} = \left(\hat{c}_1, \hat{c}_2, \ldots, \hat{c}_N, \hat{c}_1^\dagger, \hat{c}_2^\dagger, \ldots, \hat{c}_N^\dagger\right)^T$$

$$\vec{\hat{\eta}} = \left(\hat{\eta}_1, \hat{\eta}_2, \ldots, \hat{\eta}_N, \hat{\eta}_1^\dagger, \hat{\eta}_2^\dagger, \ldots, \hat{\eta}_N^\dagger\right)^T$$

Bogoliubov transform:

$$\vec{\hat{\eta}} = U\vec{\hat{c}} \qquad U = \begin{pmatrix} g & h \\ h^* & g^* \end{pmatrix} \qquad g = \frac{1}{2}(\phi+\psi) \qquad h = \frac{1}{2}(\phi-\psi)$$

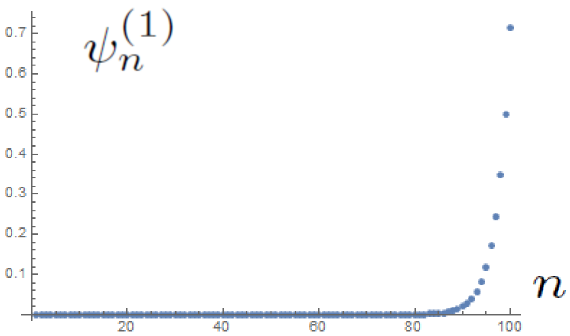
$$\begin{cases} \phi_k^T (A-B) = \Lambda_k \psi_k \\ \psi_k^T (A+B) = \Lambda_k \phi_k \end{cases}$$

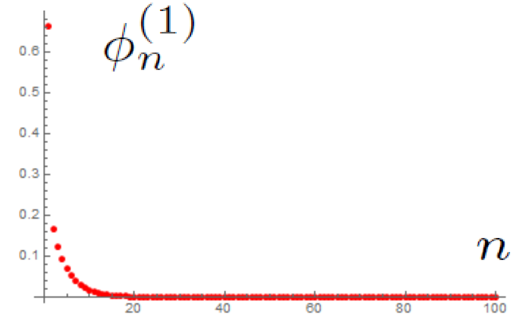
$\{h,\epsilon\}$	Λ	ψ	ϕ
$\forall\{h,\epsilon\}$	$\Lambda_\kappa = 2\sqrt{1+h^2-2h\cos\theta_\kappa}$	$\psi_n(\theta_k) = \sqrt{\frac{2}{\pi}}\frac{h\sin(n\theta_k)+\epsilon(2h+\epsilon)\sin((n-1)\theta_k)}{\sqrt{h^2+\epsilon^2(2h+\epsilon)^2+2h\epsilon(2h+\epsilon)\cos\theta_k}}$	$\phi_n(\theta_k) = \frac{2h}{\Lambda(\theta_k)}\psi_n(\theta_k) + \frac{2\epsilon\delta_{n1}}{\Lambda(\theta_k)}\psi_1(\theta_k) - \frac{2(1-\delta_{n1})}{\Lambda(\theta_k)}\psi_{n-1}(\theta_k)$
I	$\Lambda^{(1)} = \frac{2\lvert h+\epsilon\rvert(1-h^2)h^{N-1}}{\sqrt{\lvert 1+\epsilon^2+2h\epsilon\rvert}}$	$\psi_n^{(1)} = \sqrt{1-h^2}\left(h^{N-n} - \frac{(\epsilon+h)^2 h^{N+n-2}}{1+2\epsilon h+\epsilon^2}\right)$	$\phi_n^{(1)} = \frac{\sqrt{\lvert 1+2h\epsilon+\epsilon^2\rvert}\sqrt{1-h^2}}{\lvert h+\epsilon\rvert(1+2\epsilon h+\epsilon^2)}\left(1+\frac{\epsilon}{h}\left(\delta_{n1}+(1-\delta_{n1})\left(2+\frac{\epsilon}{h}\right)\right)\right)h^n$
II	$\Lambda^{(2)} = 2\lvert h+\epsilon\rvert\sqrt{\frac{1+\epsilon^2+2h\epsilon}{\epsilon(2h+\epsilon)}}$	$\psi_n^{(2)} = \frac{(-1)^n\sqrt{\left(\epsilon\left(2+\frac{\epsilon}{h}\right)\right)^2-1}}{\left(\epsilon\left(2+\frac{\epsilon}{h}\right)\right)^n}$	$\phi_n^{(2)} = \frac{2h}{\Lambda^{(2)}}\psi_n^{(2)} + \frac{2\epsilon\delta_{n1}}{\Lambda^{(2)}}\psi_1^{(2)} - 2\frac{1-\delta_{n1}}{\Lambda^{(2)}}\psi_{n-1}^{(2)}$

Orange region: distorted Majorana (boundary) fermion

$$\Lambda^{(1)} = \frac{2|h+\epsilon|(1-h^2)h^{N-1}}{\sqrt{|1+\epsilon^2+2h\epsilon|}}$$

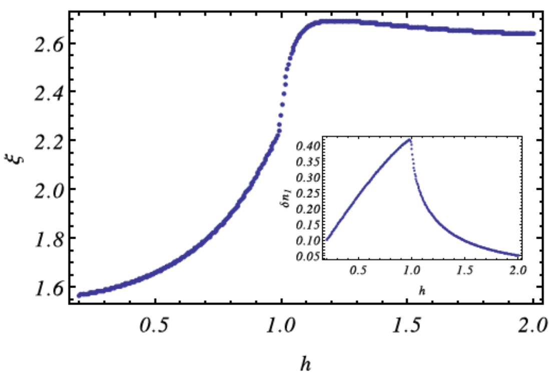
this becomes the Kitaev solution when ε → 0

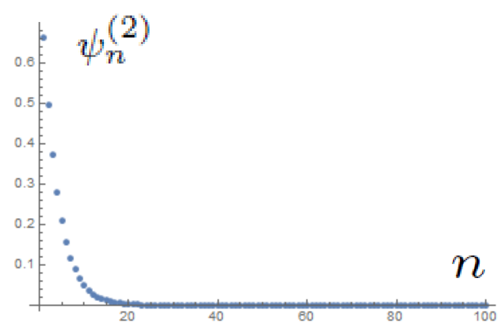


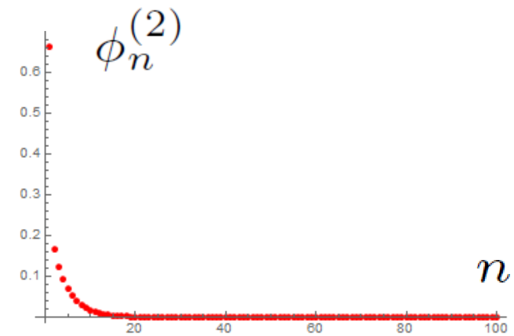


Yellow (+ red) region: localized (impurity) state

$$\Lambda^{(2)} = 2\,|h+\epsilon|\,\sqrt{\frac{1+\epsilon^2+2h\epsilon}{\epsilon(2h+\epsilon)}}$$







Green line: the edge state becomes a zero mode

Surface Magnetization

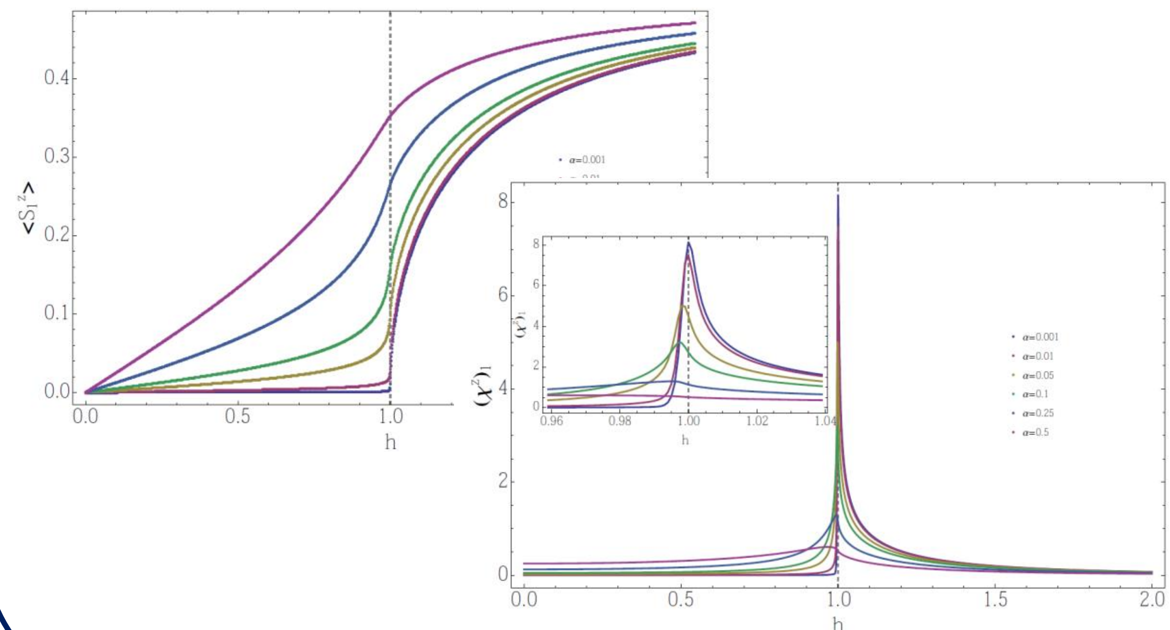
Local Magnetization:

$$\langle S_n^z \rangle = \frac{\langle \sigma_n^z \rangle}{2} = \frac{1}{2} - \langle c_n^\dagger c_n \rangle$$

Surface Magnetization detects the phase !

Surface magnetization (thermodynamics limit)

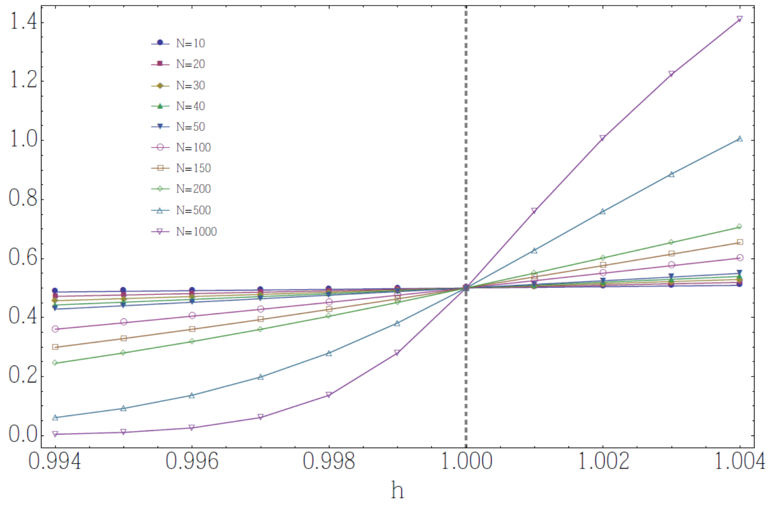
$$\lim_{\epsilon \to -h^\pm} \langle S_1^z \rangle = \mp \frac{\sqrt{h^2-1}}{2h} \Theta(h-1)$$

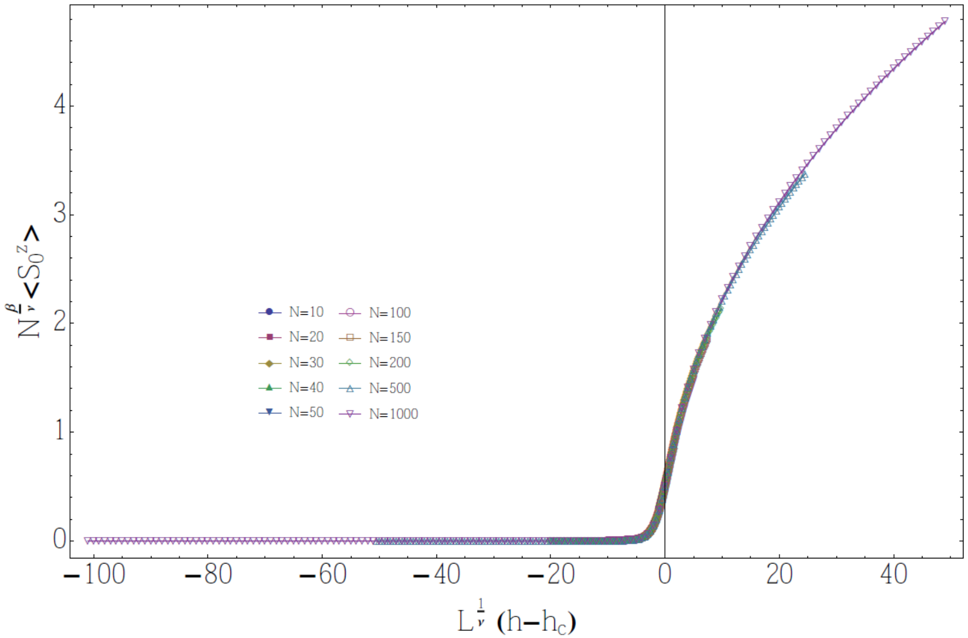


Finite size and scaling

Scaling Ansatz

$$\langle \sigma_1^z \rangle = N^{-\frac{\beta}{\nu}} f\left(\left(h - h_c\right) N^{\frac{1}{\nu}}\right)$$





Critical exponents

$$\nu = 1, \quad \beta = \frac{1}{2}$$

Universal behavior !

A special quantum quench: switching off the defect

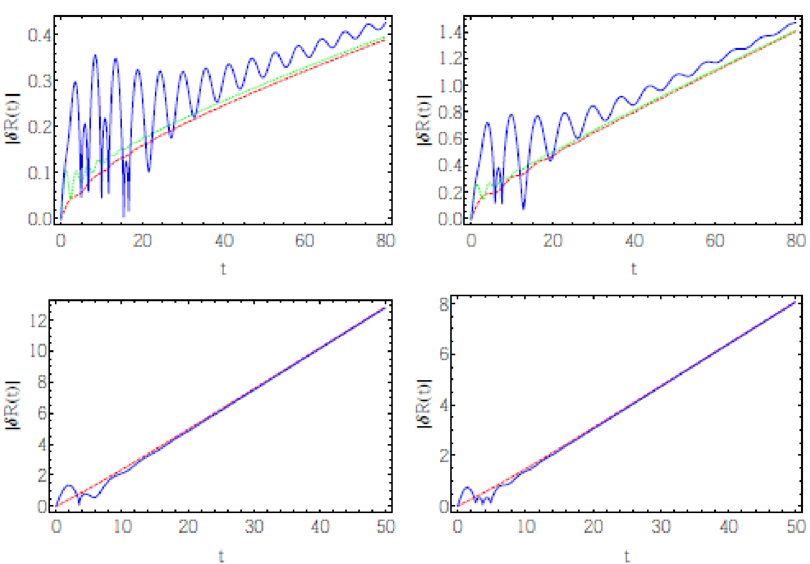
(t-dependent) local magnetization after suddenly removing the defect (ε ---> 0) :

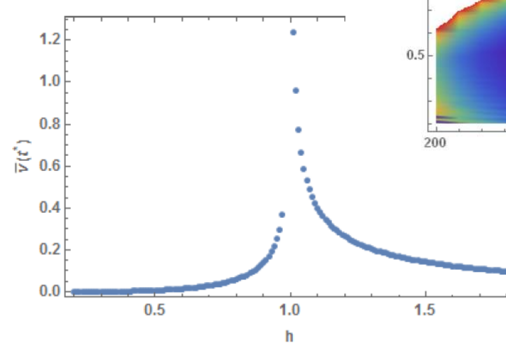
$$\delta n_i(t) \;=\; \langle c_i^\dagger(t) c_i(t)\rangle - \langle c_i^\dagger c_i\rangle_{GS'}$$

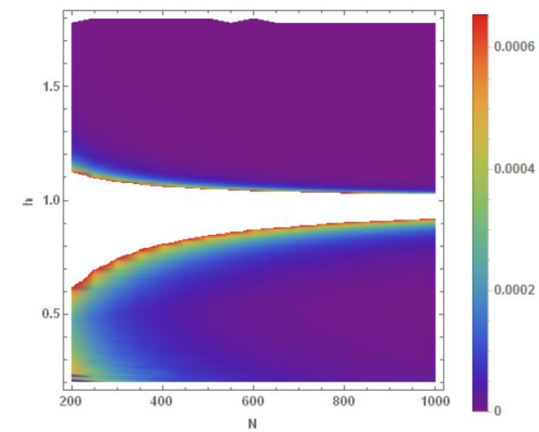
Root mean square magnetization position and speed

$$R^2(t) \;=\; \sum_{i=1}^{N} \delta n_i(t)(i-1)^2$$

$$v(t) \;=\; \frac{d}{dt}\sqrt{|R^2(t) - R^2(0)|}$$

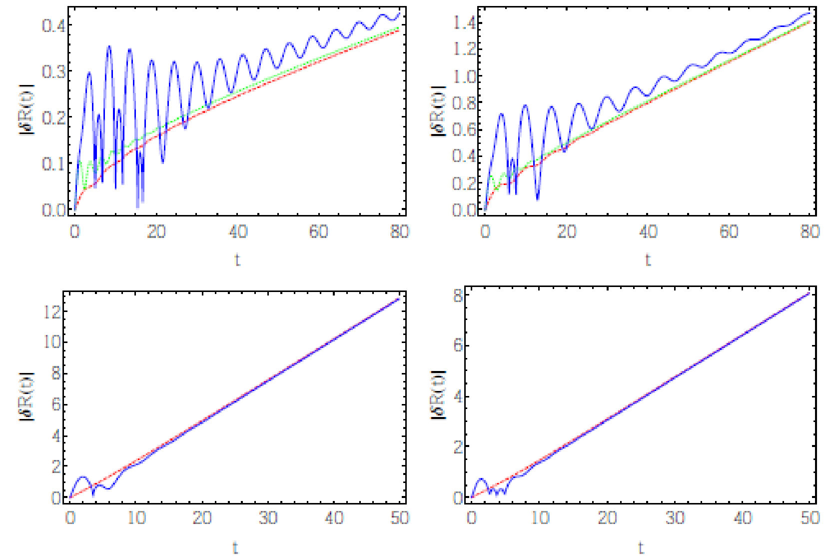




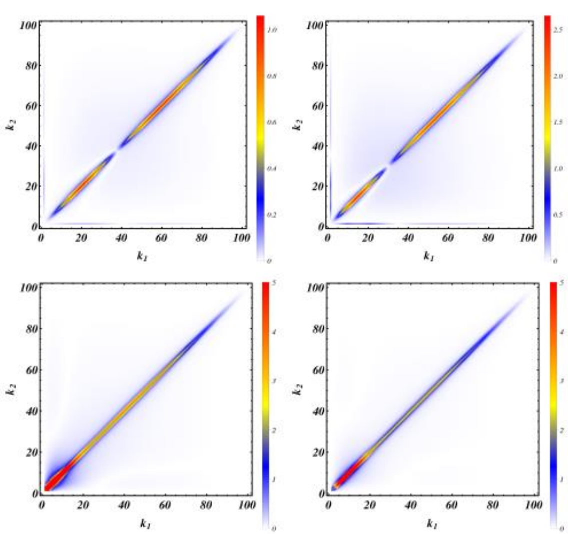


Local detection of the Majorana mode

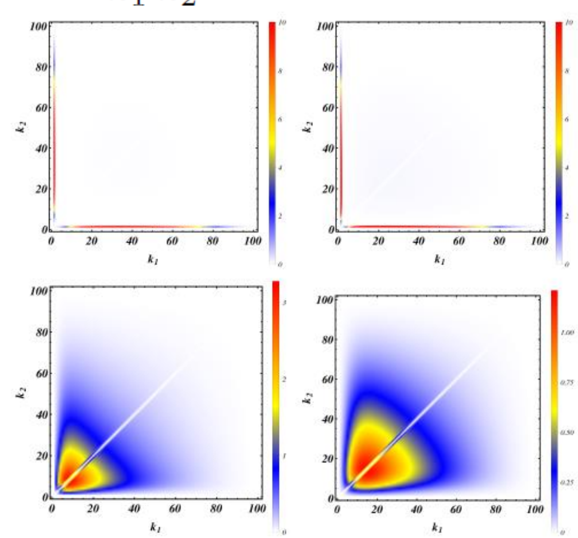
$$
\begin{aligned}
\delta n_i(t) &= \langle c_i^\dagger(t) c_i(t)\rangle - \langle c_i^\dagger c_i\rangle_{GS'} \\
&= \sum_{k_1,k_2} A^i_{k_1\,k_2} \cos\left((\Lambda'_{k_1} - \Lambda'_{k_2})t\right) \\
&+ \sum_{k_1,k_2} B^i_{k_1\,k_2} \cos\left((\Lambda'_{k_1} + \Lambda'_{k_2})t\right)
\end{aligned}
$$



$A^i_{k_1\,k_2}$



$B^i_{k_1\,k_2}$



Summary :

- ☑ Static properties of an Ising chain + local defect
- ☑ Surface Magnetization
- ☑ Disturbance propagation properties
- ☑ Role of the Majorana mode