QUANTUM PROBING OF BOSE-HUBBARD MODEL VIA MEMORY EFFECTS

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QuProCS

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OPEN QUANTUM SYSTEMS & ENTANGLEMENT

Non Markovian Quantum Dynamics
Cortona, 26 August 2015

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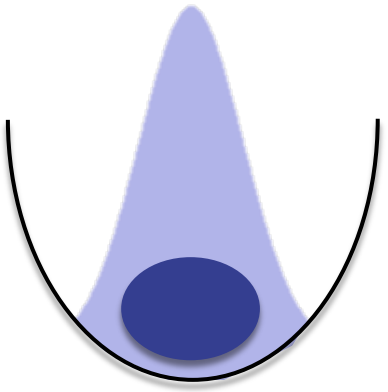
  - Density fluctuations and Density-density correlation functions

➢ Bosonic System

$$\hat{H} = \int dx\, \hat{\Phi}(x)^\dagger \hat{h}(x) \hat{\Phi}(x) + \frac{g'}{2} \int dx\, \hat{\Phi}(x)^\dagger \hat{\Phi}(x)^\dagger \hat{\Phi}(x) \hat{\Phi}(x)$$

- Expansion in Wannier States

$$\hat{\Phi}(x) = \sum_i \sum_n \omega_i^n(x) \hat{c}_i^n$$



- Bose-Hubbard Model

$$\hat{H}_{BH} = -J \sum_i (\hat{c}_{i+i}^\dagger \hat{c}_i + \hat{c}_i^\dagger \hat{c}_{i+1}) + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) - \mu \sum_i \hat{n}_i$$

- Parameters

$$U = g' \int dx\, \omega_i^{0*}(x) \omega_i^{0*}(x) \omega_i^0(x) \omega_i^0(x)$$

$$J = -\int dx\, \omega_i^{0*}(x) \hat{h}(x) \omega_{i+1}^0(x)$$

$$\hat{H}_{BH} = -J\sum_i(\hat{c}^\dagger_{i+i}\hat{c}_i + \hat{c}^\dagger_i\hat{c}_{i+1}) + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) - \mu\sum_i \hat{n}_i$$

On site interaction

$U << J$ Superfluid

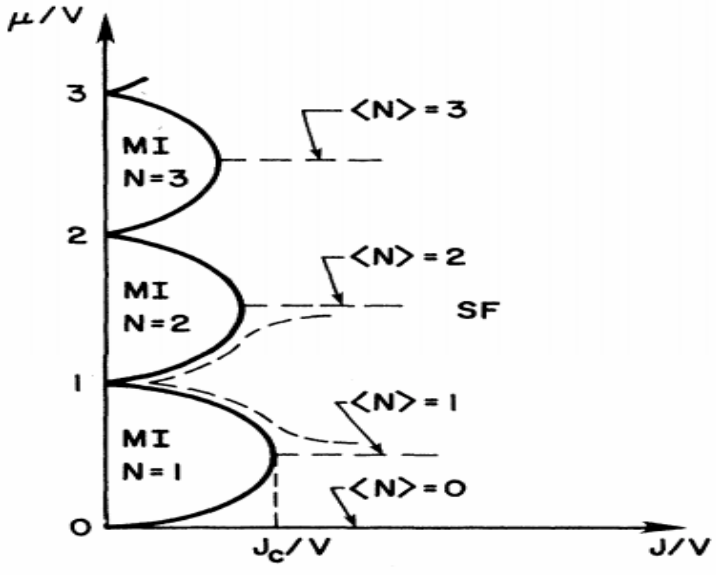
$U >> J$ Mott Insulator

$U \to \infty$ Free Fermions

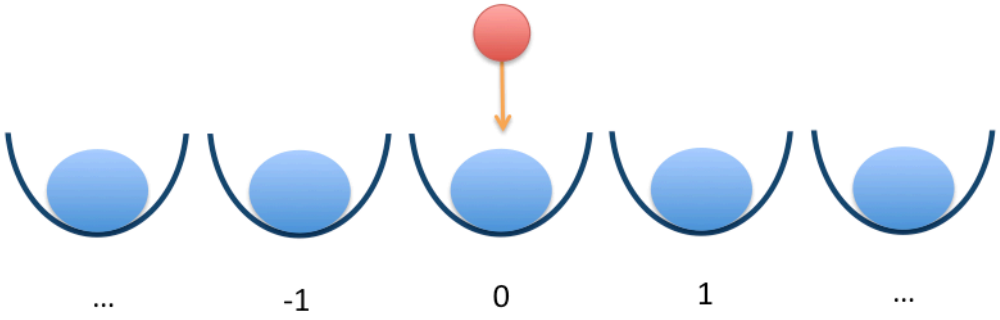
- Ground-states

$$|\Psi_{\mathbf{GS}}(U=0)\rangle = \frac{(\hat{a}^\dagger_{k=0})^N}{\sqrt{N!}}|vac\rangle$$

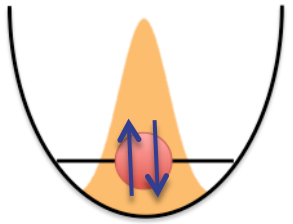
$$|\Psi_{\mathbf{GS}}(J=0)\rangle = \prod_i^{N_s}\frac{(\hat{c}^\dagger_i)^g}{\sqrt{g!}}|vac\rangle$$



[1] M. P. A. Fisher, P. B. Weichman, G. Grinstein, and Daniel S. Fisher, Phys. Rev. B 40, 546 (1989)
[2] M. A. Cazalilla, R. Citro, T. Giamarchi, E. Orignac, and M. Rigol, Rev. Mod. Phys. 83, 1405 (2011)



Probe:
Impurity Atom



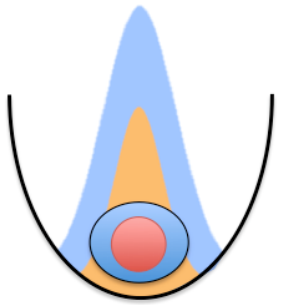
$$\Psi(x) = \frac{1}{\sqrt{2\pi}s} e^{-\frac{x^2}{2s^2}}$$

- Density-density interaction

$$\hat{H}_{int} = g\,|e\rangle\langle e| \otimes \int dx\,|\Psi(x)|^2 \hat{n}(x)$$

- Localized Probe

$$\hat{H}_{int} = g\,|e\rangle\langle e| \otimes \int dx\,|\Psi(x)|^2 |\omega_0(x)|^2 \hat{c}_0^\dagger \hat{c}_0$$



$$U_g = g \int dx |\psi(x)\omega_0(x)|^2$$

$$\hat{H}_{int} = U_g\,|e\rangle\langle e|\,\hat{c}_0^\dagger \hat{c}_0 \longrightarrow \hat{n}_0 = n_0 + \delta\hat{n}_0$$

➢ SF vs MI

- J>>U Superfluid Phase
  - *Bogoliubov Theory*[3]

$$\hat{a}_{k=0} \rightarrow \sqrt{N_0} + \delta\hat{a}_{k=0}$$

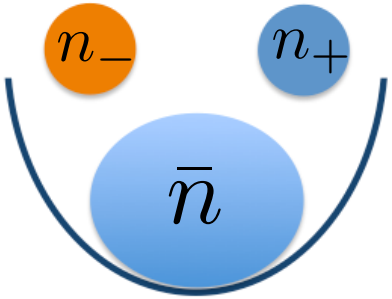
$$\hat{H}_{BH} = \sum_k \omega_k \hat{b}_k^\dagger \hat{b}_k$$

$$\delta\hat{n}_{\mathbf{0}} = \sum_k \frac{\beta_k}{N_s}(\hat{b}_k^\dagger + \hat{b}_k)$$

- J<<U Mott Insulator Phase
  - *Three States Theory*[4]

$$\{|\bar{n}\rangle, |\bar{n} \pm 1\rangle\}_{i=1\ldots N_s}$$

$$\hat{H} = \sum_{k,\sigma=\pm} \omega_{k,\sigma} \hat{\gamma}_{k,\sigma}^\dagger \hat{\gamma}_{k,\sigma}$$



$$\delta\hat{n}_{\mathbf{0}} = \hat{n}_{\mathbf{0},+} - \hat{n}_{\mathbf{0},-}$$

[3] D. van Oosten, P. van der Straten, and H. T. C. Stoof, Phys. Rev. A 63, 053601 (2001)
[4] P. Barmettler, D. Poletti, M. Cheneau, and C. Kollath, Phys. Rev. A 85, 053625 (2012)

➢ Probe dynamics

$$\rho_s(t) = \begin{pmatrix} \rho_{gg}(0) & \rho_{eg}(0)e^{\Gamma(t)+i\phi(t)} \\ \rho_{ge}(0)e^{\Gamma(t)-i\phi(t)} & \rho_{ee}(0) \end{pmatrix}$$

- Dephasing rate and decoherence function

$$\gamma(t) = U_g^2 \,\mathrm{Re} \int_0^t dt_1 \,\mathrm{Tr}[\delta\hat{n}_{\mathbf{0}}(t_1)\delta\hat{n}_{\mathbf{0}}(0)\rho_E]$$

$$\Gamma(t) = -\int_0^t dt_1 \,\gamma(t_1)$$

- Non-Markovianity measure[5]

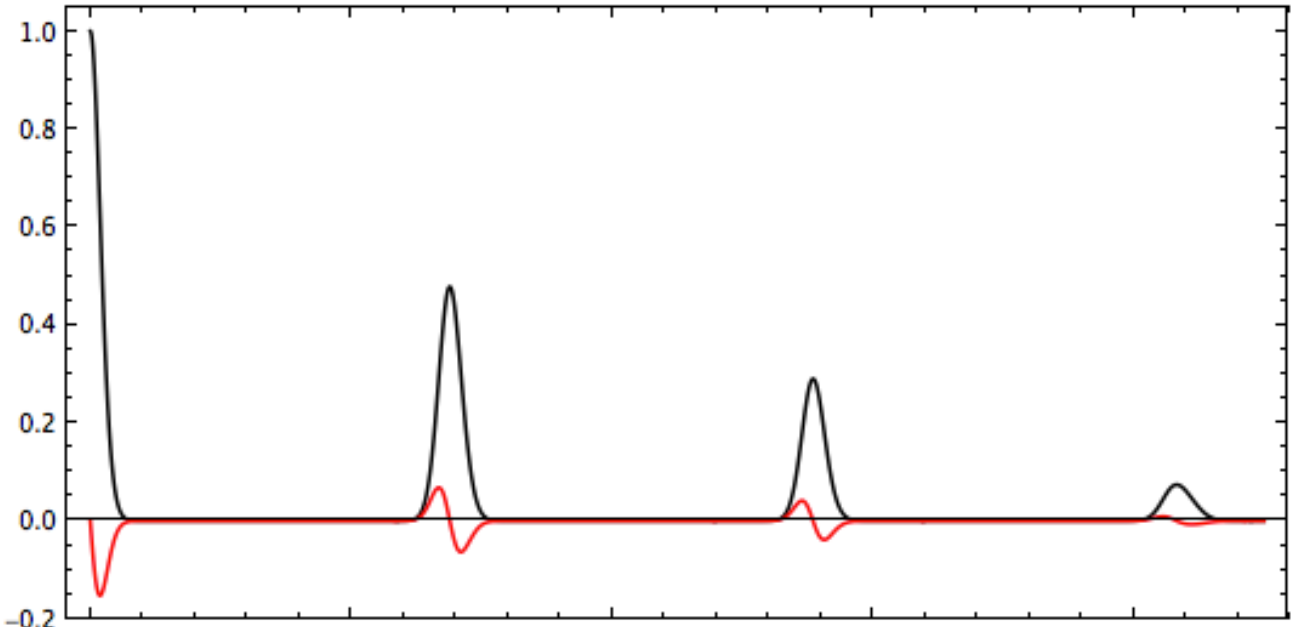
$$\mathcal{N} = \max_{\rho_{1,2}(0)} \int_{\sigma>0} \sigma(t, \rho_{1,2}(0))$$

[5] Heinz-Peter Breuer, Elsi-Mari Laine, and Jyrki Piilo, Phys. Rev. Lett. 103, 210401
[6] H.-P. Breuer, E.-M. Laine, J. Piilo, and B. Vacchini, arXiv:1505.01385 (2015)

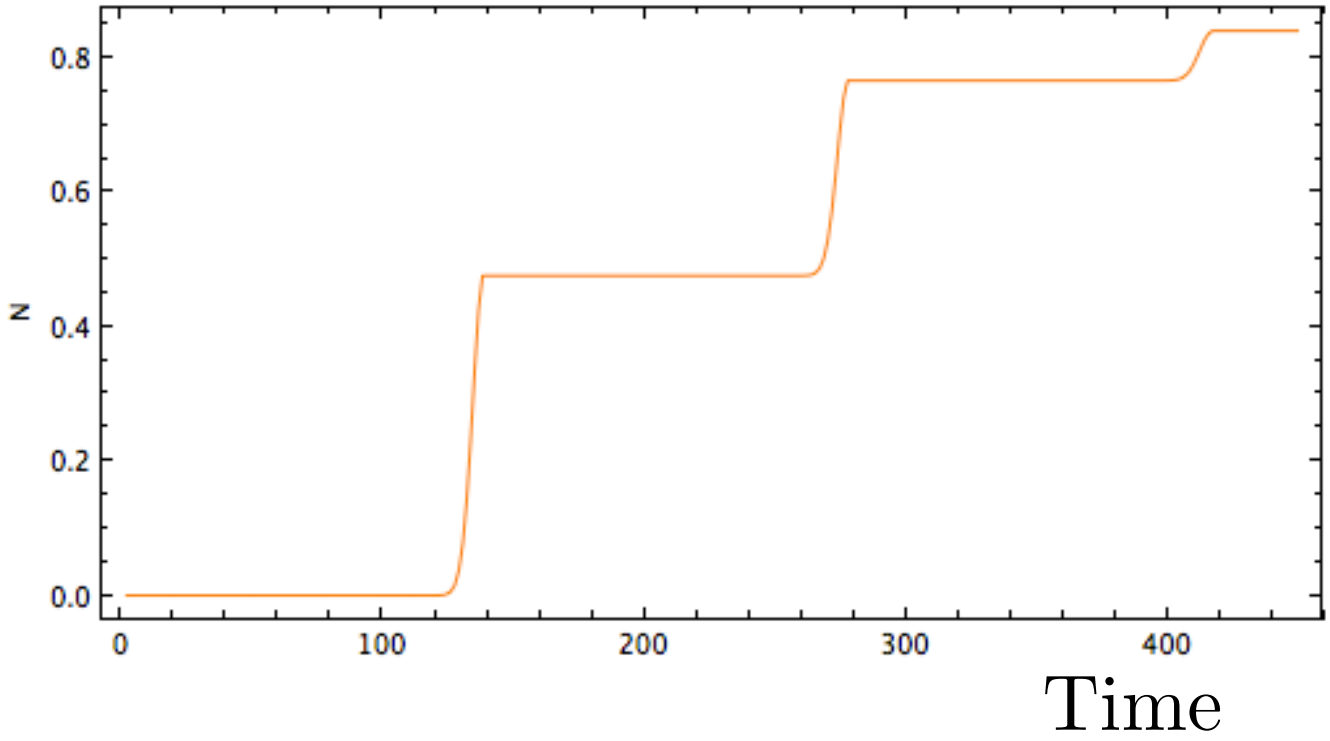
➢ Results SF

$|\rho_{eg}(t)|$ —

$\sigma(t)$ —

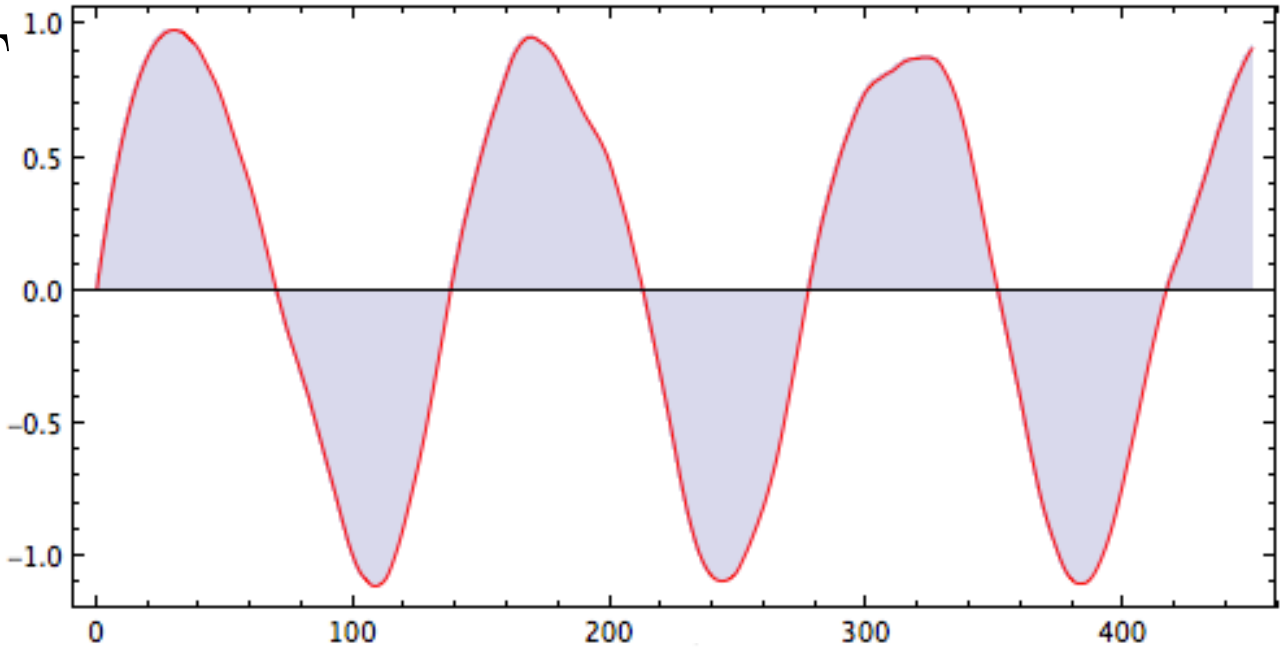


$\mathcal{N}$ —

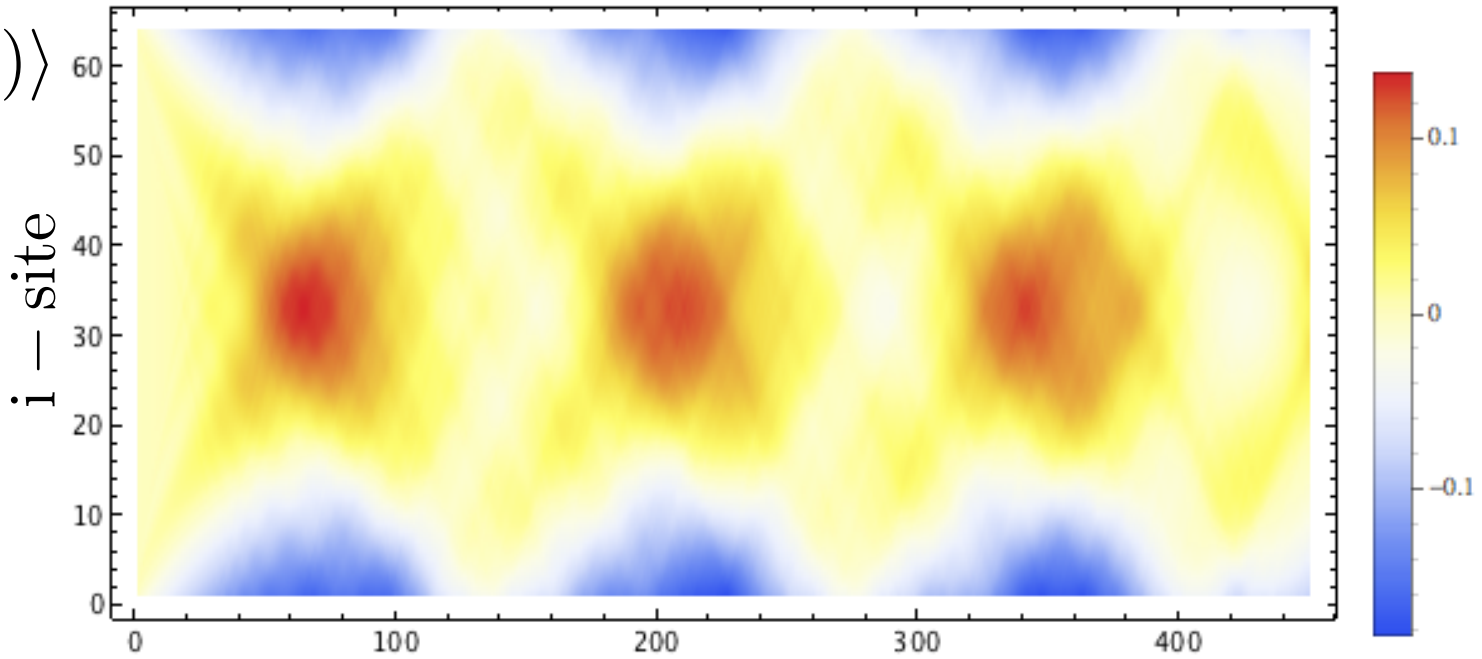


➢ Results SF

$\gamma(t)$



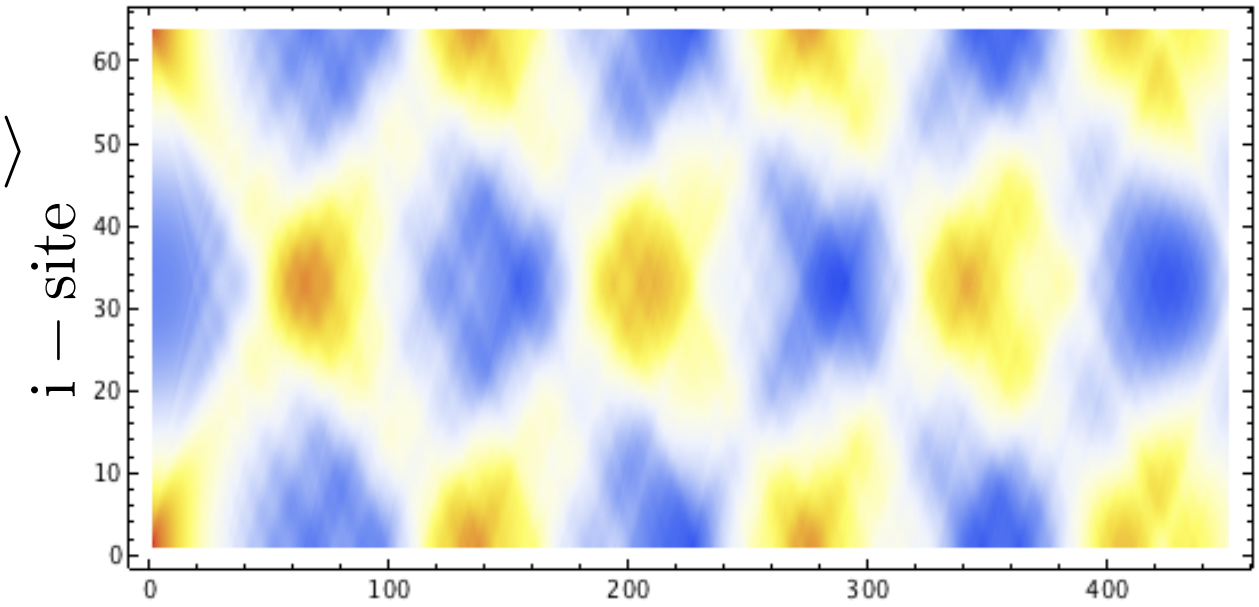
$\langle \delta \hat{n}_i(t) \rangle$



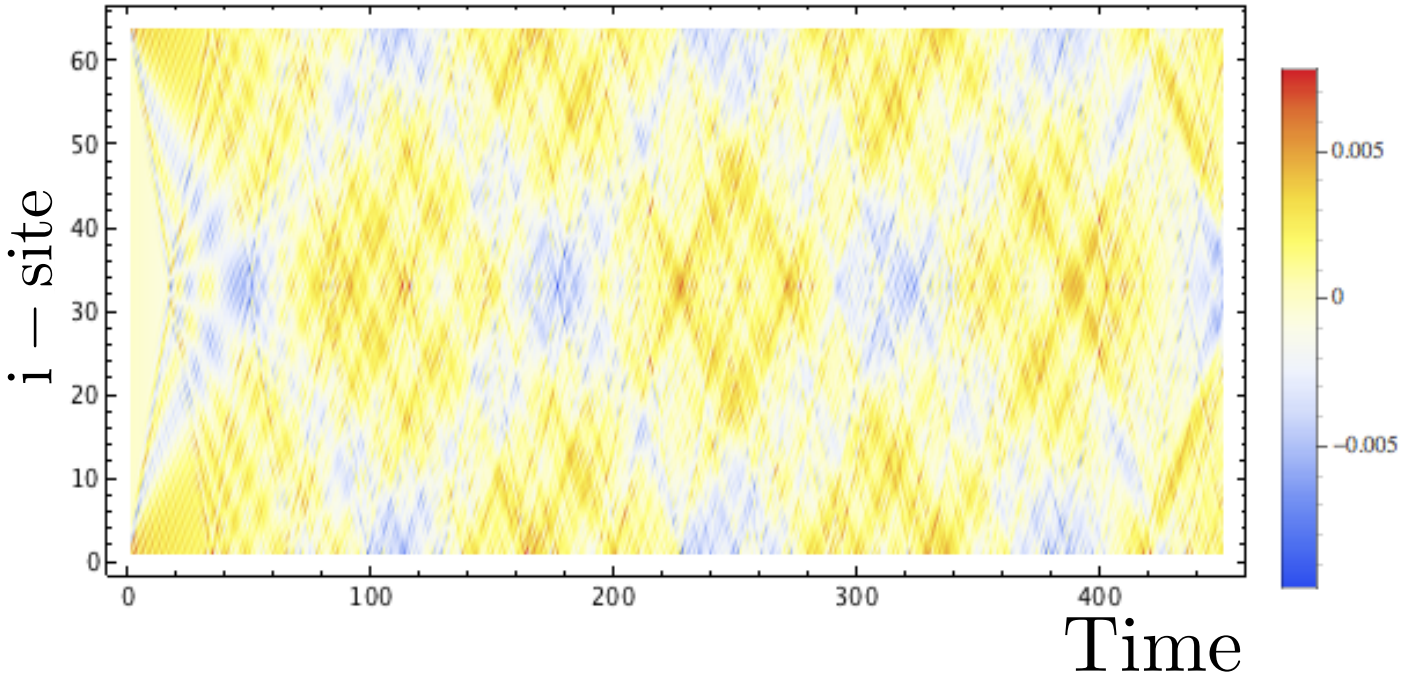
➢ Results SF

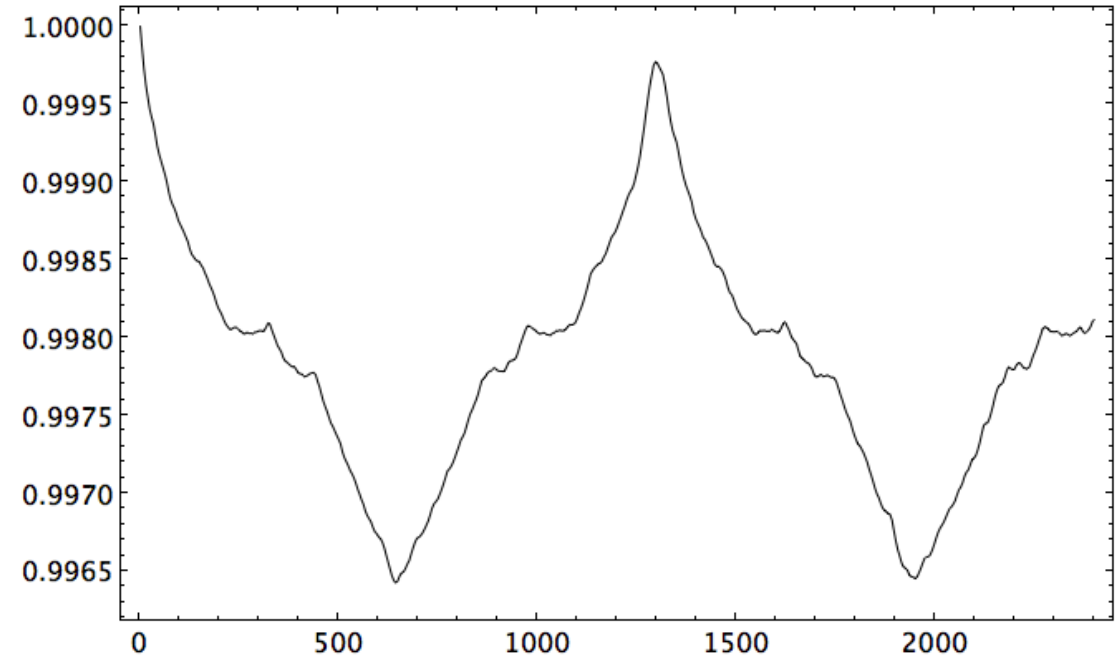
$\langle \delta\hat{n}_i(t)\delta\hat{n}_0(0)\rangle$

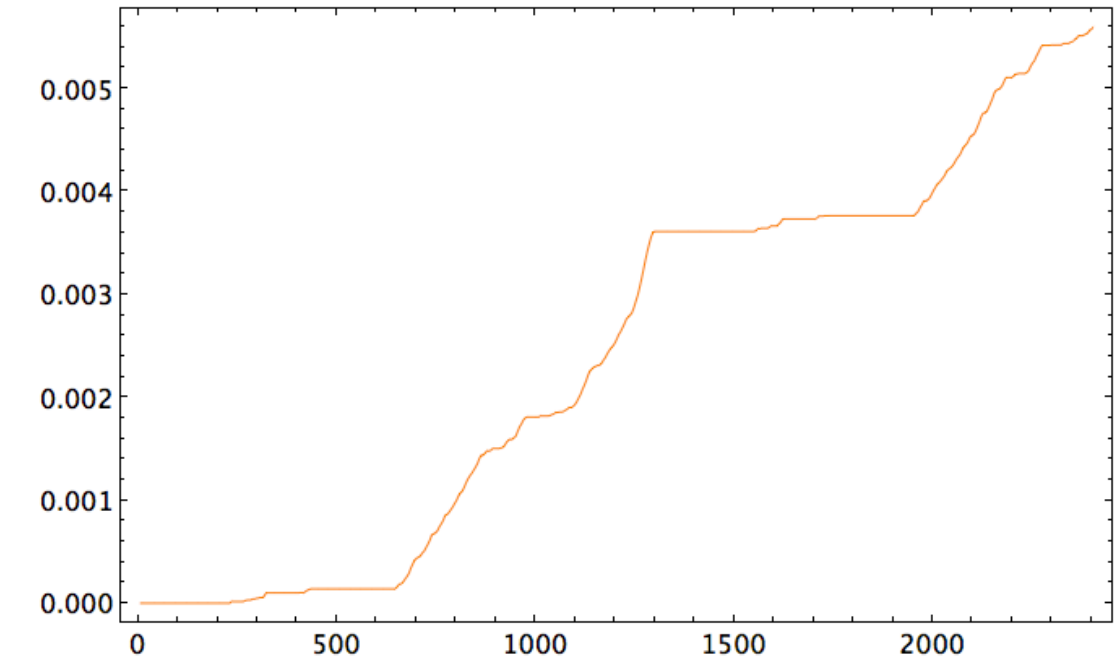
Re



Im





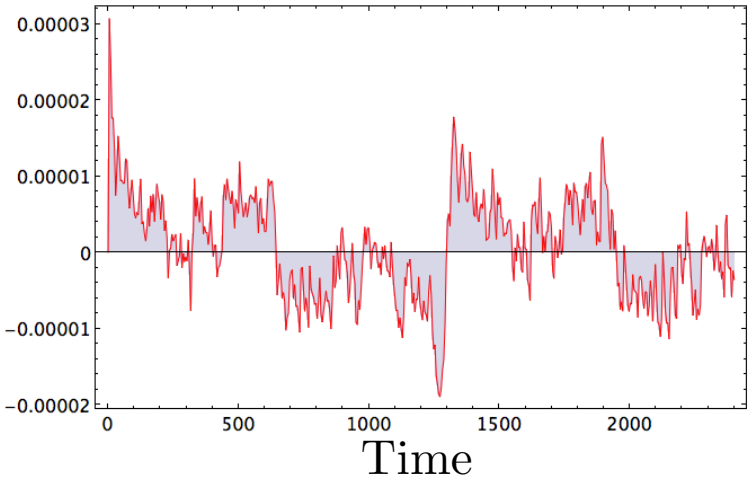


➢ Results MI

$|\rho_{eg}(t)|$ —

$\sigma(t)$ —

$\mathcal{N}$ —



Conclusions

- The decoherence in the open system dynamics detects features of the many-body environment
- Candidate as non-destructive quantum probe for ultracold atoms
- Ongoing work
  - Correlations and density fluctuations in the Mott Phase

THANK YOU
FOR YOUR ATTENTION!