Microscopic description for the emergence of collective decoherence in extended systems

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Outlook

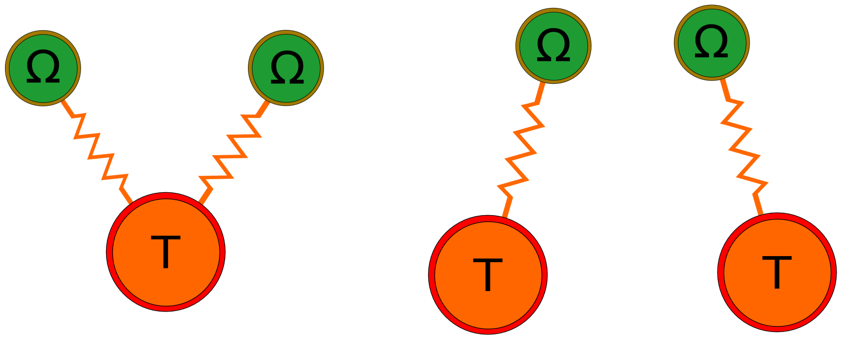
- Motivation
- Common/separate environments: noiseless subsystems
- D-dimensional crystal as environment
- Dynamical consequences for CB/SB
- Correlation length = CB to SB crossover?
- Spatial disorder
- What is a cut-off?
- Dissipation vs dephasing

Usual procedure to describe noise: pick up common (CB) or separate (SB) baths with a given spectral density. For example quantum Brownian motion:

$$H = H_S + H_B + H_I$$

$$H_I^{(CB)} = (x_1 + x_2) \sum_k g_k X_k$$

$$H_I^{(SB)} = x_1 \sum_k g_k X_k + x_2 \sum_k g_k Y_k$$



The spectral density encodes all properties interesting for the system dynamics:

$$J(\omega) = \sum_k \frac{g_k^2}{\nu_k} \delta(\omega - \nu_k)$$

The typical criterion to choose CB or SB is: if the 2 system components are closer to each other than the environment's correlation length, then choose CB.

- Why is this important?
- Is that criterion trustable?

Importance:

- With CB (dissipation or dephasing), the environment does **not** destroy all quantum information in the system, e.g.:

- Brownian motion keeps 2-mode entanglement asymptotically [Paz-Roncaglia PRL. 100, 220401 (2008)]
- Collective spin dephasing keeps alive some superpositions (e.g. $|01\rangle + |10\rangle$) [Palma-Suominem-Eckert, Proc.Roy.Soc.Lond. A452 567 (1996)]

What about the notion of distance versus correlation length?:

- Phenomenological models: spin-boson, boson-boson (with isotropic dispersions). They lack a description of bath's correlation length.
- Microscopic models: BCS, scalar field, 2 masses attached to a string, atoms radiating into electromagnetic vacuum, probes inside BEC (Jaksch).
- Jeske-Cole (PRA 87, 052138 (2013): they do analyze correlation length and do not find a general relation with CB/SB crossover

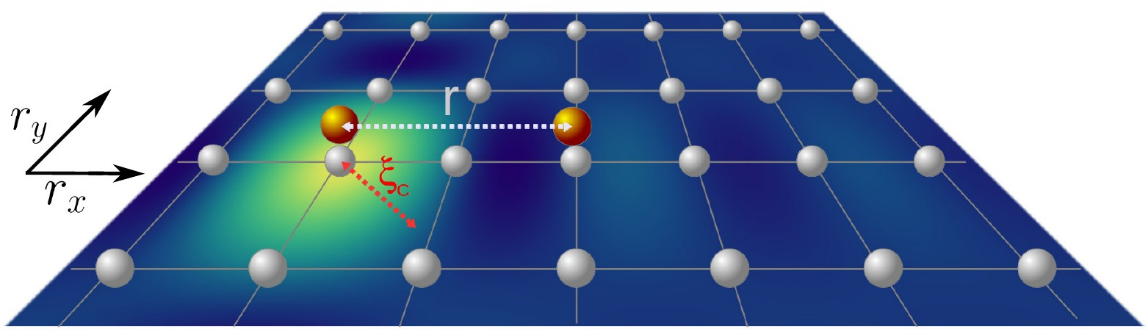
If decoherence of two distant units is sensitive to correlation length, it can be probed!

D-dimensional crystal as an environment

$$H_S = \Omega(a_1^\dagger a_1 + a_2^\dagger a_2)$$

$$H_B = \sum_{\vec{n}} \frac{P_{\vec{n}}^2}{2} + \frac{\omega_0^2 Q_{\vec{n}}}{2} + \frac{g}{2}\sum_{\vec{l}}(Q_{\vec{n}} - Q_{\vec{n}+\vec{1}})^2 = \sum_{\vec{k}} \omega_{\vec{k}}(A_{\vec{k}}^\dagger A_{\vec{k}} + 1/2)$$

$$H_{SB} = \lambda(q_1 Q_{\vec{n}} + q_2 Q_{\vec{n}'}) = \int_{\pi}^{\pi} d^D\vec{k}\,\frac{\lambda(2\pi)^{-D/2}}{2\sqrt{\Omega\omega_{\vec{k}}}}[(a_1 e^{i\vec{k}\vec{n}} + a_2 e^{i\vec{k}\vec{n}'} A_{\vec{k}}^\dagger + h.c.]$$



$$\omega_{\vec{k}} \quad = \quad \sqrt{\omega_0^2 + 4Dg(\sin^2\tfrac{k_x}{2} + \sin^2\tfrac{k_y}{2} + \ldots + \sin^2\tfrac{k_D}{2})}$$

For T=0

$$\dot{\rho}_S = -i[H_S + H_{LS}, \rho_S] + \sum_{j,l=1,2} \Gamma_{j,l}(a_j \rho_S a_l^\dagger - \frac{1}{2}\{a_l^\dagger a_j, \rho_S\})$$

$$\Gamma_{1,1}(t) = \Gamma_{2,2}(t) = \frac{\lambda^2}{2\Omega(2\pi)^D} \int_{-\pi}^{\pi} d^D\vec{k} \frac{1}{\omega_{\vec{k}}} \frac{\sin[t(\Omega - \omega_{\vec{k}})]}{\Omega - \omega_{\vec{k}}}$$

$$\Gamma_{1,2}(\vec{r}, t) = \Gamma_{2,1}(\vec{r}, t) = \frac{\lambda^2}{2\Omega(2\pi)^D} \int_{-\pi}^{\pi} d^D\vec{k} \frac{1}{\omega_{\vec{k}}} \frac{\sin[t(\Omega - \omega_{\vec{k}})]}{\Omega - \omega_{\vec{k}}} \cos(\vec{k} \cdot \vec{r})$$

Weak coupling (slow dissipation)

$$\Gamma_{1,1}(t) = \Gamma_{2,2}(t) \quad \xrightarrow{t\to\infty} \quad \frac{\lambda^2}{2\Omega^2(2\pi)^D}$$

$$\Gamma_{1,2}(\vec{r}, t) = \Gamma_{2,1}(\vec{r}, t) \quad \xrightarrow{t\to\infty} \quad \frac{\lambda^2}{2\Omega^2(2\pi)^D} \int_{-\pi}^{\pi} d^D\vec{k} \, \underline{\delta(\Omega - \omega_{\vec{k}})} \cos(\vec{k} \cdot \vec{r})$$

Resonance condition **selects** the manifold of momenta providing `communication' between probes: cross-dissipation is a result of resonant interference of crystal phonons.

Dynamical consequences:

The dissipator $\sum_{j,l=1,2} \Gamma_{j,l}(a_j \rho_S a_l^\dagger - \frac{1}{2}\{a_l^\dagger a_j, \rho_S\})$

becomes diagonal with $a_\pm = (a_1 \pm a_2)/\sqrt{2}$
(by a t-independent variable transform, whenever $\Gamma_{1,1} = \Gamma_{2,2}$)

$\sum_{j=\pm} \Gamma_\pm(a_j \rho_S a_j^\dagger - \frac{1}{2}\{a_j^\dagger a_j, \rho_S\})$ and $\Gamma_\pm = \Gamma_{1,1} \pm \Gamma_{1,3}$

When the cross-damping $\Gamma_{1,2} = \Gamma_{1,1}$, the mode a_- does **not** decay. Therefore a 2-mode squeezed state can have asymptotic survival of entanglement.

With the lighter condition $|\Gamma_{1,2}| \lesssim \Gamma_{1,1}$ we can have entanglement decaying at the very low rate $\Gamma_- \ll \Gamma_+$.

CB vs SB: Particular cases for the cross-damping term

1D:
periodically oscillating between CB, SB, and “anti”-CB, with a period given by the probe frequency (as related to the dispersion in the crystal)

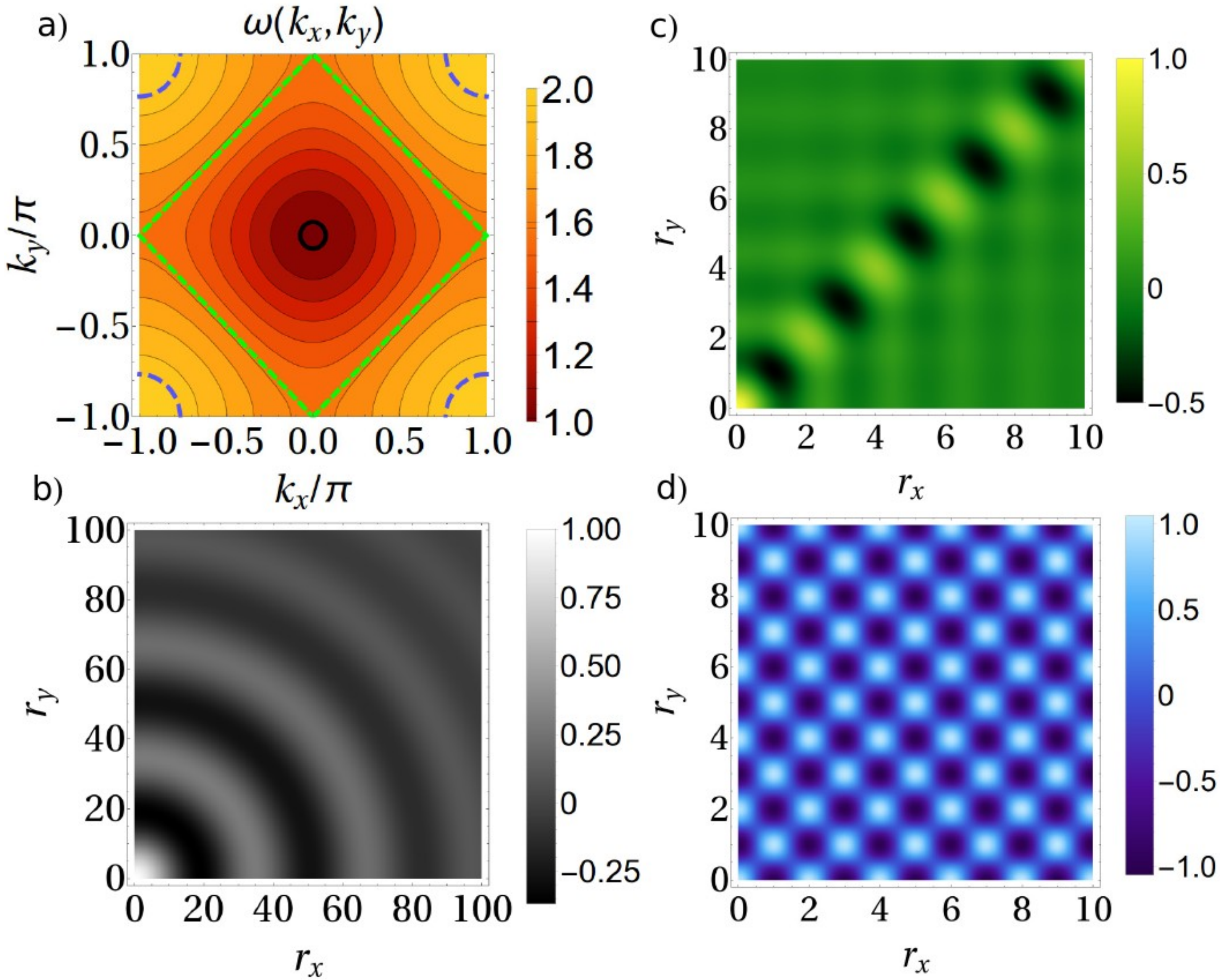
$$\Gamma_{1,2}^{(1D)}(x) \propto \cos(k_\Omega x) \qquad , \qquad \omega(k_\Omega) \hat{=} \Omega$$

Isotropic D-dimensions: (probes resonant to very small momenta)

$$\omega_{\vec{k}} \simeq \omega_{|\vec{k}|} = \sqrt{\omega_0^2 + 2Dg|\vec{k}|^2} \qquad \Gamma_{1,2}^{(2D)}(\vec{r}) \propto J_0(|\vec{k}_\Omega| r) \qquad \Gamma_{1,2}^{(3D)}(\vec{r}) \propto \mathrm{sinc}(|\vec{k}_\Omega| r)$$

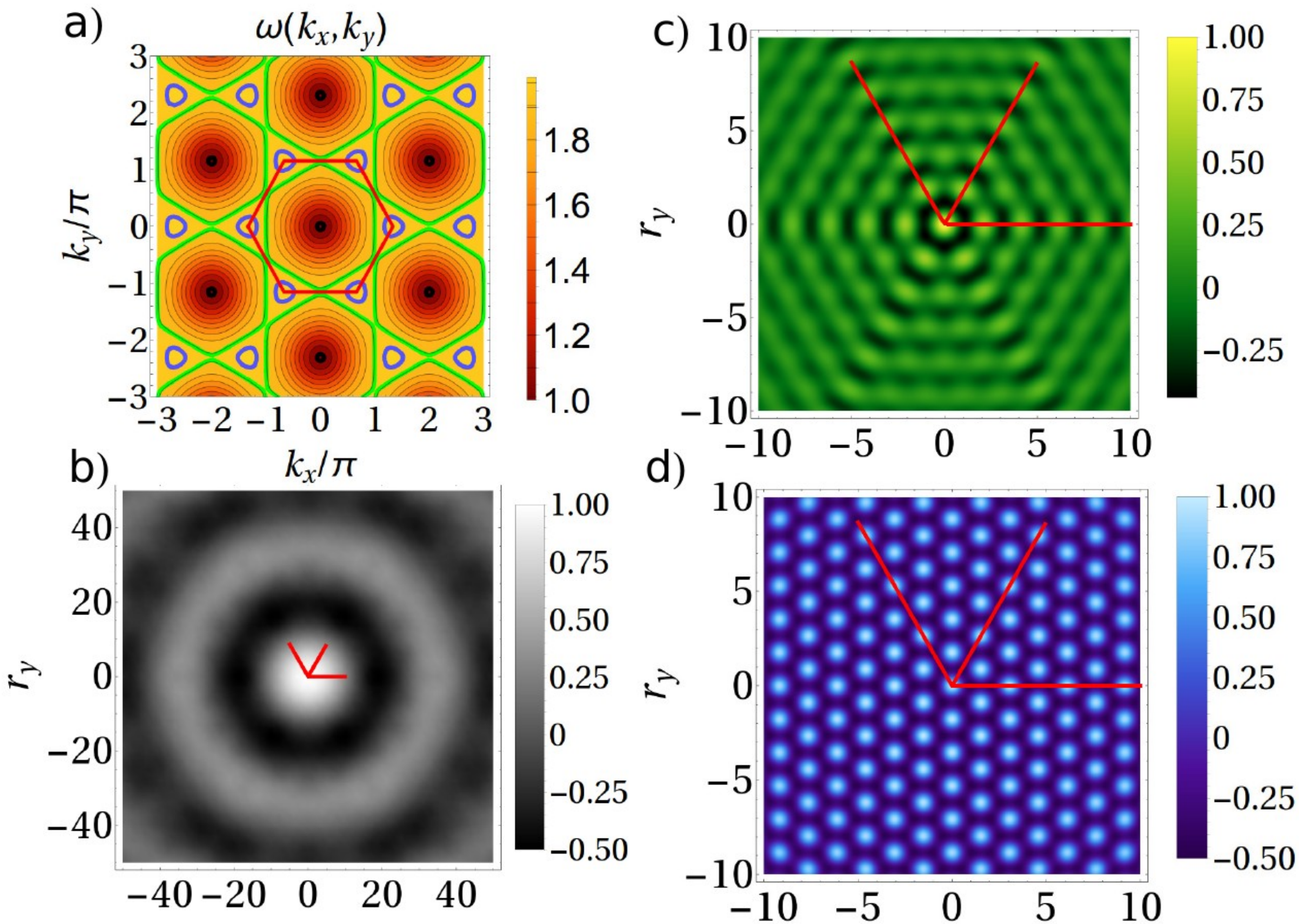
Becomes SB for distances $|\vec{r}| > 1/|\vec{k}_\Omega|$

Probes resonant with higher momenta, e.g. 2D case:



You can have CB at long distances, in directions established by the symmetry of the crystal

Triangular crystal case:



Same behaviour, different symmetry directions

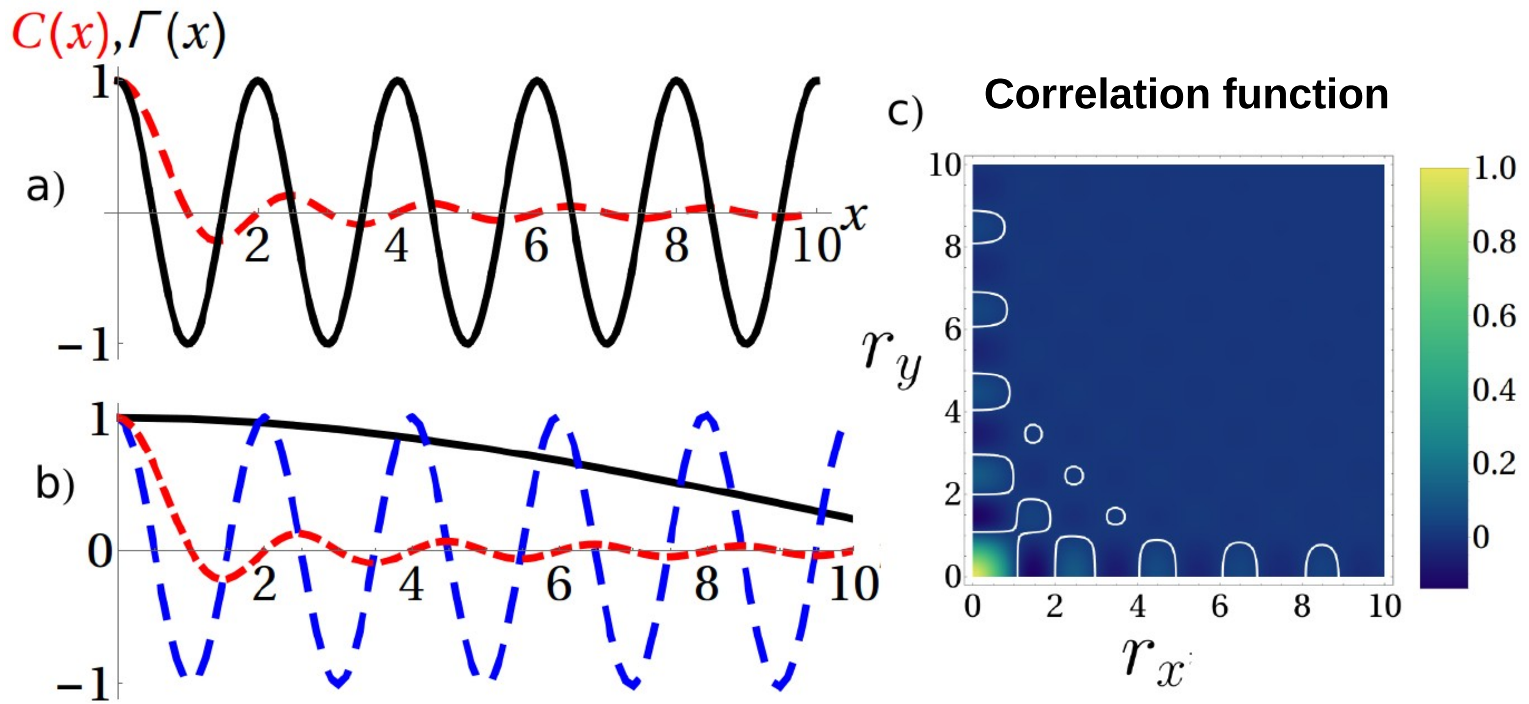
What is the **correlation length** of the crystal?

$$C(\vec{r}) = \langle Q_0 Q_{\vec{r}} \rangle = (2\pi)^{-D} \int_{-\pi}^{\pi} d^D\vec{k}\cos(\vec{k}\cdot\vec{r})[N_{\vec{k}}+1/2]/\omega_{\vec{k}} \quad \xrightarrow{T=0} \quad (2\pi)^{-D} \int_{-\pi}^{\pi} d^D\vec{k}\cos(\vec{k}\cdot\vec{r})/2\omega_{\vec{k}}$$

i.e. it is “transmitted” by **all** phonon momenta (no resonance condition imposed)

1D

2D along x



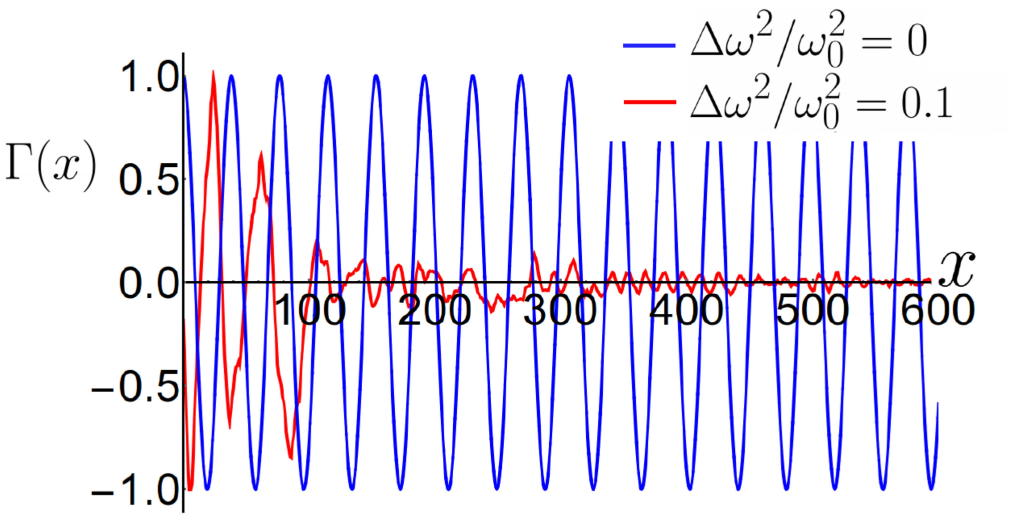
With **spatial disorder** in the crystal

$$\Gamma_{1,1}(t) = \Gamma_{2,2}(t) = \frac{\lambda^2}{2\Omega(2\pi)^D}\int_{-\pi}^{\pi} d^D\vec{k}\,\frac{1}{\omega_{\vec{k}}}\frac{\sin[t(\Omega-\omega_{\vec{k}})]}{\Omega-\omega_{\vec{k}}}|f_{\vec{n},\vec{k}}|^2$$

$$\Gamma_{1,2}(\vec{r},t) = \Gamma_{2,1}(\vec{r},t) = \frac{\lambda^2}{2\Omega(2\pi)^D}\int_{-\pi}^{\pi} d^D\vec{k}\,\frac{1}{\omega_{\vec{k}}}\frac{\sin[t(\Omega-\omega_{\vec{k}})]}{\Omega-\omega_{\vec{k}}}f_{\vec{n},\vec{k}}f^*_{\vec{n}',\vec{k}}$$

the crystal eigenfunctions $f_{\vec{n},\vec{k}}$ are not plane waves anymore and get localized, so cross-damping becomes impossible further away than the localization length.

E.g. **1D** :



Now let's take **non-point-like probes**

$$H_{SB} = \lambda \sum_{\vec{R}} g(\vec{R})(q_1 Q_{\vec{n}+\vec{R}} + q_2 Q_{\vec{n}'+\vec{R}})$$

(Probe interacts with a finite crystal area)

And now the cross-dissipation reads

$$\Gamma_{1,2}^{(D)}(\vec{r}) = \frac{\lambda^2}{2\Omega^2(2\pi)^D} \int_{\pi}^{\pi} d^D\vec{k} \cos(\vec{k}\cdot\vec{r}) \Phi(\vec{k}) \delta(\Omega - \omega_{\vec{k}})$$

$$\Phi(\vec{k}) = \sum_{\vec{R},\vec{R}'} g(\vec{R}) g(\vec{R}') \cos[\vec{k}(\vec{R}-\vec{R}')]$$

For example:

$$g(\vec{R}) \propto \exp(-|\vec{R}|^2/2\sigma^2) \longrightarrow \Phi(\vec{k}) \propto \exp(-|\vec{k}|^2\sigma^2)$$

thus, the integral is limited to momenta higher than $1/\sigma$
introducing in this way a natural (hi-freq) cutoff.

$\Rightarrow$ Probe insensitive to wavelengths shorter than “its own size”

Dephasing vs damping

$$H_{SB} = \sum_{i=1,2} S^{(i)} B^{(i)}$$

Info on
- The eigenfreqs othe environment
- How 1 and 2 couple to each eigenmode

Damping

cross-damping rate:

$$[S^{(i)}, H_S] \neq 0 \longrightarrow \int d\tau e^{i\Omega_\alpha \tau} \left\langle B^1(\tau) B^2(0) \right\rangle$$

Resonant filtering

Dephasing

$$[S^{(i)}, H_S] = 0 \longrightarrow \int d\tau \left\langle B^1(\tau) B^2(0) \right\rangle$$

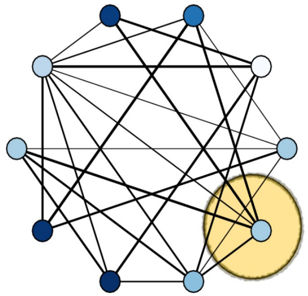
(notice that the spectral density, i.e. the description of how each probe interacts with a given crystal eigenfrequency is inside those integrals)

If we are to probe correlation lengths, dephasing might be the only way (we have to arrange the coupling strengths so as to mimic the integral in the correlation function).

CONCLUSIONS

- CB/SB transition NOT related to correlation function of the crystal.
- Possibility to engineer CB for distant system units (along crystal symmetry axes).
- Distant CB limited by spatial disorder.
- A natural high-frequency cut-off appears from a finite-length system.
- Conservation of coherence/entanglement achievable for CB (even distant) cases.
- Dephasing vs. damping can be understood from the simple resonance condition. Dephasing can be (probably) engineered for correlation length probing.

Dissipation in extended environments



Finite microscopic models of network environment

→ **non-uniform bosonic-chain baths**

- Non-Markovianity

Vasile, Galve, Zambrini, PRA 89, 022109 (2014)

- energy flow between system and environment

Galve, Zambrini, IJQI 1560022, 12 (2014)

→ **bosonic and spin baths**

- Quantum Darwinism and non-Markovianity

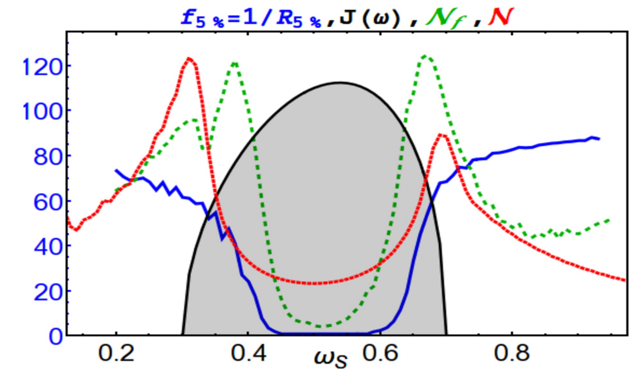
Galve, Zambrini, Maniscalco, Scientific Reports 6, 19607 (2016)

Giorgi, Galve, Zambrini, PRA 92, 022105 (2015)

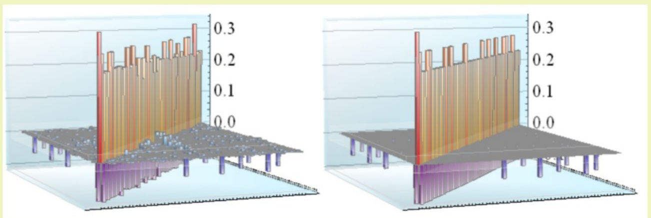
→ **Bosonic network environment**

- Engineering and probing of the unknown structure

Nokkala,Galve,Zambrini,Maniscalco,Piilo, 1503.04635



network probing



reconstructed and real adiacency matrix A

of a small world network