Non-equilibrium thermodynamics in Coulomb Crystals

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Outline

- Intro to Coulomb Crystals
- Non-equilibrium thermodynamics tools
- Results

Coulomb Crystals

Trapped Ions

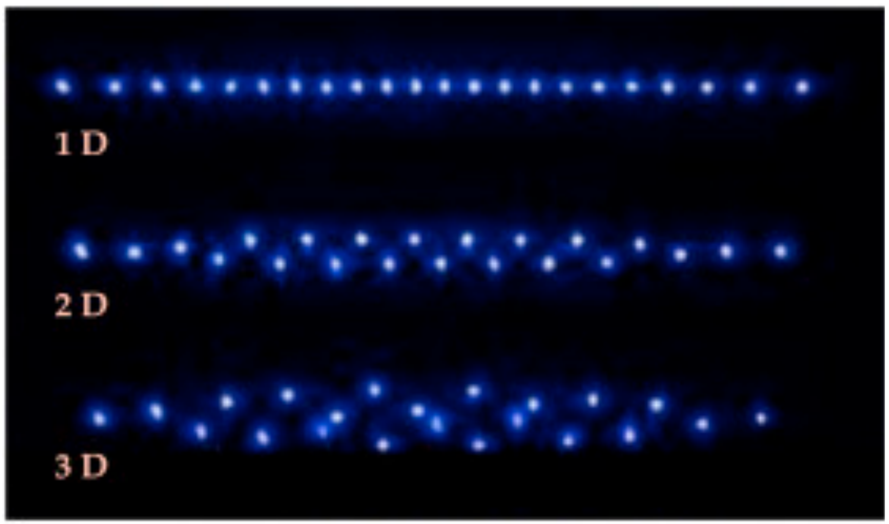
$$H_0 = \sum_{j=1}^{L} \left[\frac{P_{x,j}^2 + P_{y,j}^2}{2M} + \frac{M\omega_0^2}{2} Y_j^2 + V_L(X_j) \right] + \frac{Q^2}{8\pi\epsilon_0} \sum_{i \neq j} \left[(X_i - X_j)^2 + (Y_i - Y_j)^2 \right]^{-1/2}$$

Interplay **confining potential**

and **Coulomb repulsion**

Coulomb Crystals

Structural phase transition



Linear Phase $\omega_0 > \omega_c$

ZigZag Phase $\omega_0 < \omega_c$

H. L. Partner et al., Physica B: Condensed Matter, Volume 460, 2015, 114–118,
http://dx.doi.org/10.1016/j.physb.2014.11.051

Semi-classical approach

Equilibrium Positions

ZigZag Phase

$$y_j = (-1)^j b/2$$

Linear Phase

$$y_j = 0$$

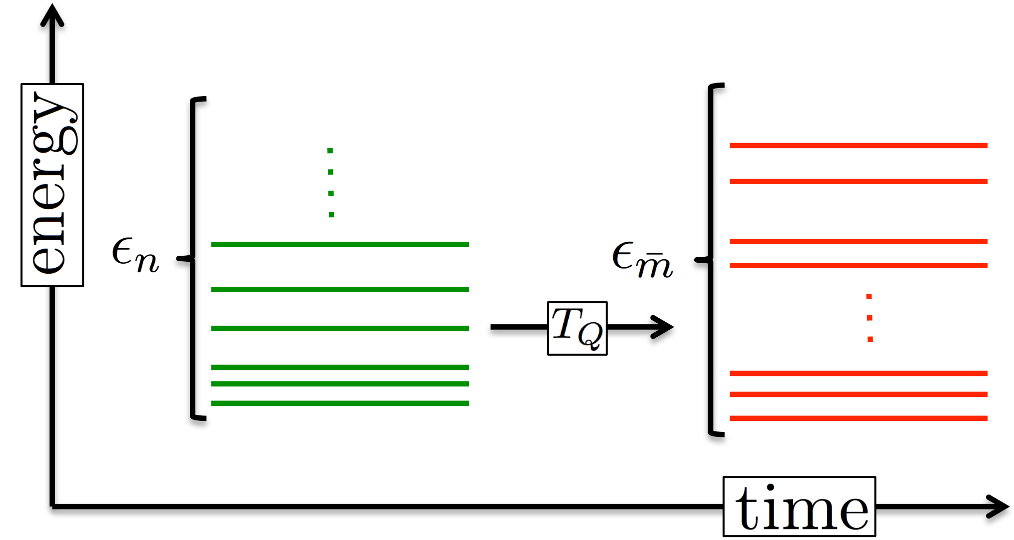
Critical Point

$$\omega_c = \sqrt{7\zeta(3)/2}$$

Excitations linear phase

$$\omega_k = \sqrt{\omega^2 - 4\sum_{j=1}^{L/2} \frac{1}{j^3} \sin^2 \frac{jk}{2}}$$

Tools



Work distribution

$$P_F(W) \equiv \sum_{n,\bar{m}} p_n^0 p_{\bar{m}|n}^{t_f} \delta[W - (\epsilon_{\bar{m}} - \epsilon_n)]$$

Characteristic function

$$\chi_F(t) = \mathrm{Tr}[e^{iH_f t} U(t_f, 0)^\dagger e^{-iH_i t} U(t_f, 0) \rho_i]$$

Crooks-Tasaki relation

$$\frac{P_F(W)}{P_B(-W)} = e^{\beta(W-\Delta F)}$$

Moments

$$M^{(n)} = (-i)^n \frac{\partial^n \ln \chi_F(t)}{\partial t^n}\bigg|_{t=0}$$

$$M^{(1)} = \langle W \rangle$$

Measure of Irreversibility

$$\langle W_{IRR} \rangle = \langle W \rangle - \Delta F$$

Same phase quench

Transverse frequency quench

$$\hat{H}_i = V + \sum_k \hat{P}_k^2 + \frac{\omega_k^2(t)}{2}\hat{Q}_k^2$$

Time-dependency of the quench

$$\omega_k^2(t) = \omega_k^2(0) - \frac{t}{T_Q}(\omega_k^2(0) - \omega_k^2(T_Q))$$

Same phase quench

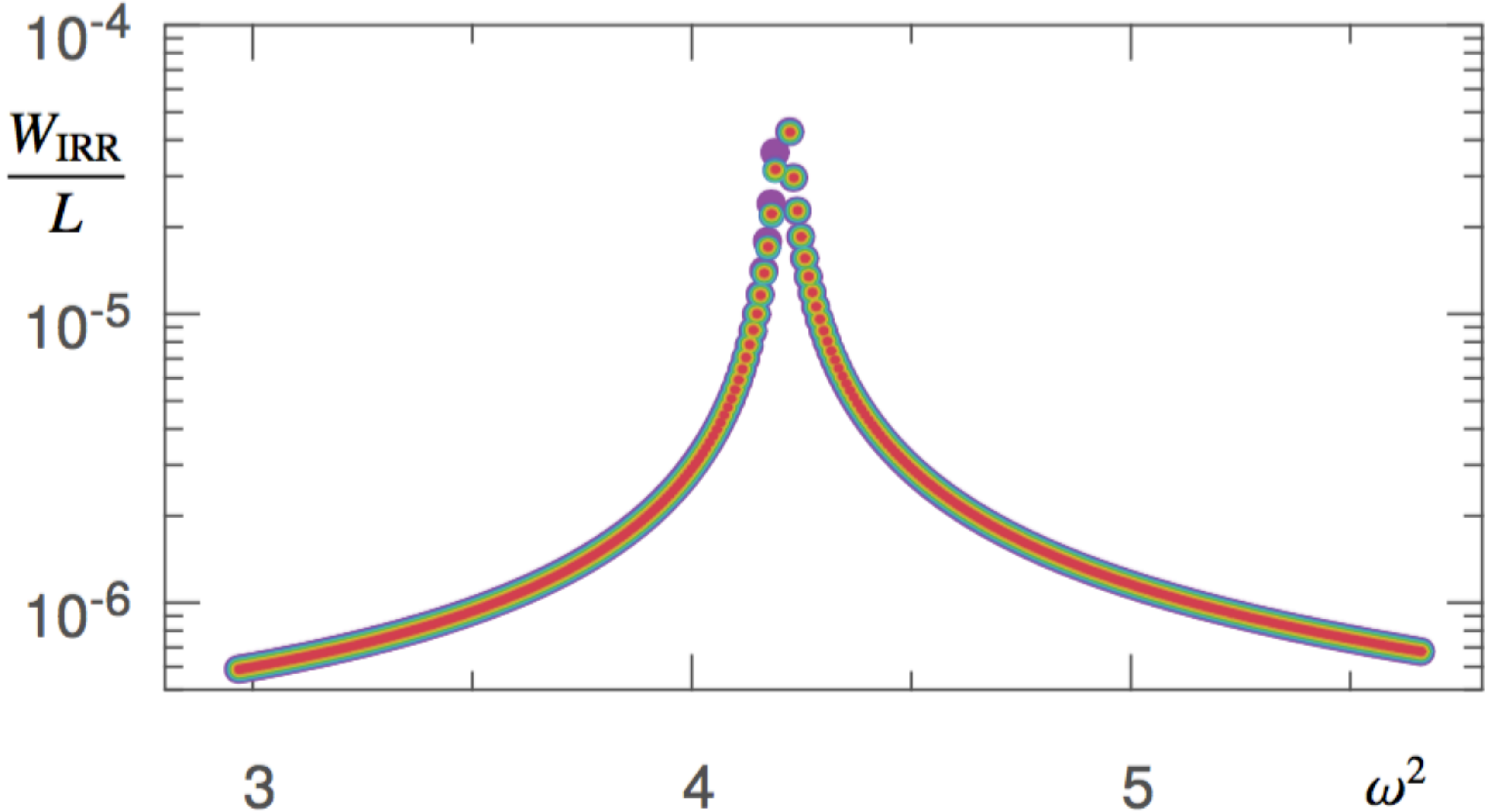
Characteristic function

$$\chi_F(t) = \prod_k \frac{\sqrt{2}(1 - e^{-\beta\omega_k(0)})e^{it\frac{(\omega_k(T_Q)\omega_k(0))}{2}}}{\sqrt{D_k(T_Q)}}$$

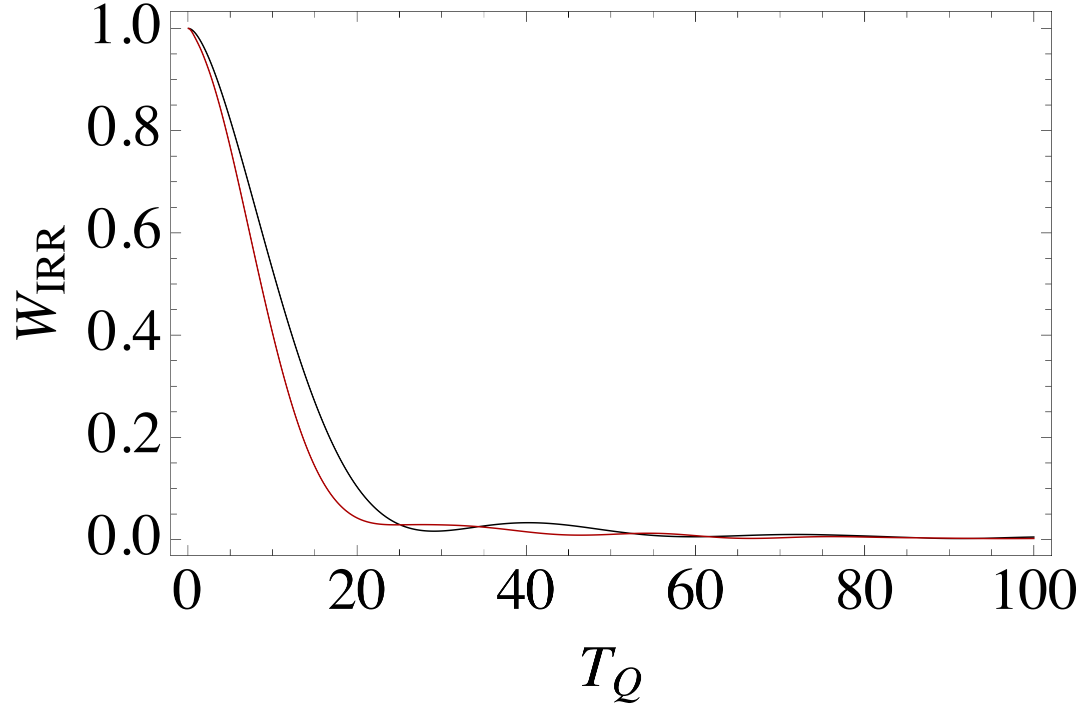
Irreversible work

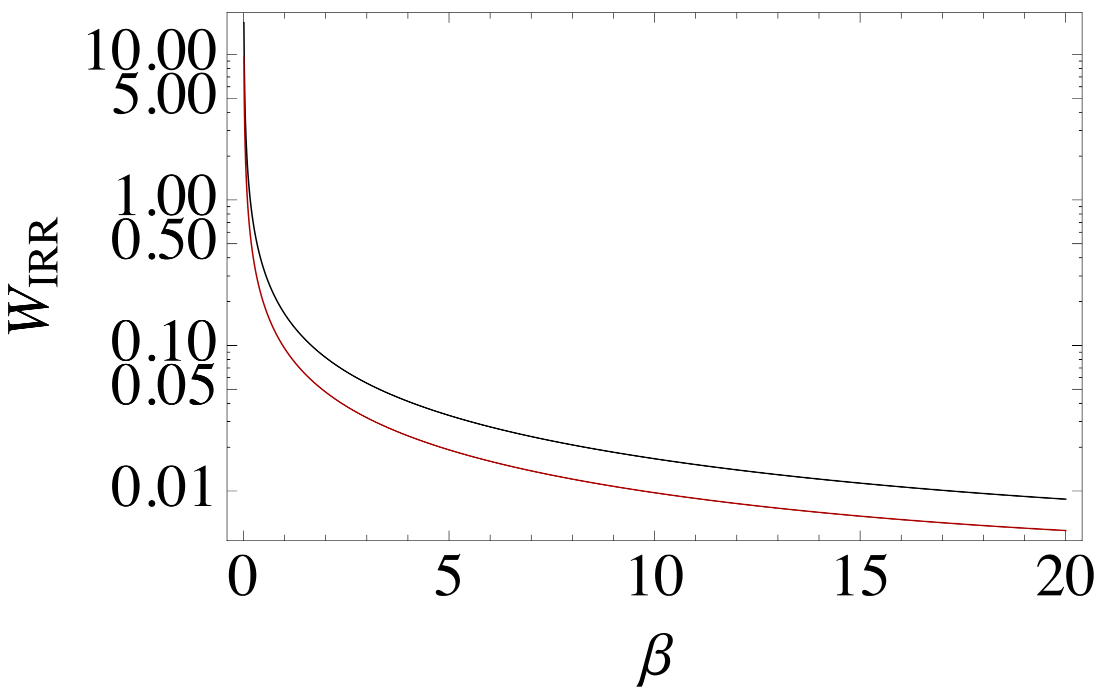
$$\langle W_{IRR}\rangle = \sum_k \frac{1}{2}(\Omega_k\omega_k(T_Q) - \omega_k(0))\coth(\beta\omega_k(0)/2)$$
$$- \frac{1}{\beta}\ln\frac{\sinh\left(\beta\omega_k(T_Q)/2\right)}{\sinh(\beta\omega_k(0)/2)})$$

Sudden Quench



Quenching time





Temperature

Anharmonic model: DMRG

Short range model

$$H(\omega)=\frac{1}{2}\sum_{j=1}^{L}\left(-g^2\frac{\partial^2}{\partial y_j^2}+(\omega^2-h_1)y_j^2+h_2(y_j+y_{j+1})^2+h_3y_j^4\right)$$

Effective Planck constant g $\quad g=\sqrt{\frac{4\pi\epsilon_0\hbar^2}{MaQ^2}}$

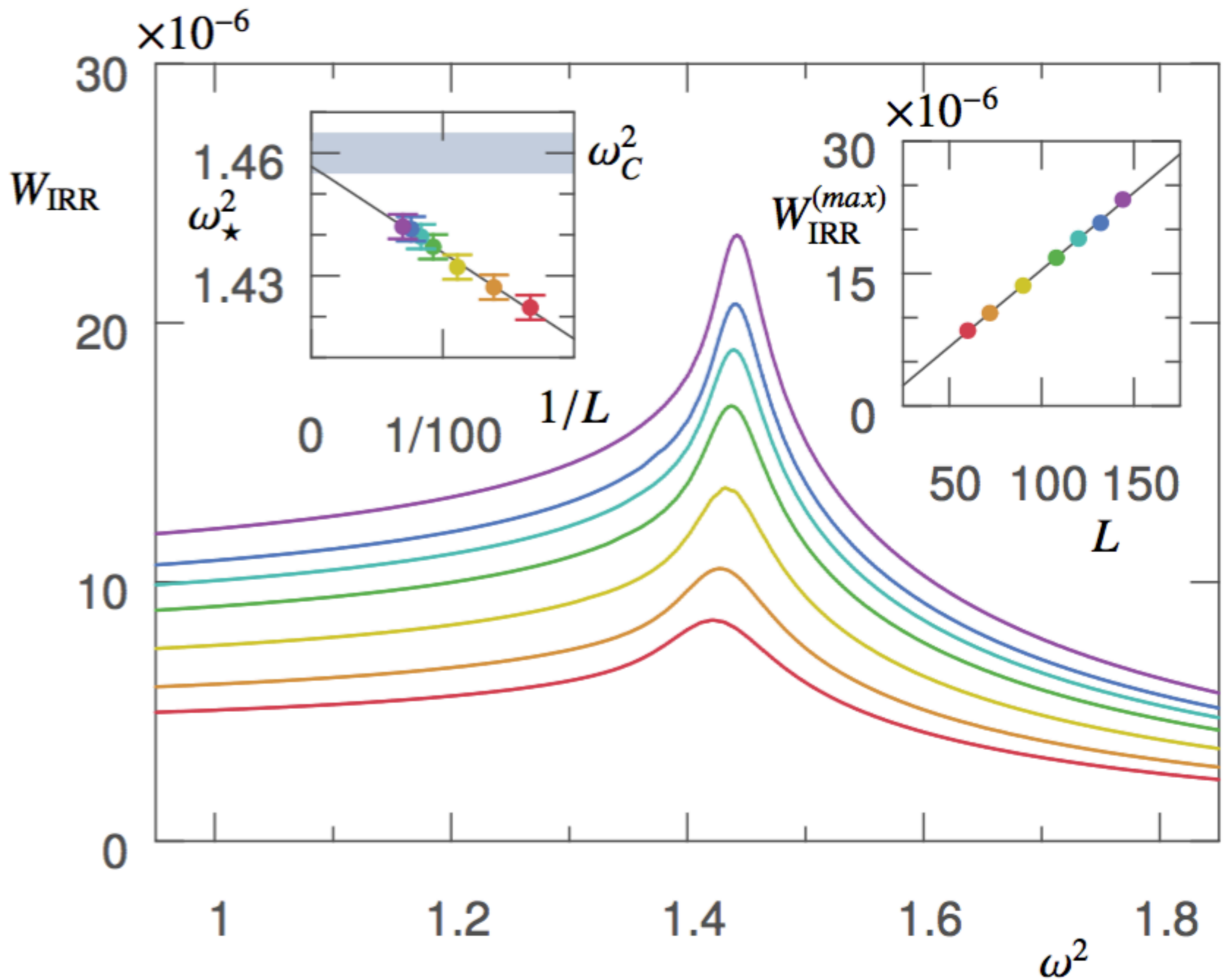
Average work

$$\langle W\rangle = E_G(\omega_f)+\frac{1}{2}(\omega_f^2-\omega_i^2)\,\mathcal{Y}^2(\omega_i)-E_G(\omega_i)$$

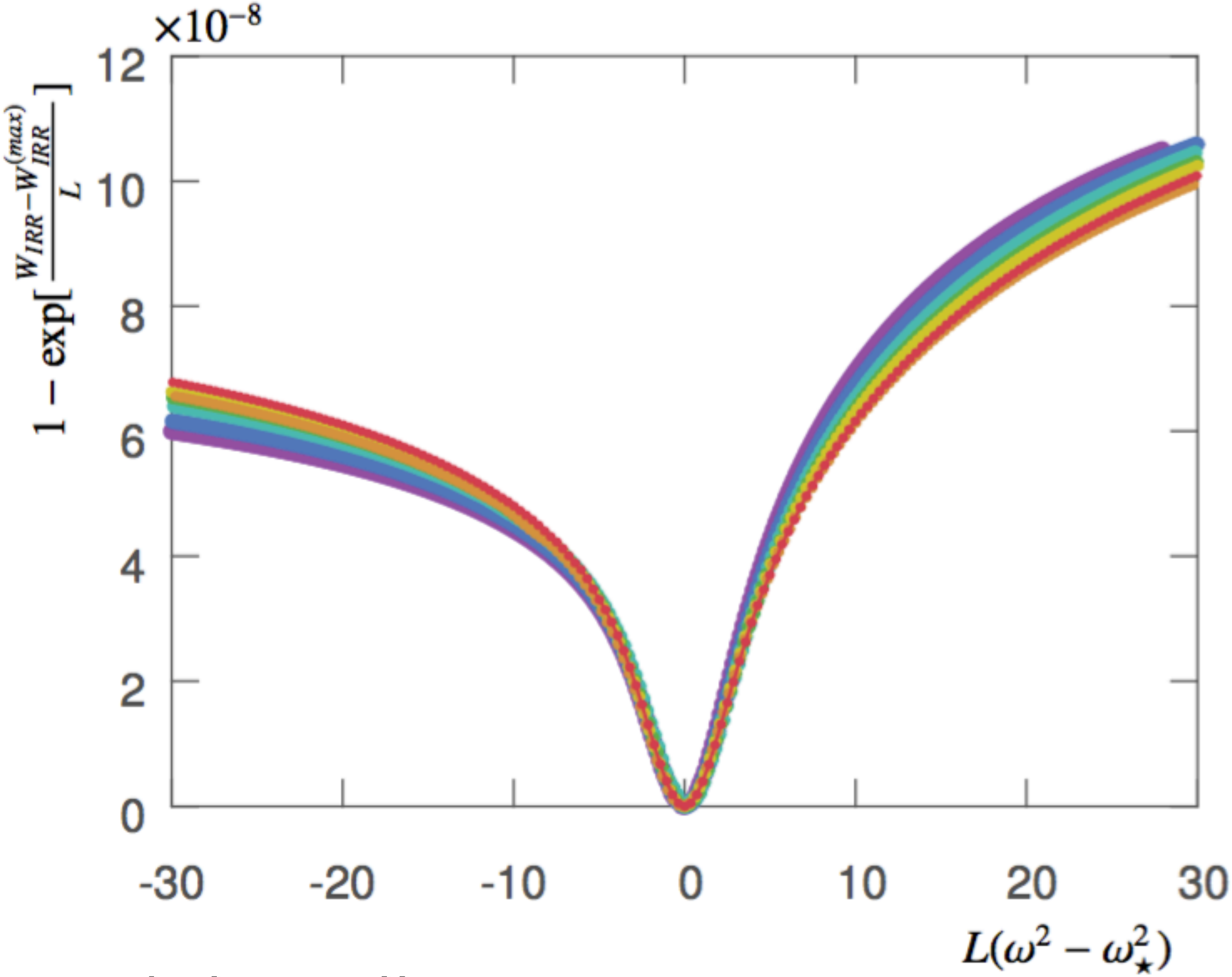
$$\mathcal{Y}^2(\omega)=\sum_j\langle\Psi_G(\omega)|y_j^2|\Psi_G(\omega)\rangle$$

P. Silvi, T.Calarco, G. Morigi, and S.Montangero, Phys. Rev. B 89, 094103

Results: Zero temperature



Scaling: $1 - e^{\frac{W_{IRR} - W_{IRR}^{(max)}}{L}} = g(L^{\frac{1}{\nu}}(\omega^2 - \omega_\star^2))$



Ising model scaling

S. Paganelli and T. J. G. Apollaro, Int. J. Mod. Phys. B 31, 1750065 (2017)

Conclusions and Future Prospectives

- Characterization of the Irreversibility
- Same scaling of the Ising model
- Single ion as a probe

Thank you for the attention!