Momentum Resolved Spectroscopy Using Atomic Quantum Probes

QuProCS

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arXiv:1511.00833

OQS&E
OPEN QUANTUM SYSTEMS & ENTANGLEMENT

Outline

- What to probe?

Outline

- What to probe?
- How is it measured so far?

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- What to probe?
- How is it measured so far?
- New method

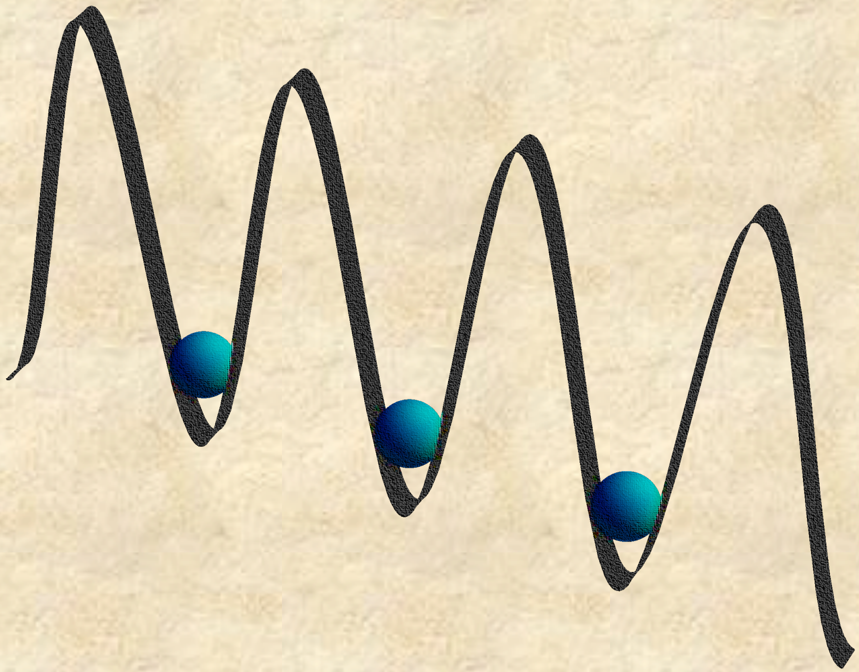
Bosons on optical lattice (1D)

Bose Hubbard

$$\hat{H} = -J \sum_{\langle i,j \rangle} \hat{c}_i^\dagger \hat{c}_j + \frac{U}{2} \sum_i \hat{c}_i^\dagger \hat{c}_i^\dagger \hat{c}_i \hat{c}_i - \mu \sum_i \hat{c}_i^\dagger \hat{c}_i$$

Superfluid Phase

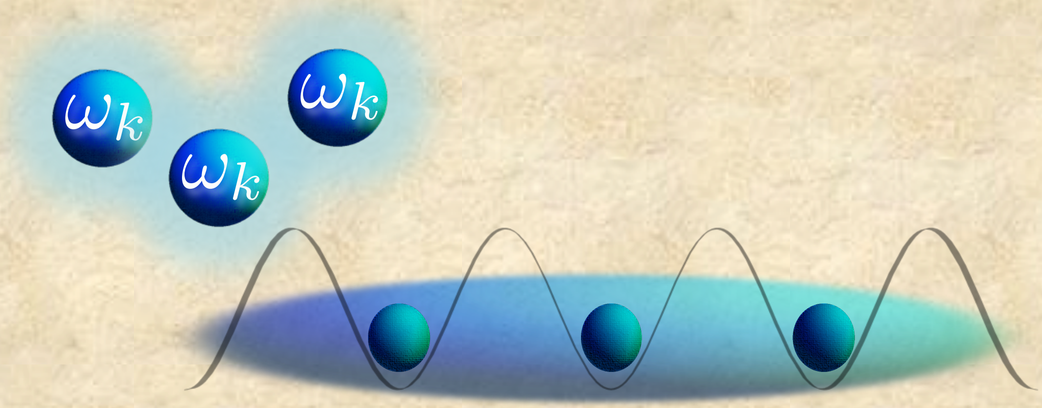
$$\hat{H} = \sum_k \omega_k \hat{b}_k^\dagger \hat{b}_k$$



Superfluid

Bogoliubov modes

$$\begin{cases} \hat{b}_{\mathbf{k}} = u_{\mathbf{k}}\hat{c}_{\mathbf{k}} - v_{\mathbf{k}}\hat{c}^{\dagger}_{-\mathbf{k}} \\ \hat{b}^{\dagger}_{\mathbf{k}} = u_{\mathbf{k}}\hat{c}^{\dagger}_{\mathbf{k}} - v_{\mathbf{k}}\hat{c}_{-\mathbf{k}} \end{cases}$$



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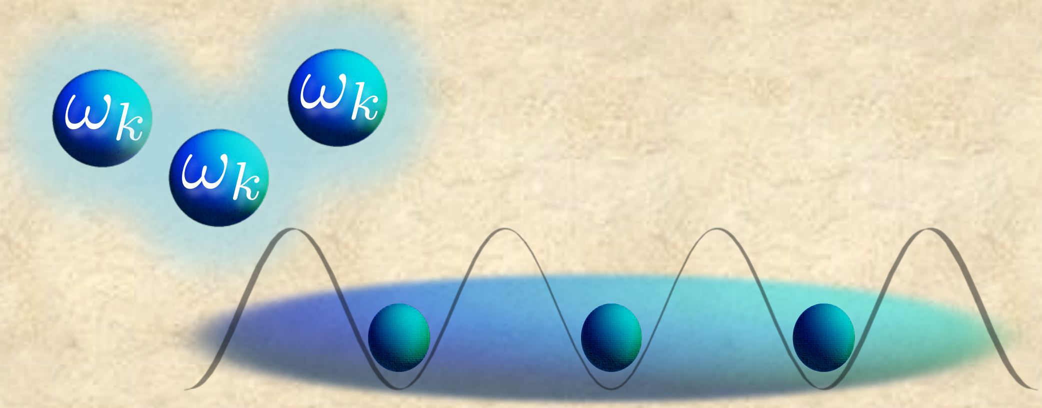
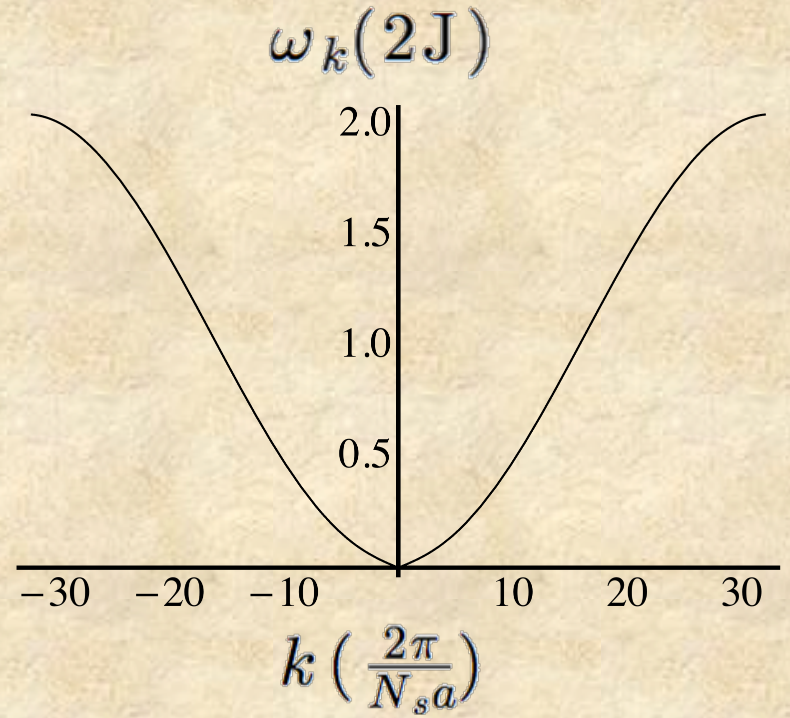


Spectrum of Excitations

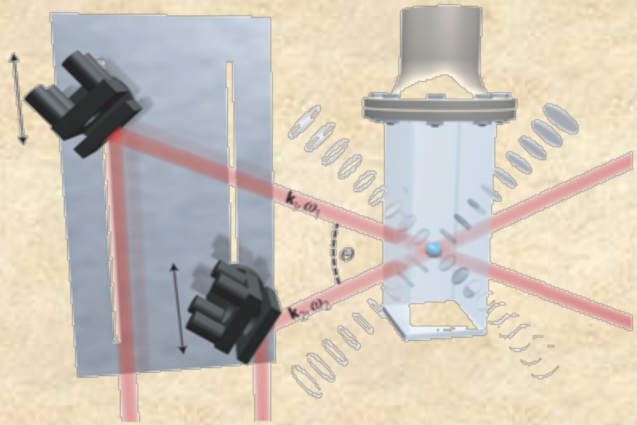
$$\omega_k = \sqrt{\epsilon_k^2 + 2Un_0\epsilon_k}$$

$$\epsilon_k = 2J(1 - \cos(ka))$$

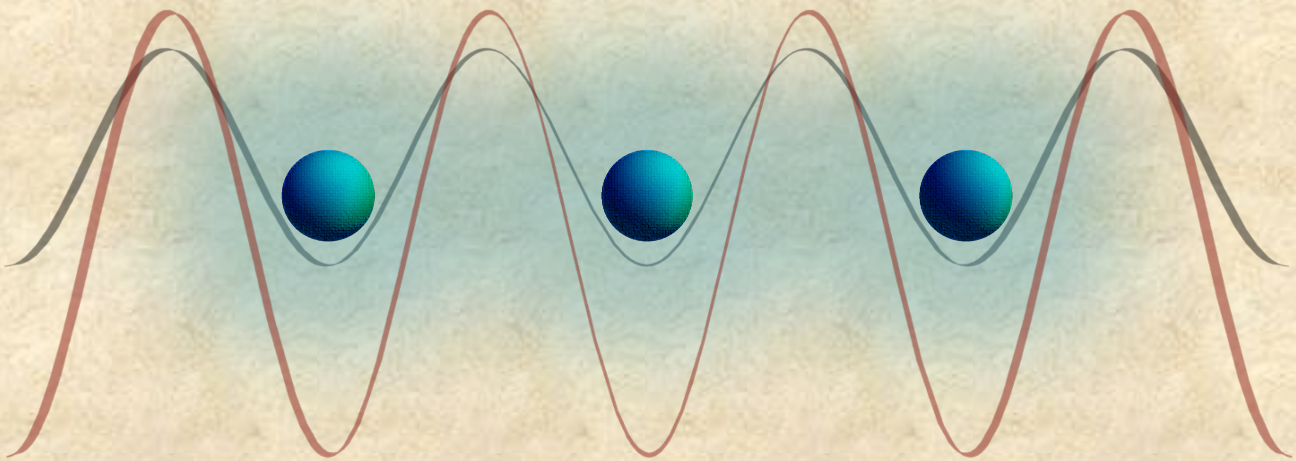


Experimental techniques

Bragg Spectroscopy[1]



Modulation of the depth of the optical lattice[2]

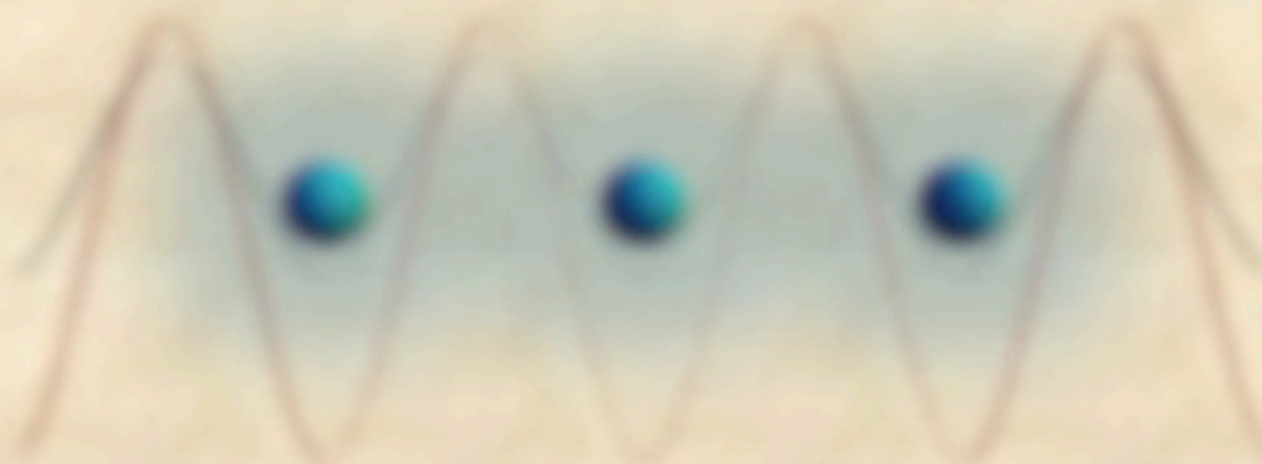


1) P. T. Ernst et al., Nature Physics **6**, 56 - 61 (2010)
2) C. Schori et al., Phys. Rev. Lett **93**, 240402 (2004)

Experimental techniques

Bragg Spectroscopy[1]

Effective but destructive!



New Probing Scheme

New Probing Scheme

<u>Less invasive</u>

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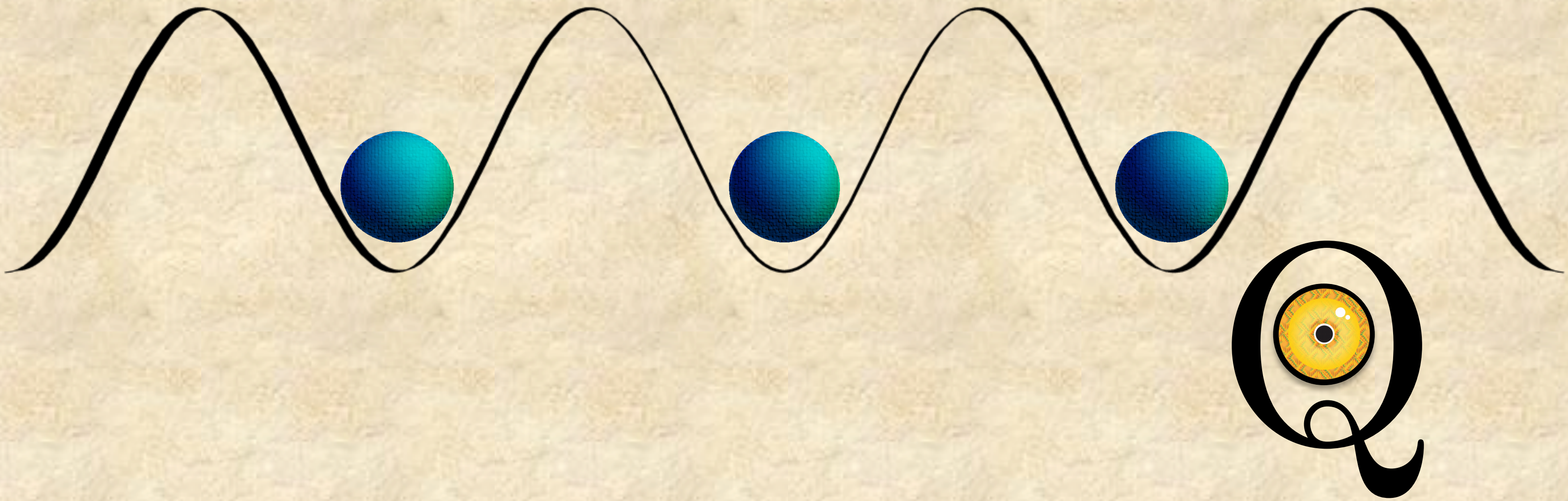
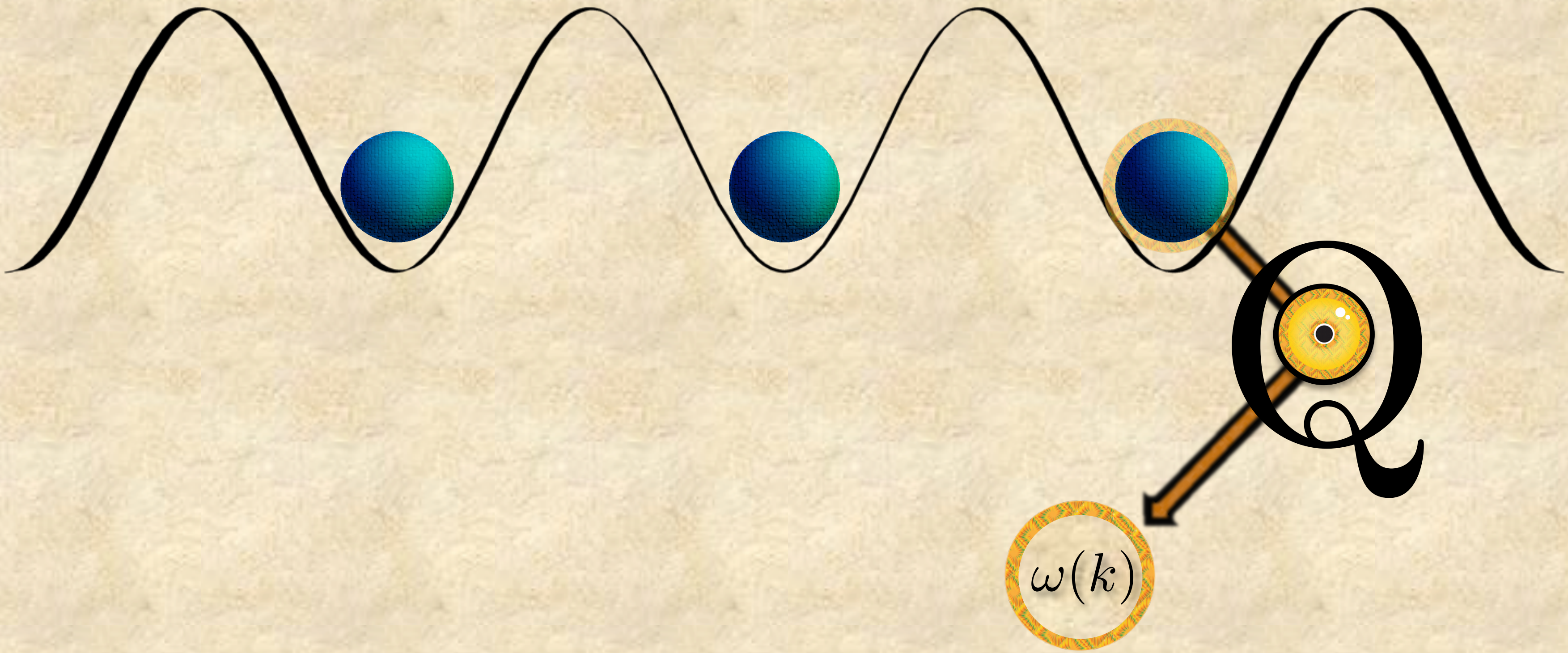
Less invasive



New Probing Scheme

Less invasive



New Probing Scheme

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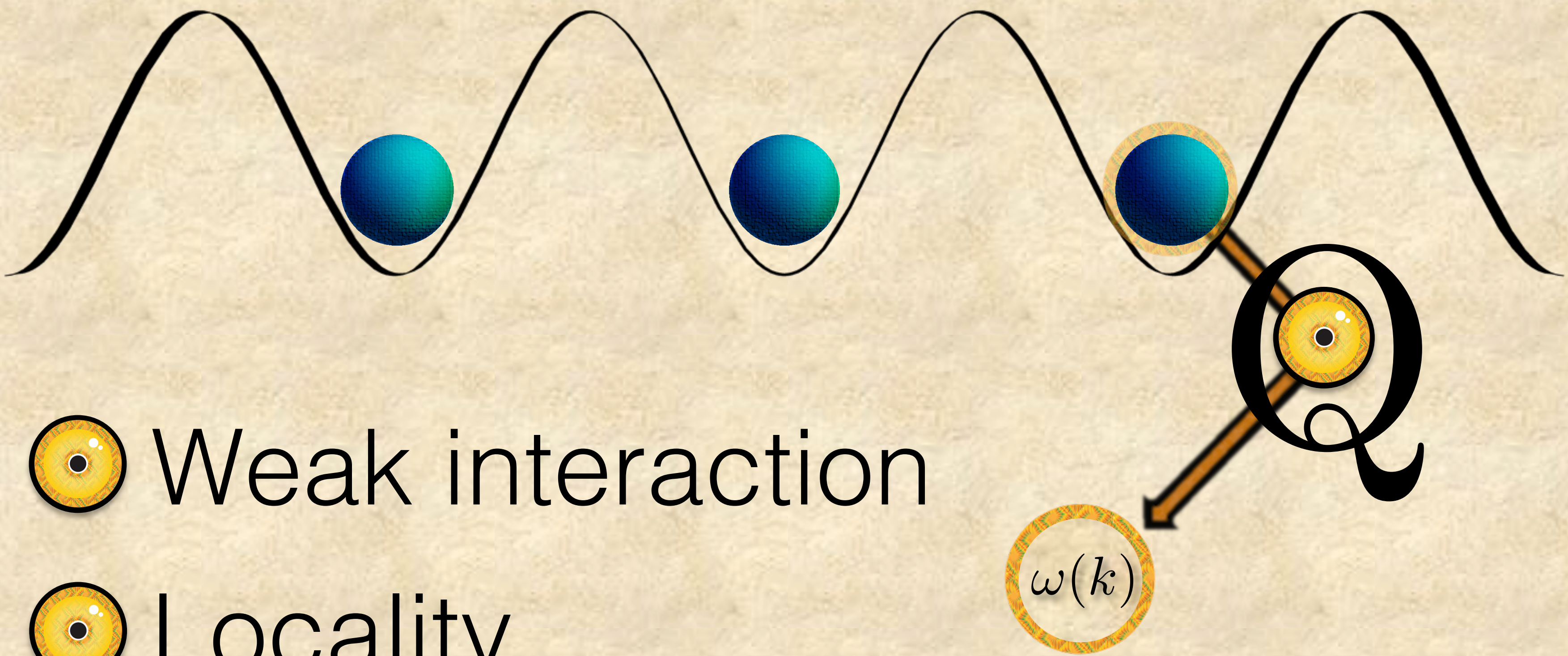

- Weak interaction
- Locality

Probe

$$\hat{H}_p = \sum_{\bar{n}} \nu_{\bar{n}} \left|\bar{n}\right\rangle \left\langle\bar{n}\right|$$

Probe

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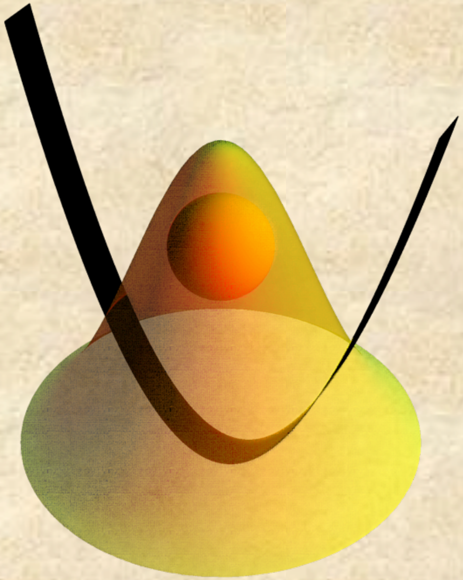
Impurity atom in a 3D Harmonic trap

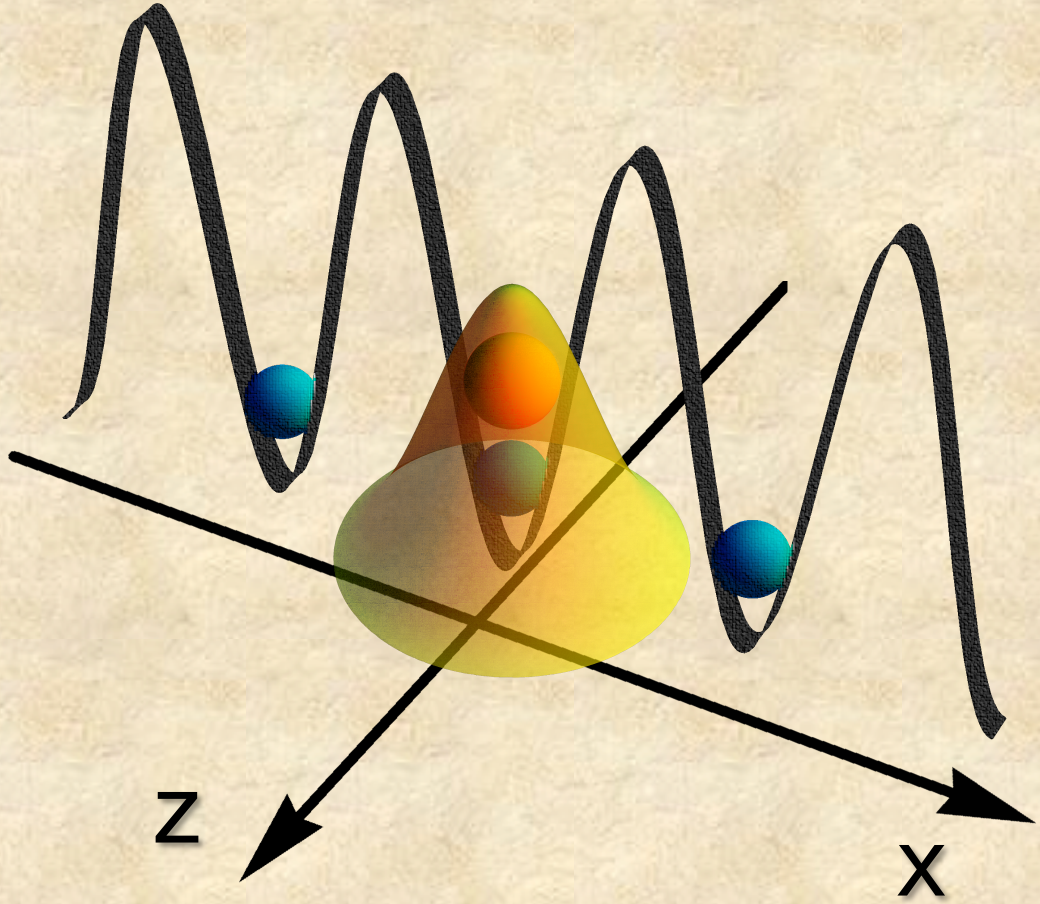
$$\psi_{\mathbf{0}}(x) \simeq e^{-\frac{x^2}{2x_0^2}} e^{-\frac{y^2}{2y_0^2}} e^{-\frac{z^2}{2z_0^2}}$$



- Controllable (ν_x, ν_y, ν_z)

Step (I): Preparation

$$\rho(0) = |\bar{0}\rangle\langle\bar{0}| \otimes \rho_\beta$$



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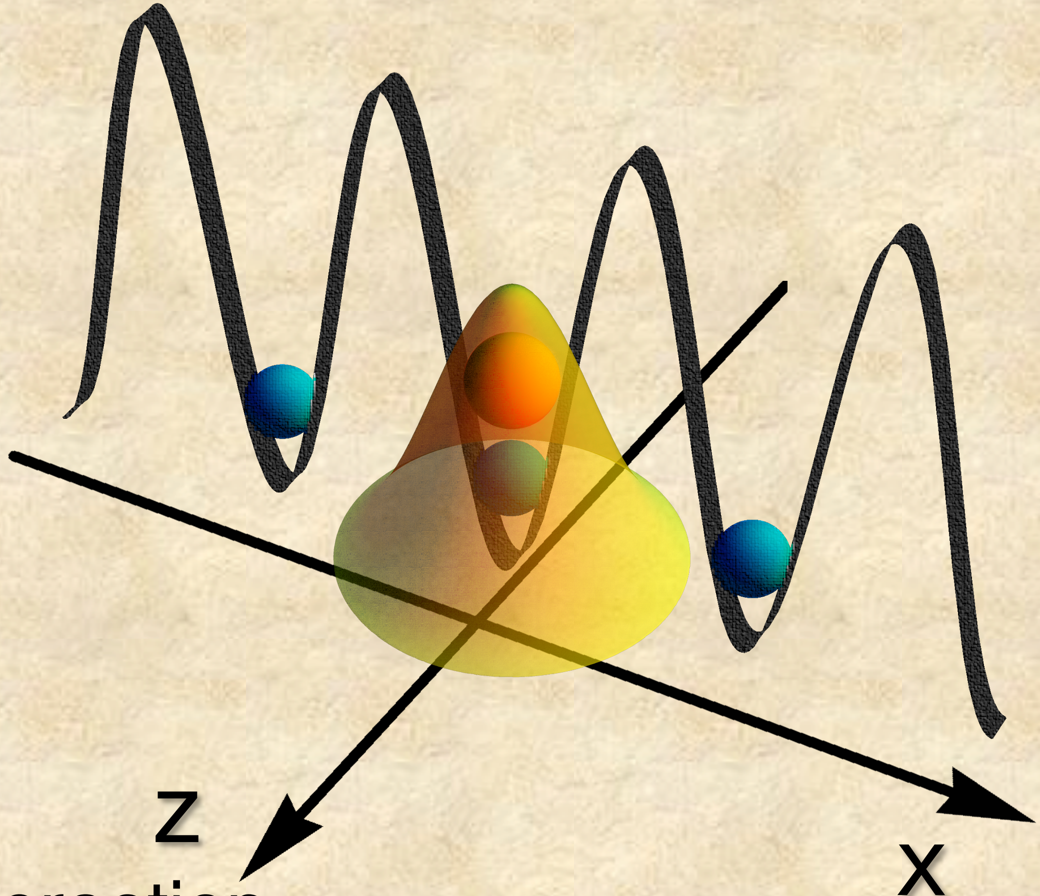

Density-density interaction

$$\hat{H}_{int} = g \sum_{\substack{\bar{n},\bar{m}\\ i,j}} \int \mathrm{d}\mathbf{x}\ \psi^*_{\bar{n}}(\mathbf{x})\psi_{\bar{m}}(\mathbf{x})\omega^*_i(x)\omega_j(x)\, |\bar{n}\rangle\langle\bar{m}| \otimes \hat{c}^\dagger_i \hat{c}_j$$

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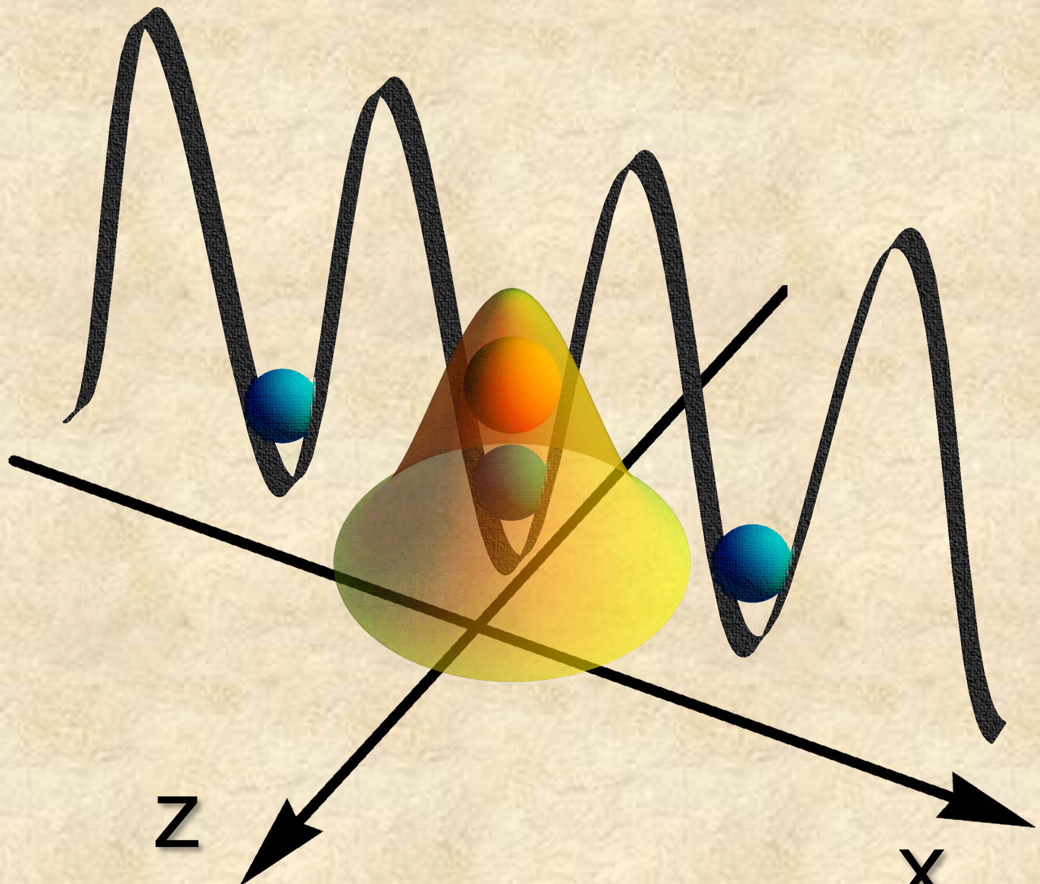

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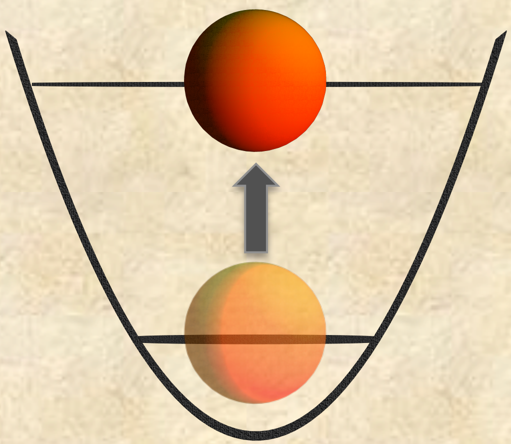
$$\hat{H}_{int} = g \sum_{\substack{\bar{n},\bar{m} \\ i,j}} \int \mathrm{d}\mathbf{x}\ \psi^*_{\bar{n}}(\mathbf{x}) \psi_{\bar{m}}(\mathbf{x}) \omega^*_i(x) \omega_j(x) \, |\bar{n}\rangle \langle\bar{m}| \otimes \hat{c}^\dagger_i \hat{c}_j$$

Superfluid

$$\hat{c}^\dagger_i \hat{c}_j \simeq n_0 + \sqrt{\frac{n_0}{N_s}} \sum_{k\neq 0} \left[\Big(u_k e^{ikx_i} + v_k e^{ikx_j} \big) \hat{b}^\dagger_k + \big(u_k e^{-ikx_j} + v_k e^{-ikx_i} \Big) \hat{b}_k \right]$$

Step (II):
Measure the transition probability

$$\Gamma^{\mathbf{I}}_{\bar{0}\to(0,0,n_z)} = \mathrm{Tr}_{\mathrm{B}} \langle 0, 0, n_z | \, \rho(t) \, | 0, 0, n_z \rangle$$



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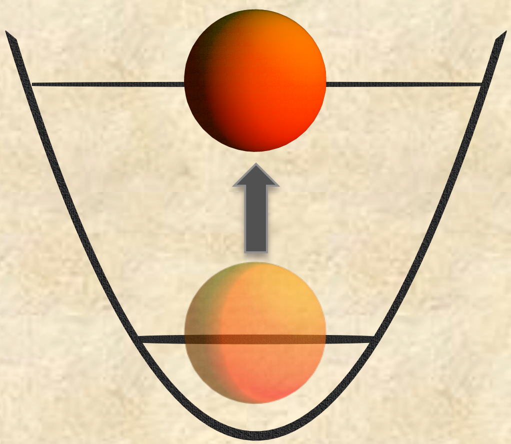

Perturbative approach

$$\hat{U}(t) = \hat{\mathbb{I}} + g\hat{U}^{(1)}(t) + g^2\hat{U}^{(2)}(t) + \dots$$

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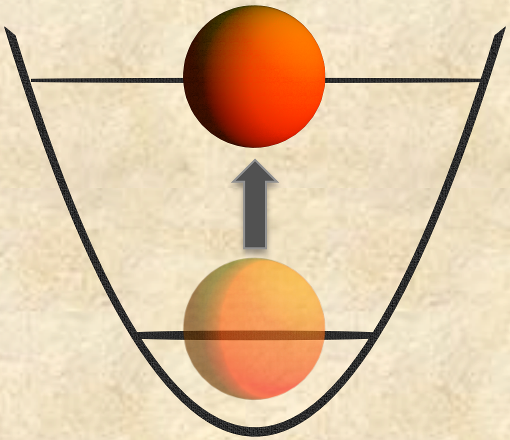

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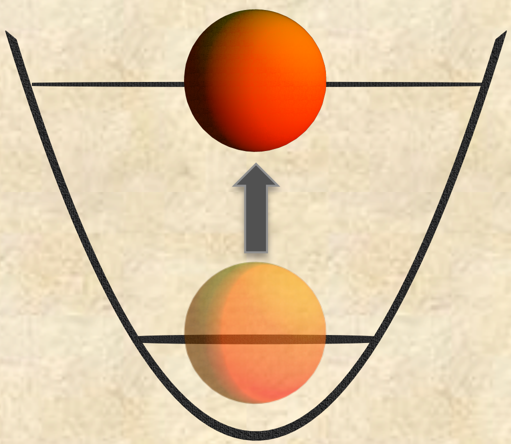

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Relevant

$$\langle 0,0,n_z|\, \hat{H}_{int}\, |\bar{0}\rangle = g\sum_{i,j}\int \mathrm{d}\mathbf{x}\; \psi^*_{(0,0,n_z)}(\mathbf{x})\psi_{(0,0,0)}(\mathbf{x})\omega_i^*(x)\omega_j(x)\hat{c}_i^\dagger\hat{c}_j$$

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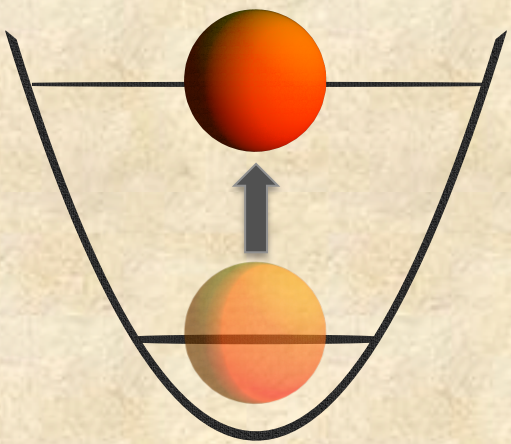

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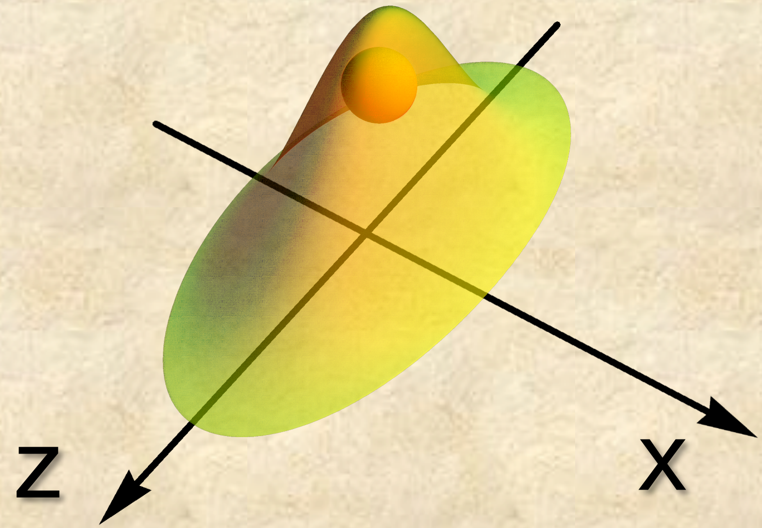
Step (III):
Vary ν_z



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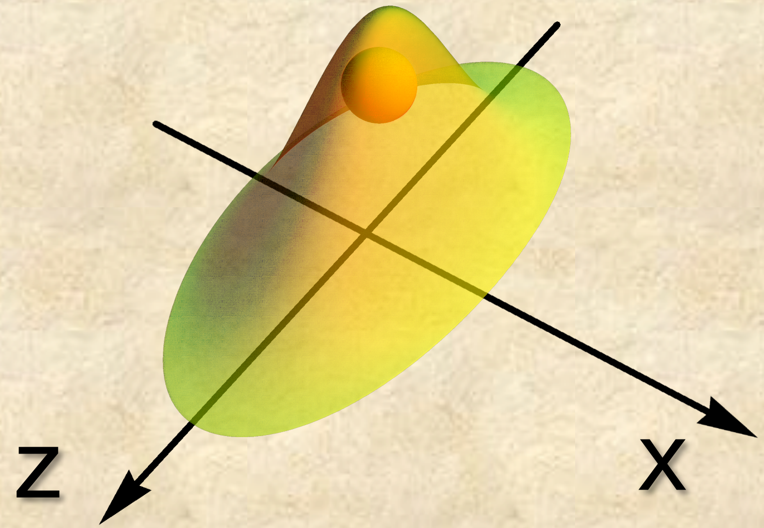
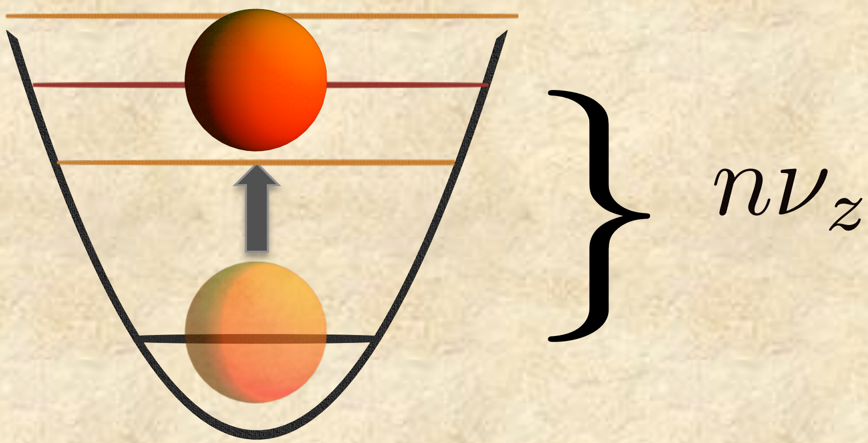


Probe Energy levels



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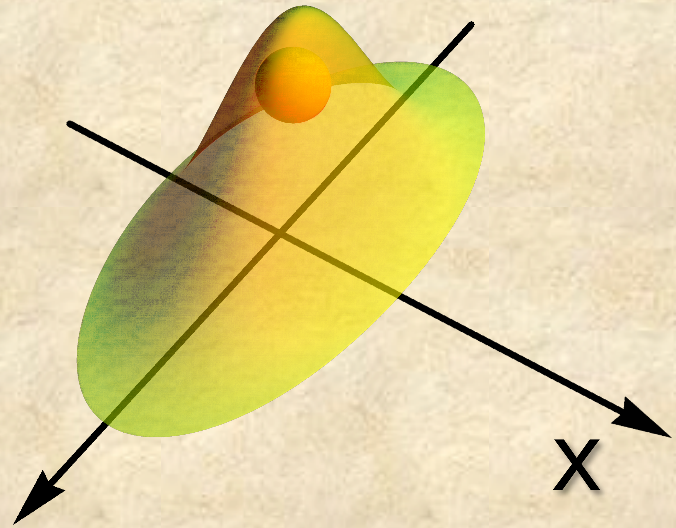

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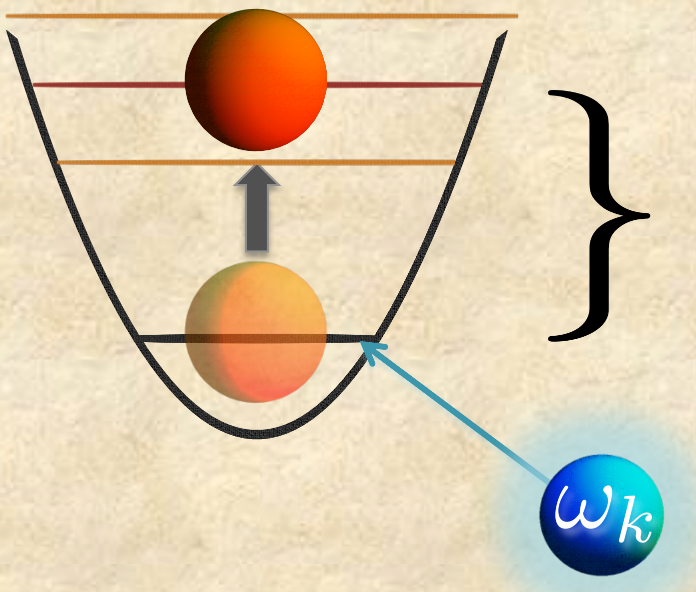
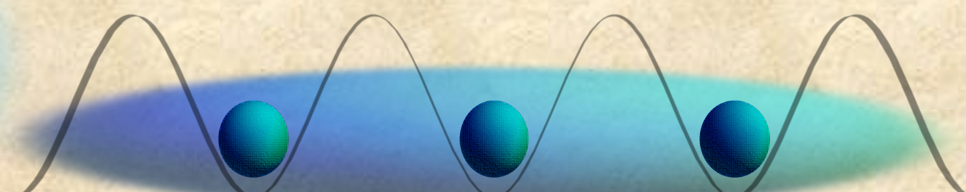


$$n\nu_z = \omega_k$$

Resonance!

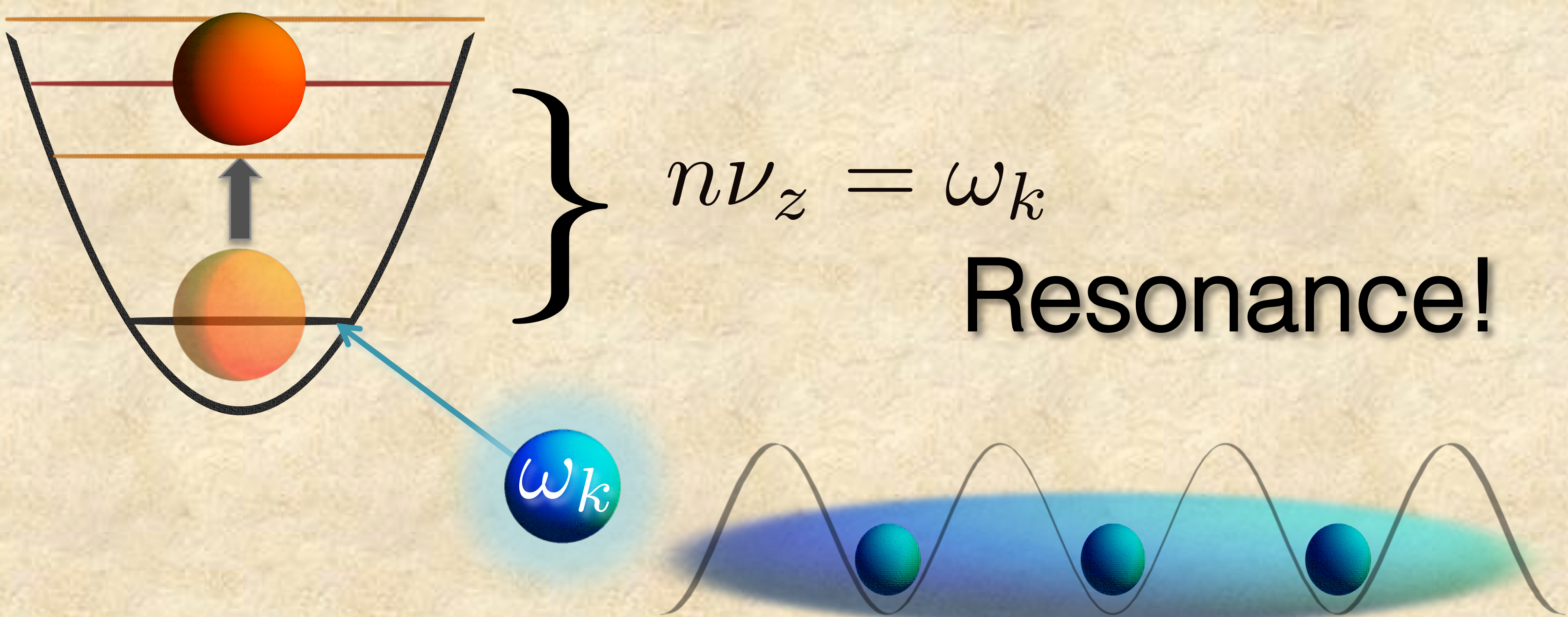


Transition Probability

$$\Gamma^{\mathbf{I}}_{0\to(0,0,n_z)} = g^2_{\mathbf{I},n}\nu\left[2n_0^2\frac{1-\cos(n\nu t)}{n^2\nu^2} + \sum_k\left(\Gamma_k^+(n\nu,t)+\Gamma_k^-(n\nu,t)\right)\right]$$

$$\Gamma_k^{\pm}(n\nu,t) = 2\beta_k^2\frac{1-\cos\left[(n\nu\mp\omega_k)t\right]}{(n\nu\mp\omega_k)^2}\left[\frac{1\mp1}{2}+n(\omega_k)\right]$$

Probe Energy levels



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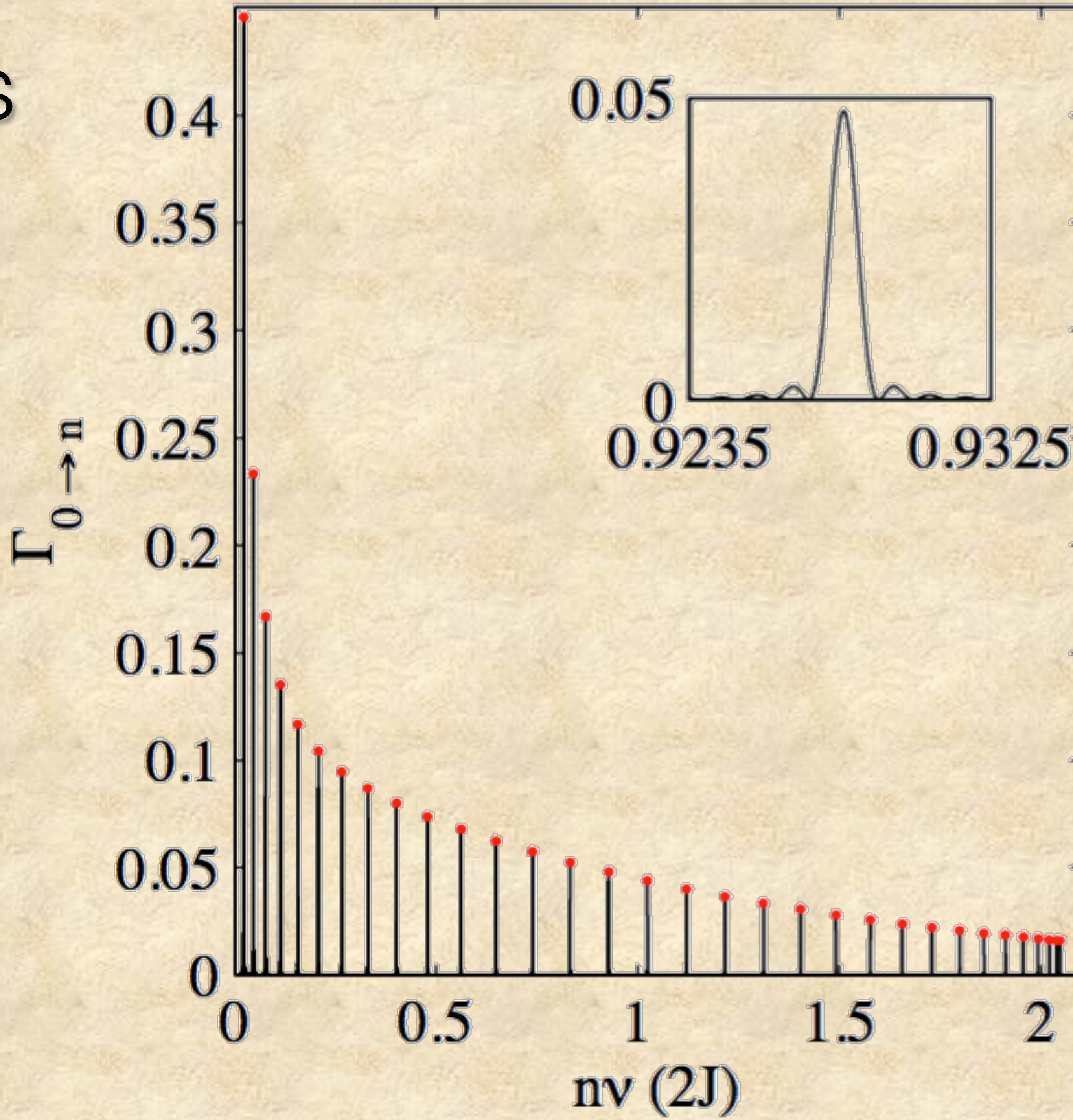
Tunability

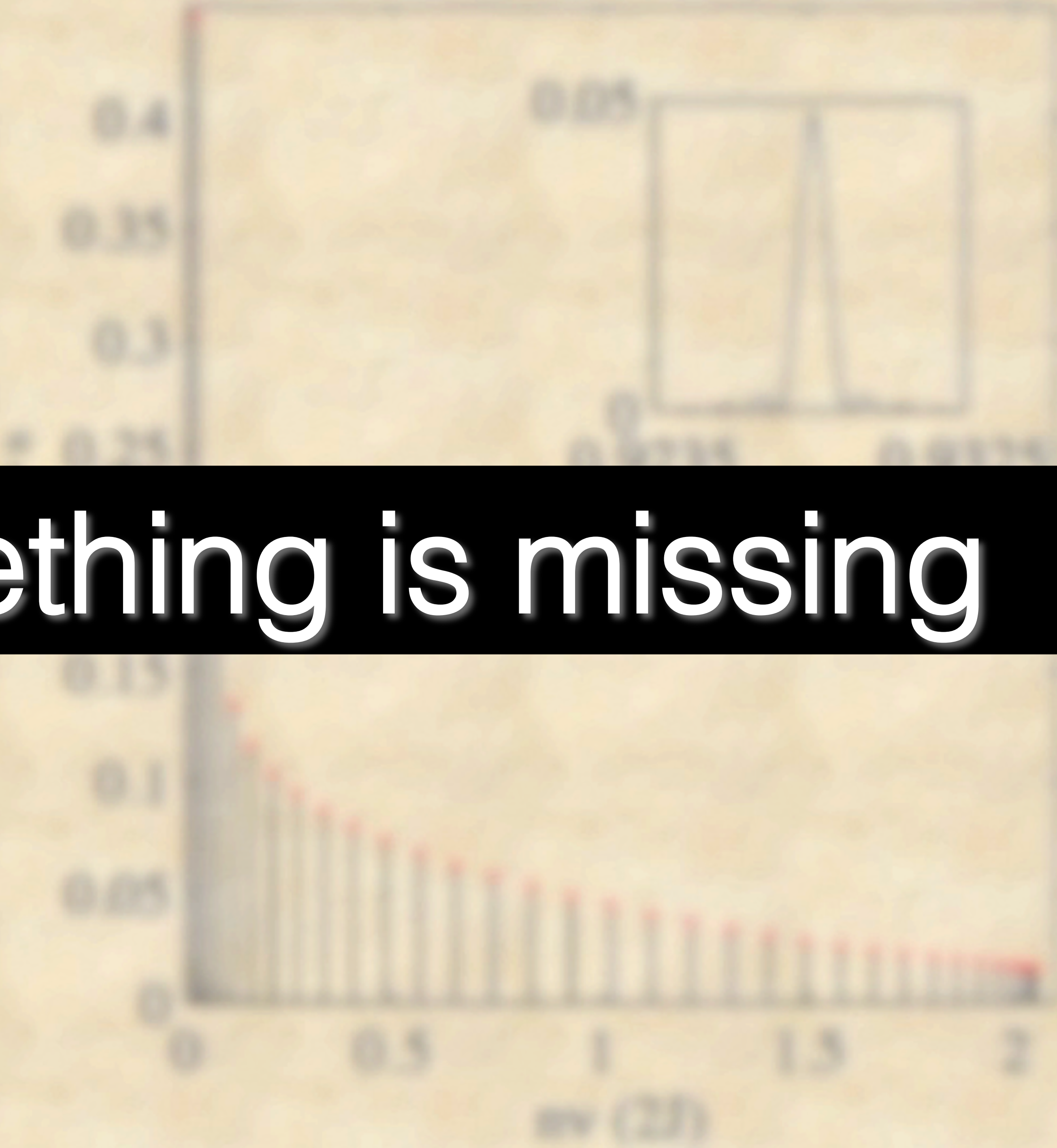
$$g^2_{\mathbf{I},n} = \frac{g^2}{\nu}\underbrace{(\int dx\,\psi^2_{n_x=0}(x)\omega_0^2(x))^2}_{\text{X}}\underbrace{(\sqrt{m}\frac{\gamma_0}{\pi}\sqrt{\nu_y})^2}_{\text{Y}}\underbrace{((-1)^{n_z}\sqrt{m}\frac{\gamma_{n_z}^{1/2}\gamma_0^{1/2}}{\pi}\sqrt{\nu})^2}_{\text{Z}}$$

Results



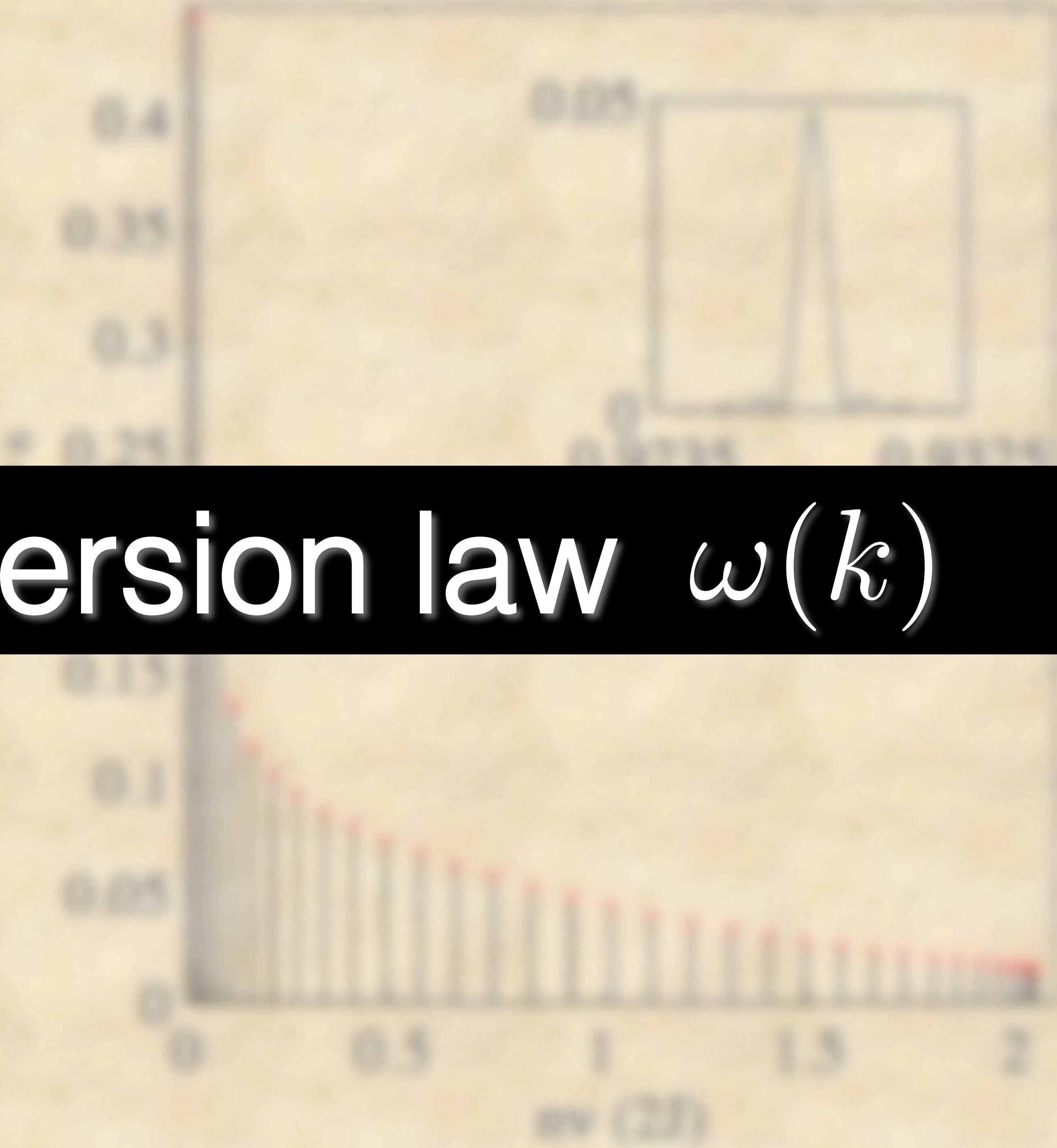
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
$\Gamma_{0\to n}$
0.05
0
0.9235
0.9325
0
0.5
1
1.5
2
nν (2J)

Results



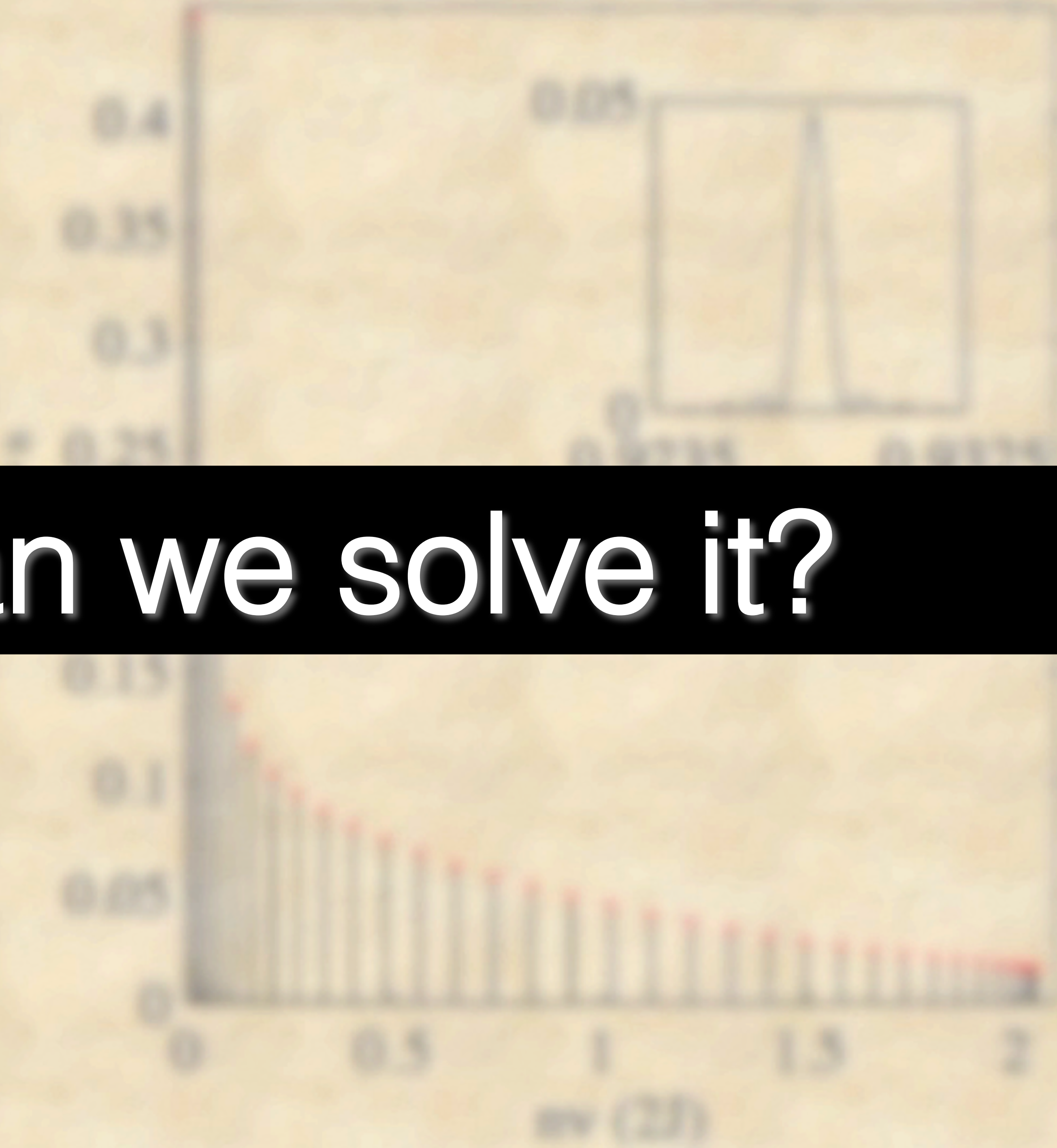
Something is missing

Results



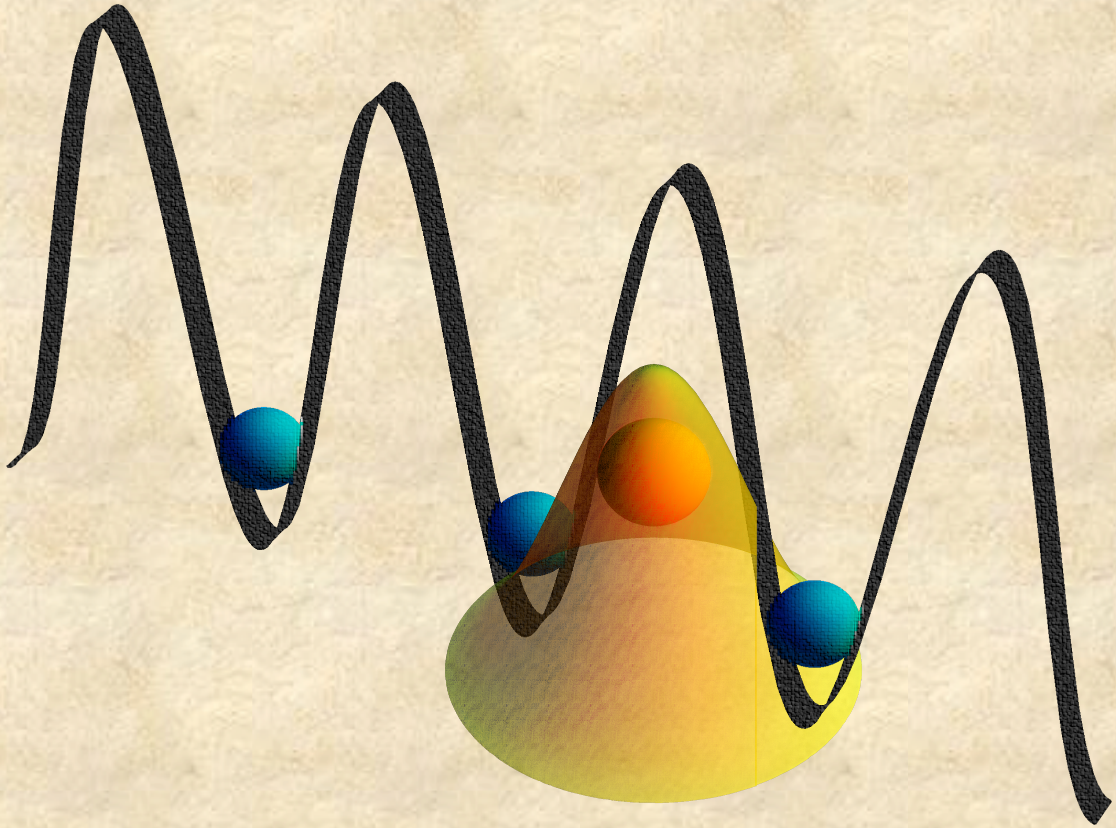
Dispersion law $\omega(k)$

Results

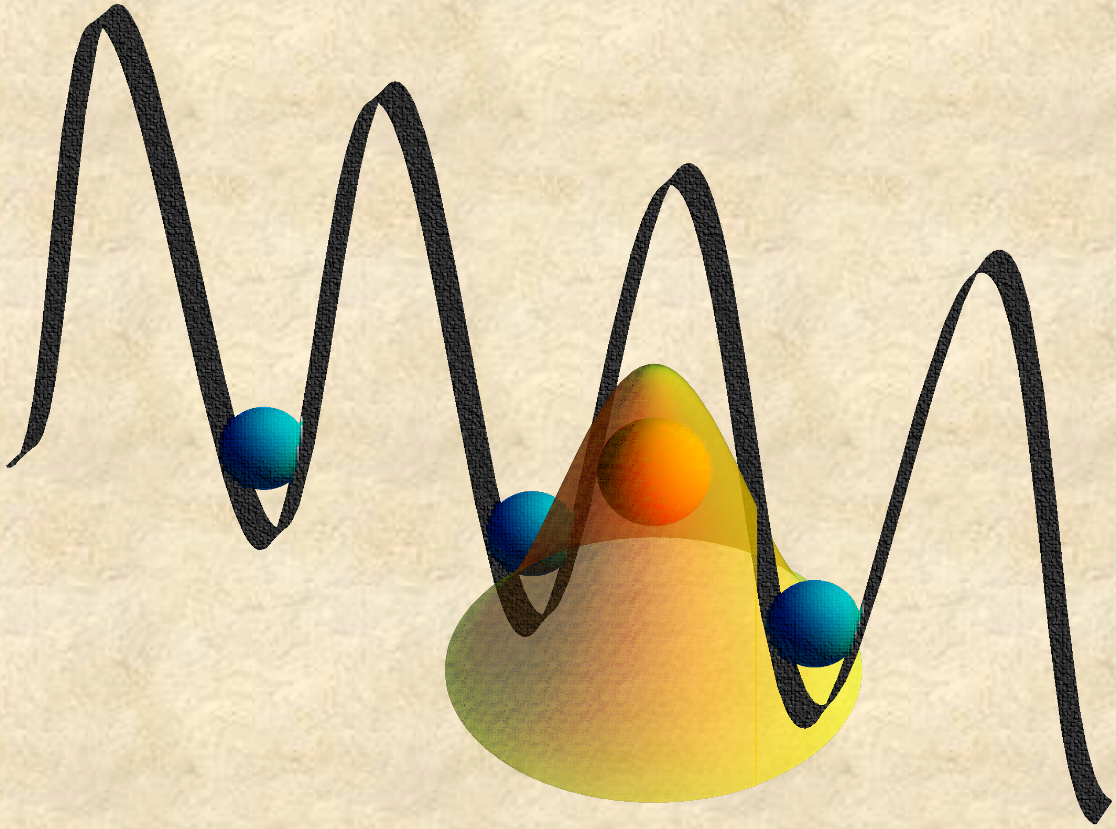


Can we solve it?

Step (IV):



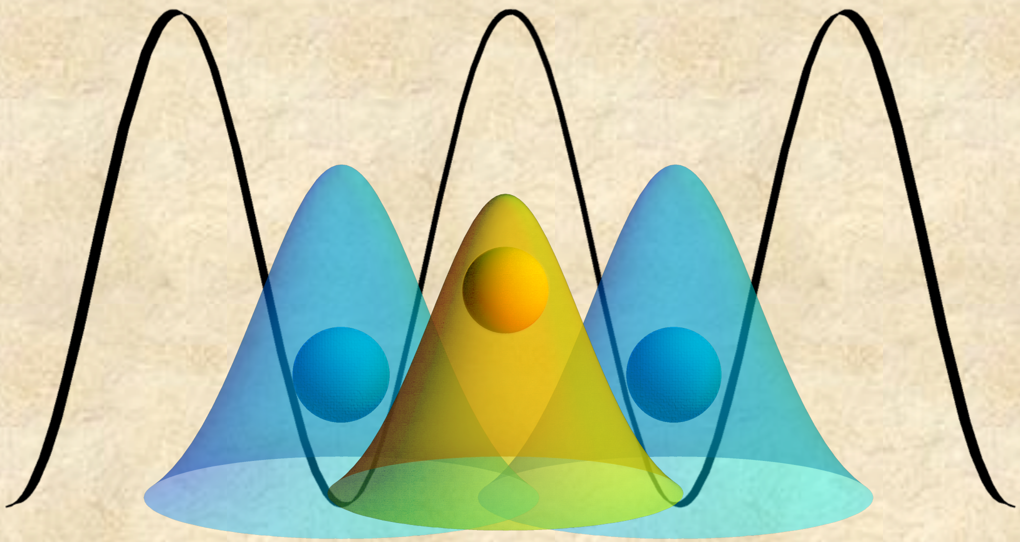
Step (IV):



Move the apparatus

Relevant

$$\langle 0,0,n_z|\,\hat{H}_{int}\,|\bar{0}\rangle \simeq g\sum_{i,j=0}^{1}\int \mathrm{d}\mathbf{x}\;\psi^*_{(0,0,n_z)}(\mathbf{x})\psi_{(0,0,0)}(\mathbf{x})\omega_i^*(x)\omega_j(x)\hat{c}_i^\dagger\hat{c}_j$$



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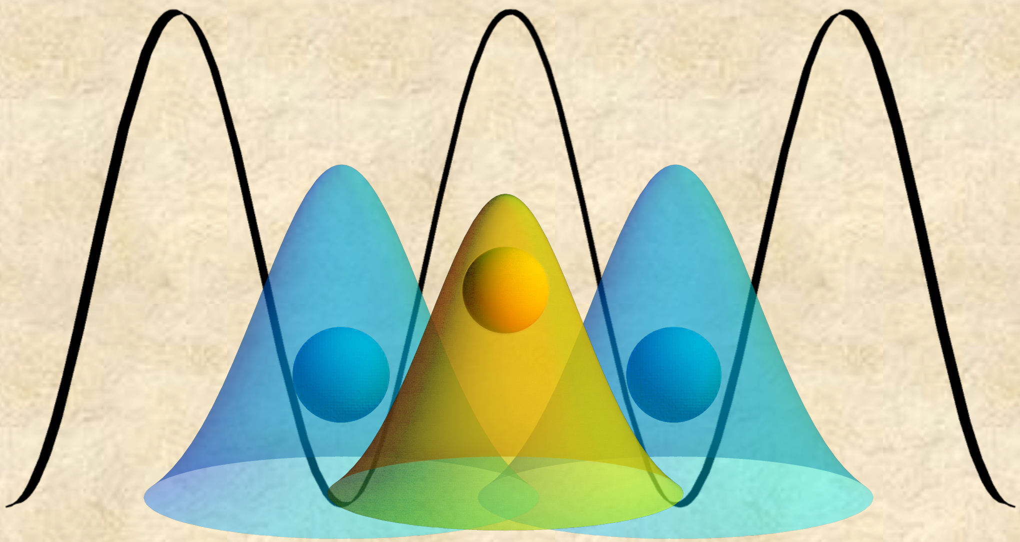
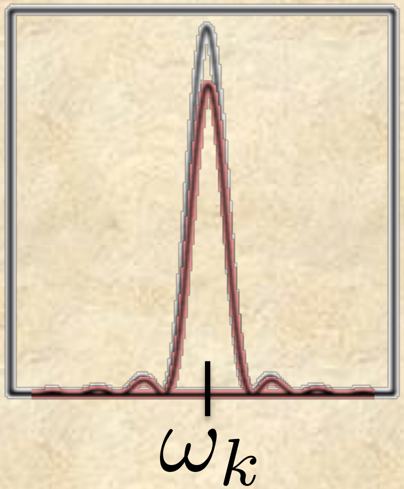


Transition Probability

$$\Gamma^{\mathbf{II}}_{\bar{0}\to(0,0,n_z)} = g^2_{\mathbf{II},n}\nu\left\{4n_0^2\frac{1-\cos(n\nu t)}{n^2\nu^2}+\sum_k(1+\cos(ka))[\Gamma_k^+(n\nu,t)+\Gamma_k^-(n\nu,t)]\right\}$$

$$g^2_{\mathbf{II},n} = 2\frac{g^2}{\nu}\left(\int dx\,\psi^2_{n_x=0}(x-\frac{a}{2})(\omega_0^2(x)+\omega_0(x)\omega_1(x))\right)^2\left(\sqrt{m}\frac{\gamma_0}{\pi}\sqrt{\nu_0}\right)^2\left((-1)^{n_z}\sqrt{m}\frac{\gamma_{n_z}^{1/2}\gamma_0^{1/2}}{\pi}\sqrt{\nu}\right)^2$$

Resonant Probabilities



$$\Gamma^{\mathbf{I}}_{\bar{0}\to(0,0,n_z)} \simeq 2\, g^2_{\mathbf{I},n}\, \beta_k^2 n(\omega_k) T_f^2$$

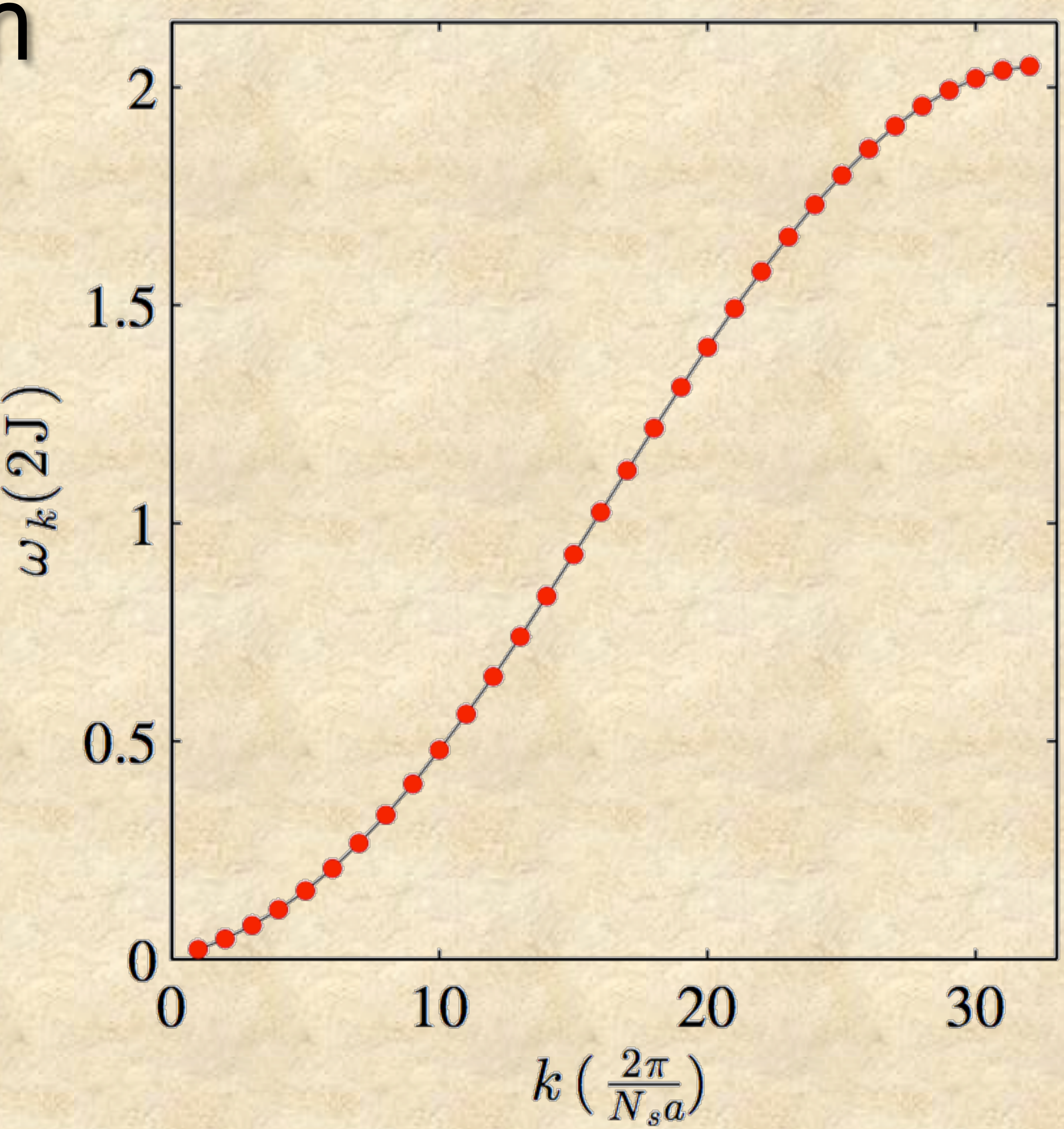
$$\Gamma^{\mathbf{II}}_{\bar{0}\to(0,0,n_z)} \simeq 2\, g^2_{\mathbf{II},n}\, \beta_k^2 n(\omega_k) T_f^2 (1+\cos(ka))$$

Resonant Probabilities

Ratio

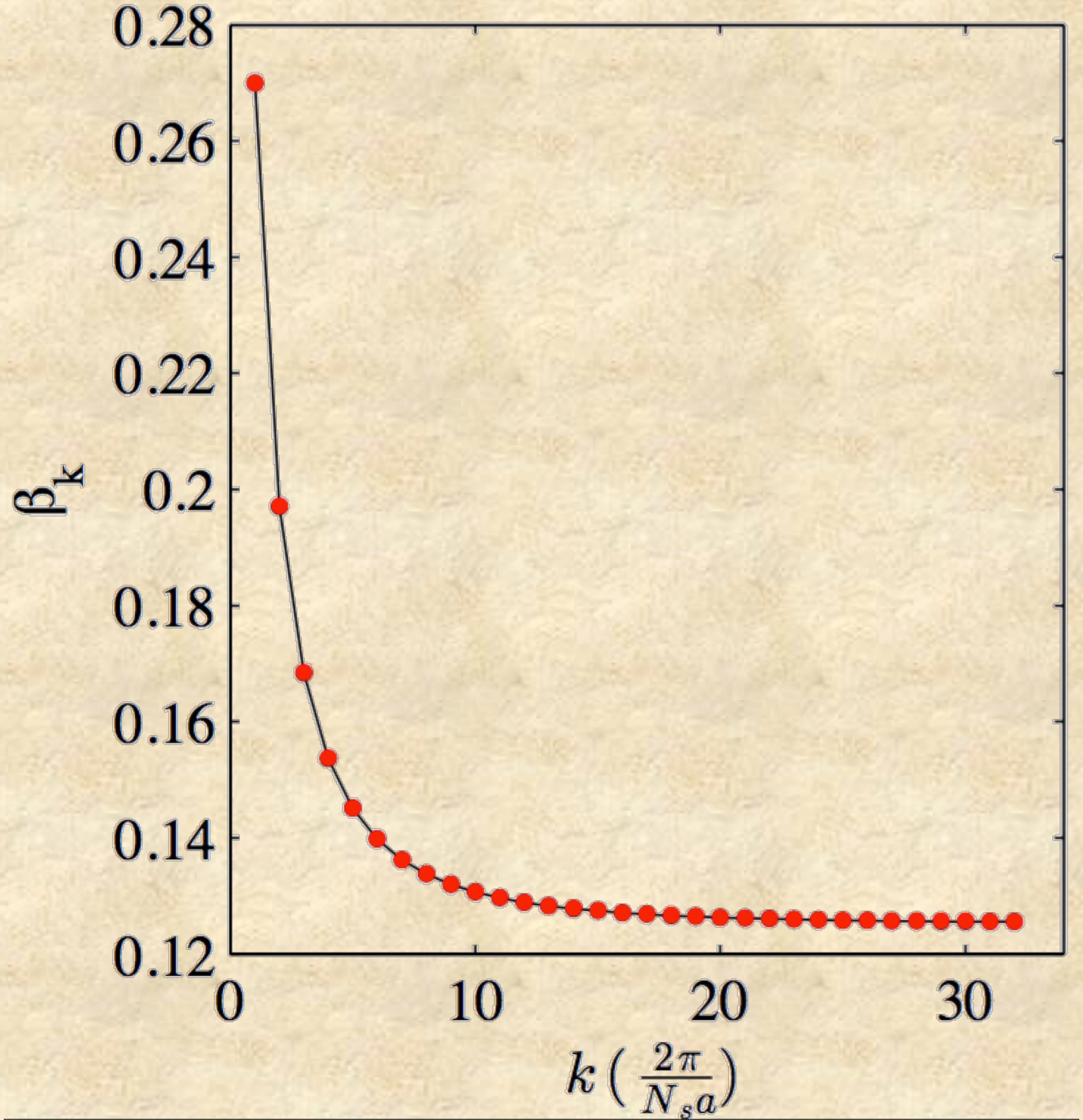
$$\frac{\Gamma^{\mathbf{II}}}{\Gamma^{\mathbf{I}}}\frac{g^{\mathbf{I}}}{g^{\mathbf{II}}} = [1 + \cos(ka)]$$

Spectrum

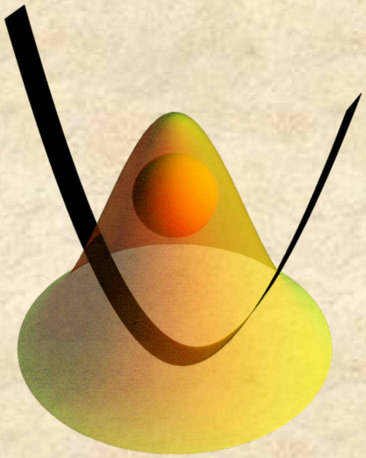


K-mode coupling

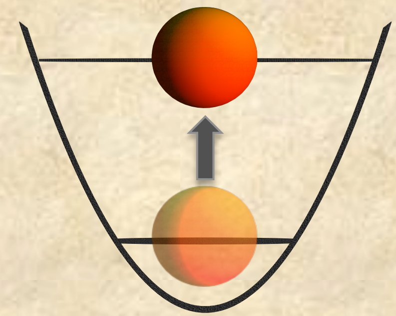
$$\Gamma^{\mathbf{I}}_{\bar{0}\to(0,0,n_z)} \simeq 2\; g^2_{\mathbf{I},n}\; \beta^2_k n(\omega_k) T^2_f$$

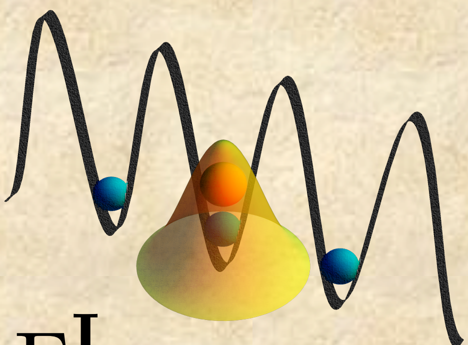


Quantum Probe

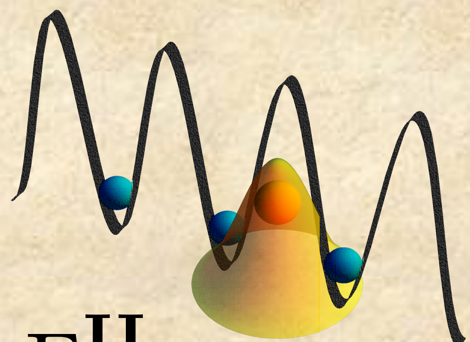


$$\psi_{\mathbf{0}}(x) \simeq e^{-\frac{x^2}{2x_0^2}} e^{-\frac{y^2}{2y_0^2}} e^{-\frac{z^2}{2z_0^2}}$$





$$\Gamma^{\mathbf{I}}_{\bar{0}\to(0,0,n_z)}$$



$$\Gamma^{\mathbf{II}}_{\bar{0}\to(0,0,n_z)}$$

$$\omega(k)$$

Outlook

- New **MRS** protocol using atomic probes
- Test the robustness of the protocol in a realistic situation
- Design multiprobes scheme for the detection of quantum correlations
- Investigate other geometries

Thank you for the attention!