

# Quantum probing of complex systems

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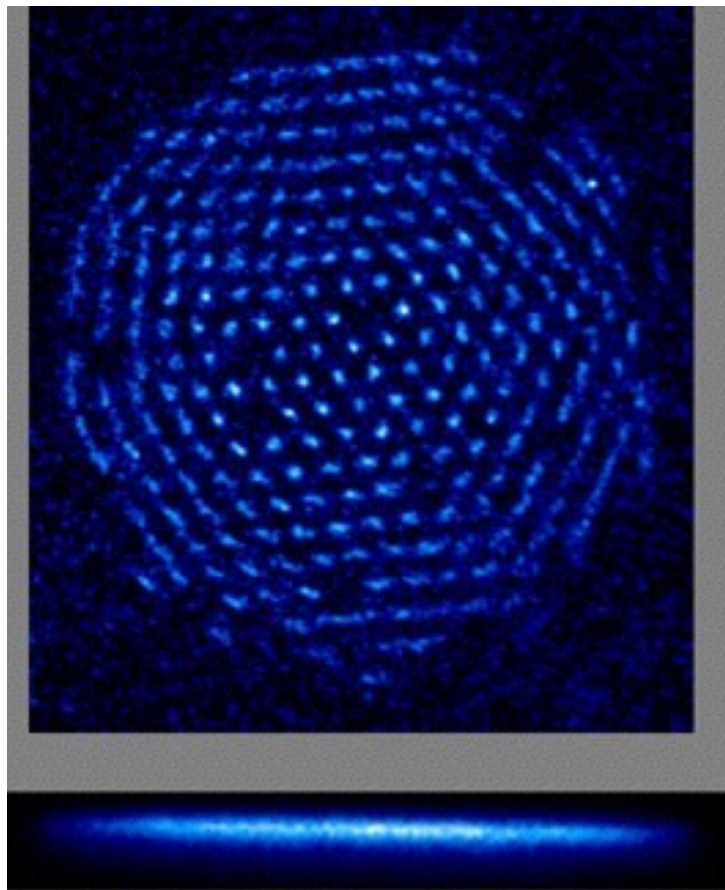


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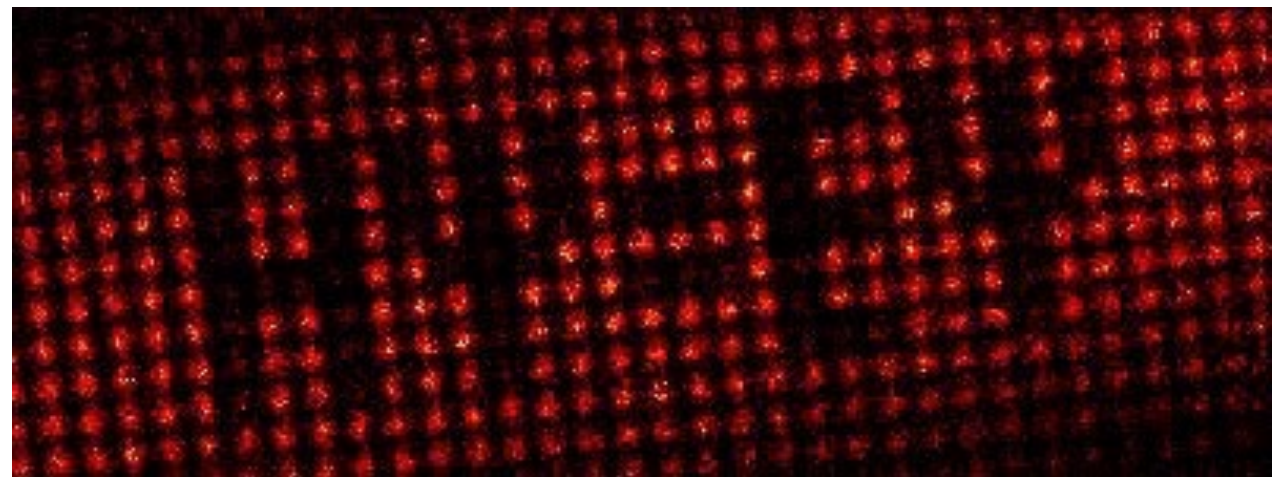


# complex systems

ionic crystals



cold atoms in optical lattices



photonic systems

complex system = really messy Hamiltonian!  
too many degrees of freedom, non-integrable, etc..



classical response theory...can be rather invasive

can we find a way to probe a complex system without  
disturbing it too much?

a well-defined, controllable quantum system

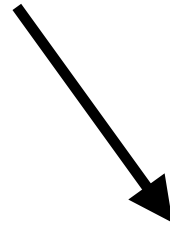


$V_{P-CS}$  = engineered interaction that would induce minimum disturbance on CS

**Q: can we map some features of CS onto the dynamics of the single quantum system? Also, if this is possible, what kind of information can we extract and how?**

tools from

- many-body theory
- open quantum system theory
- statistical physics
- quantum information theory



to investigate

- excitation spectra
- correlations/fluctuations
- decoherence
- non equilibrium properties

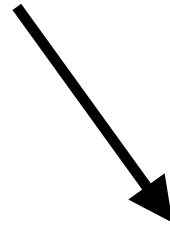


**controllable quantum simulators**



tools from

- many-body theory
- open quantum system theory
- statistical physics
- quantum information theory

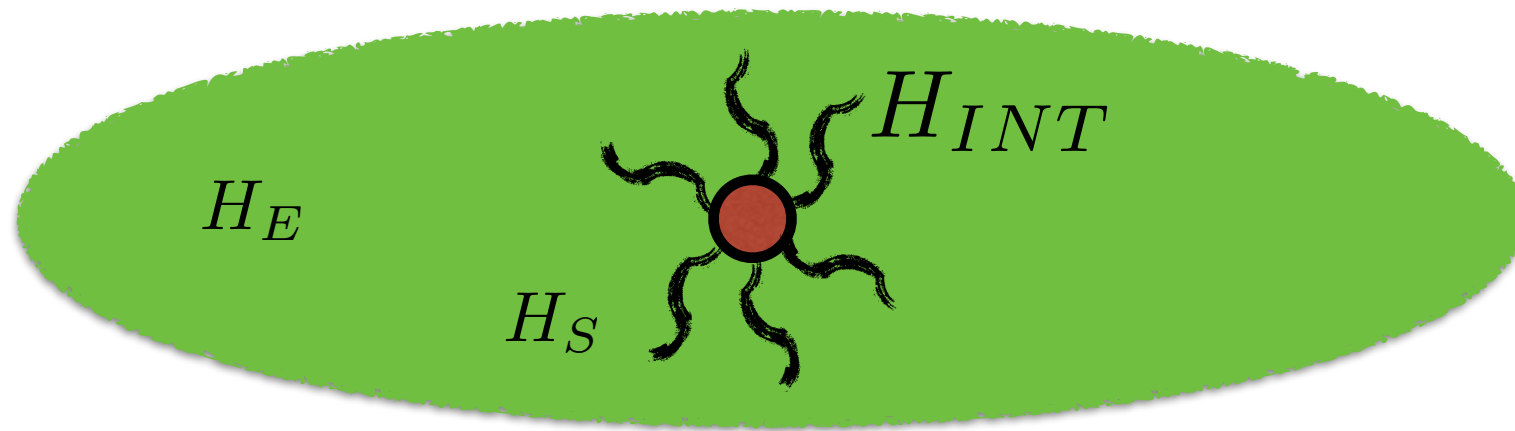


to investigate

- excitation spectra
- correlations/fluctuations
- decoherence
- non equilibrium properties



**controllable quantum simulators**



open quantum system

- dissipation
- decoherence

in a Markovian open system (no memory effects) all the quantum properties are lost within a certain time-scale

$$\frac{d}{dt}\rho_S = -i[H, \rho_S] + \sum_j \gamma_j \left( A_j \rho_S A_j^\dagger - \frac{1}{2} \{A_j^\dagger A_j, \rho_S\} \right)$$

- $A_j$  jump operators
- $\gamma_j \geq 0$
- spontaneous decay
- thermalisation of an e.m. mode in an optical cavity
- random walk



a tool to detect non-Markovian behaviour is the trace distance between quantum states

$$D(\hat{\rho}_1, \hat{\rho}_2) \equiv \frac{1}{2} \|\rho_1 - \rho_2\|_1$$

$$\|\hat{O}\|_1 = \text{Tr} \sqrt{\hat{O} \hat{O}^\dagger}$$



$$D(\rho_1(t), \rho_2(t)) \leq D(\rho_1(0), \rho_2(0)) \quad \forall \rho_1, \rho_2 \forall t \geq 0.$$

**contractivity property**

$$\sigma(t, \rho_{1,2}(0)) \equiv \frac{d}{dt} D(\rho_1(t), \rho_2(t))$$

if  $\sigma(t, \rho_{1,2}(0)) > 0$  for some  $\rho_1(0), \rho_2(0), t$   
the dynamics is non-Markovian

a possible measure for the  
degree of non-Markovian  
character is

$$\mathcal{N} = \max_{\rho_{1,2}(0)} \int_{\sigma > 0} dt \sigma(t, \rho_{1,2}(0))$$

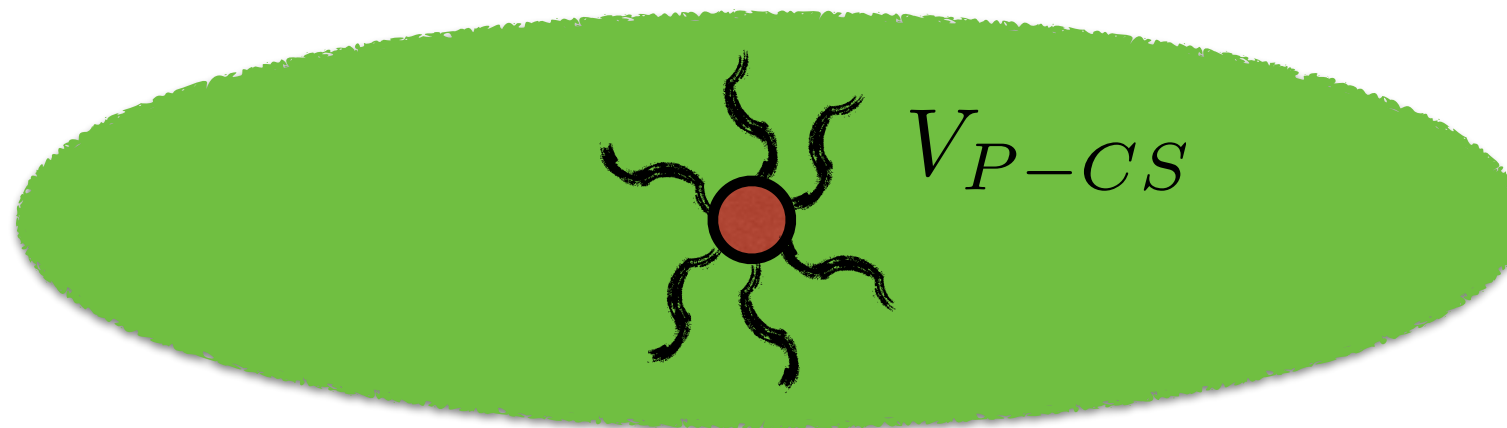
H.-P. Breuer, E.-M. Laine, and J. Piilo, Phys. Rev. Lett.  
**103**, 210401 (2009).

analytical results for 2-level systems

H.-P. Breuer, E.-M. Laine, J. Piilo, and B.  
Vacchini, arXiv:1505.01385 (2015)

how is this relevant for studying  
complex/many-body systems?

a well-defined, controllable quantum system

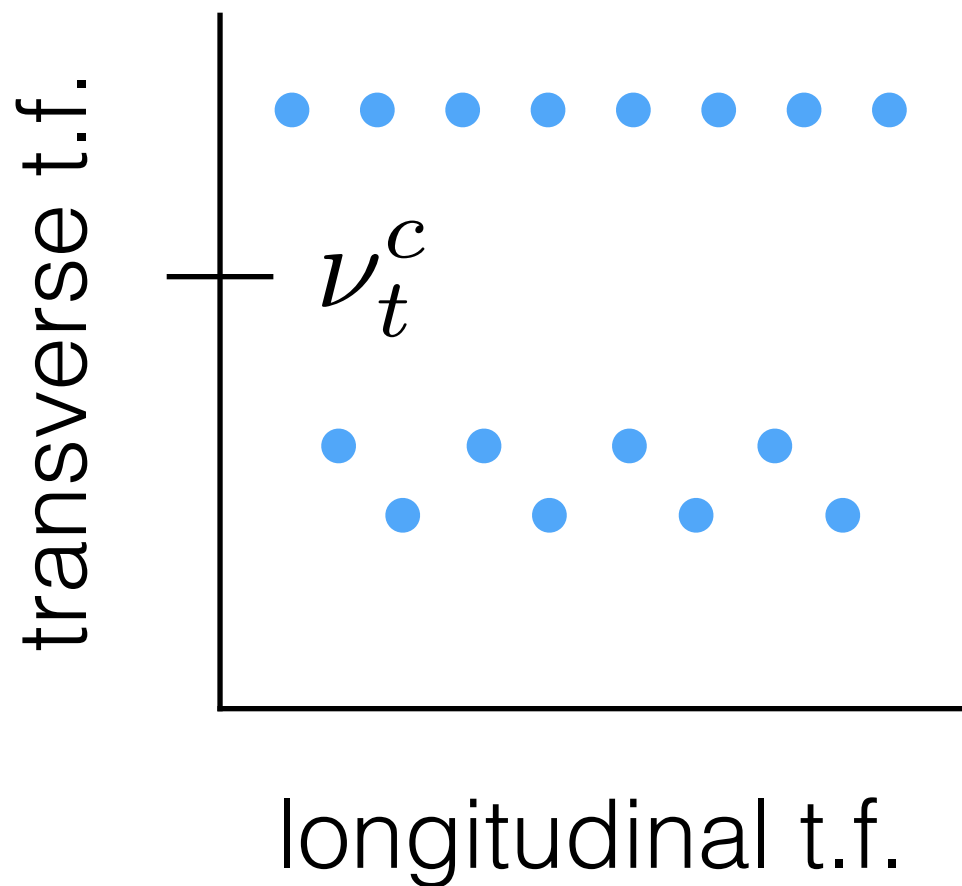


$V_{P-CS}$  = engineered interaction that would induce minimum disturbance on CS

**the complex/many-body system acts as an environment for the single controllable quantum system! By studying the open system dynamics we can infer some of the environment properties!**

critical phenomena

# example 1: Coulomb crystals in magnetic traps



- harmonic confinement (magnets)
- Coulomb repulsion (ions)

features:

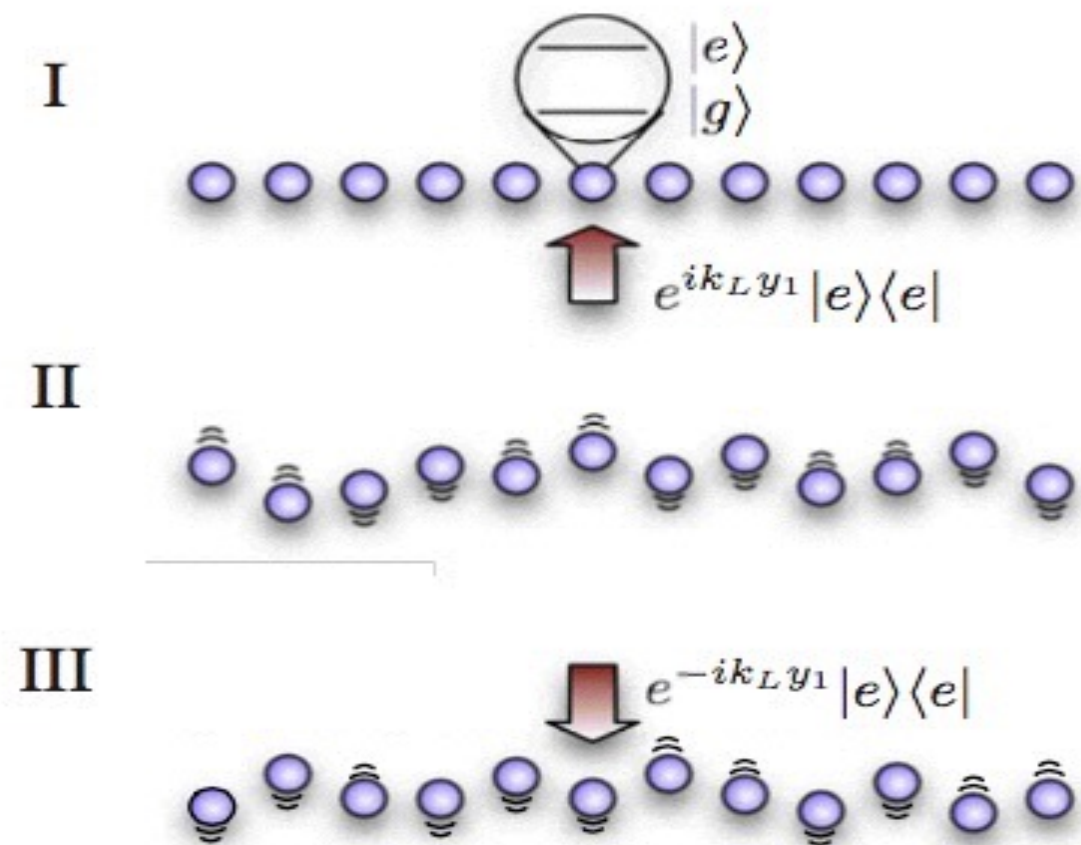
- it is structural
- second order with a closing gap
- same universality class as Ising
- it is quantum (?)

G. De Chiara, T. Calarco, S. Fishman, and G. Morigi, Phys. Rev. A **78**, 043414 (2008)

can we find a quantum probing scheme for this phase transition? what do we get out of it?

# probing protocol: Ramsey interferometry with a single qubit

the dynamics of the qubit is both dissipative and decoherent and (quite a mess)



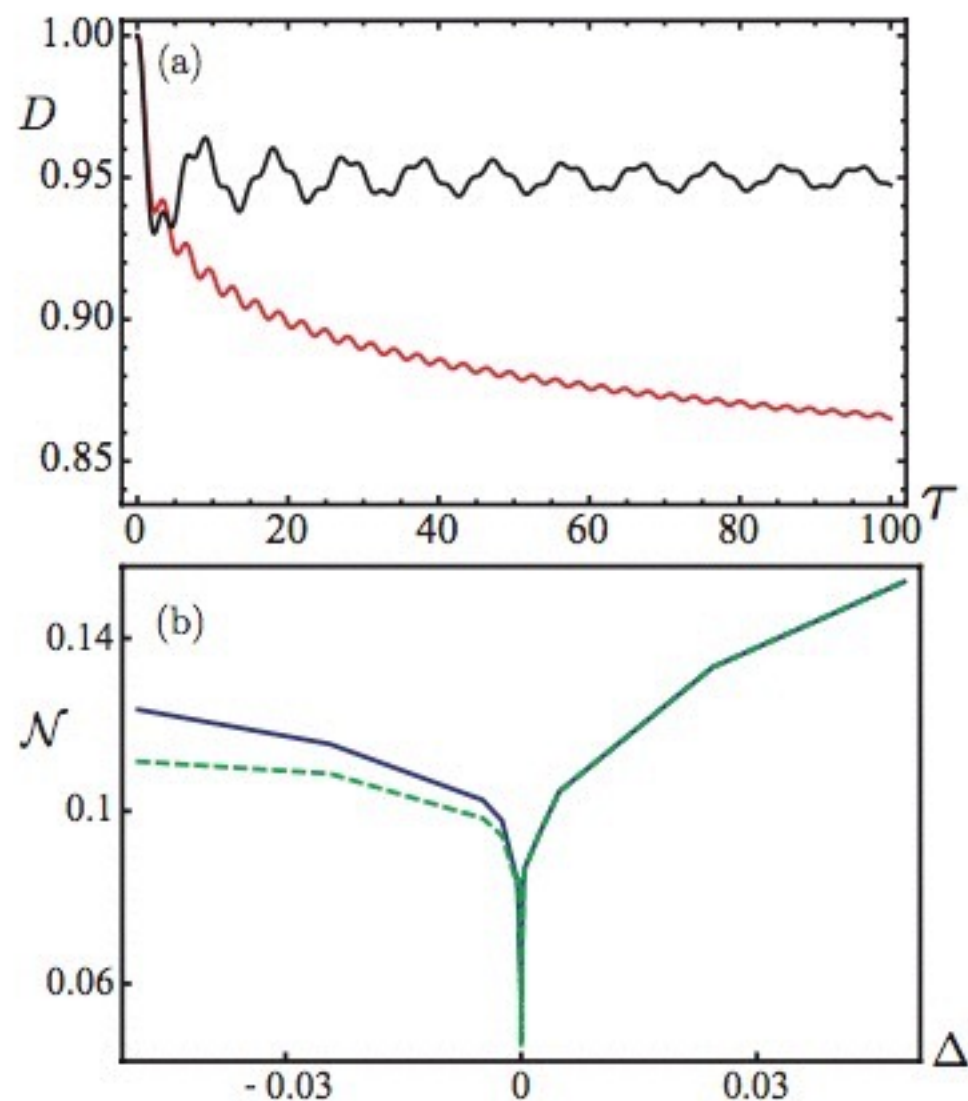
however, we can calculate the non-Markovianity properties and, even better, connect them the visibility of the Ramsey fringes (experimental shortcut to measure non-Markovianity)



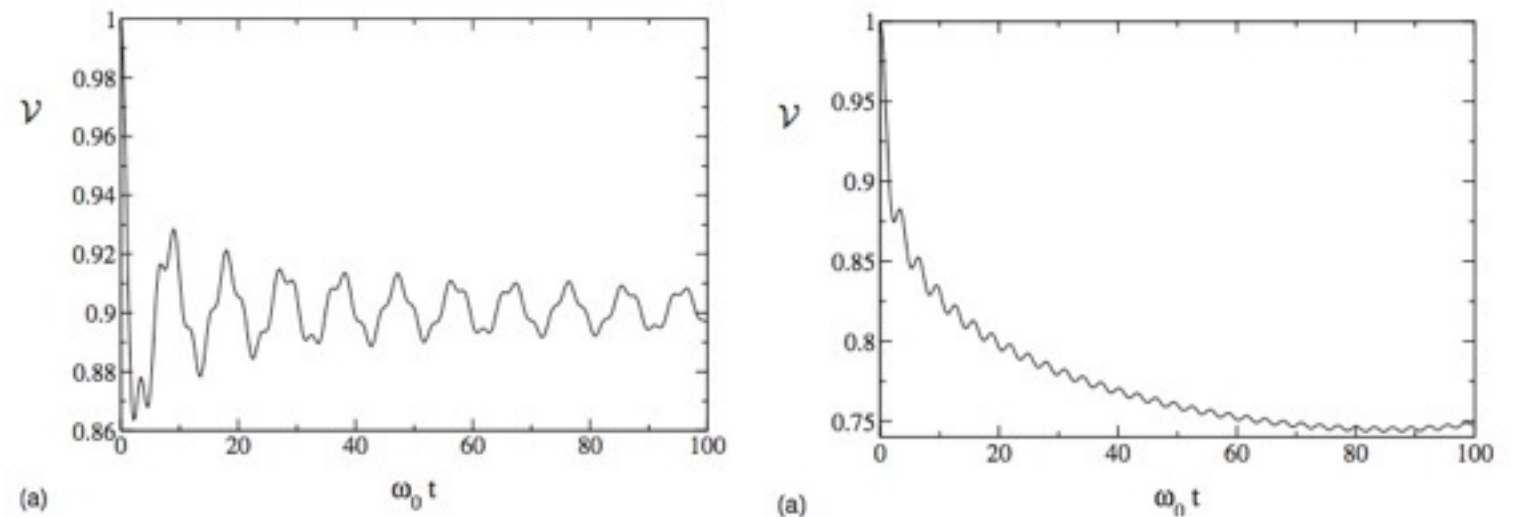
$$D_{\text{opt}}(t) = \frac{1}{4} |1 + 2 \cos B(t) [\mathcal{V}(t) - \xi^4 \mathcal{V}^{-1}(t)] + \mathcal{V}^4(t) + 2\xi^4|$$

the optimal trace distance is a function of the visibility fringes!

M. Borrelli, P. Haikka, G. De Chiara, and S. Maniscalco, Phys. Rev. A **88**, 010101 (R) (2013)



near the critical point the dynamics becomes almost non-Markovian



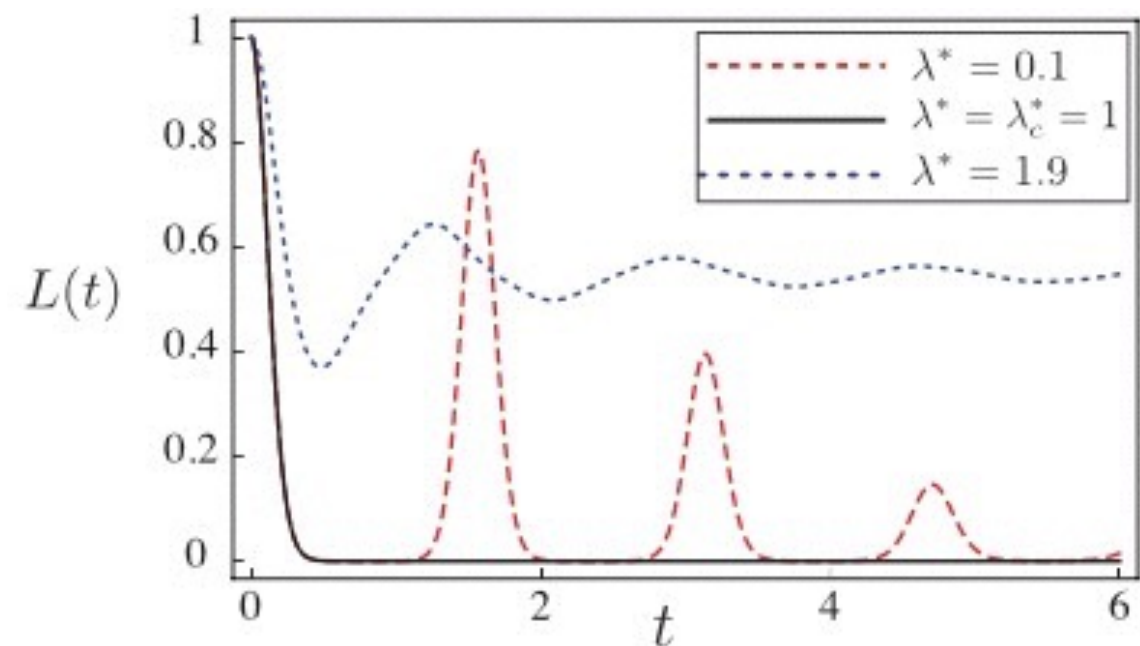
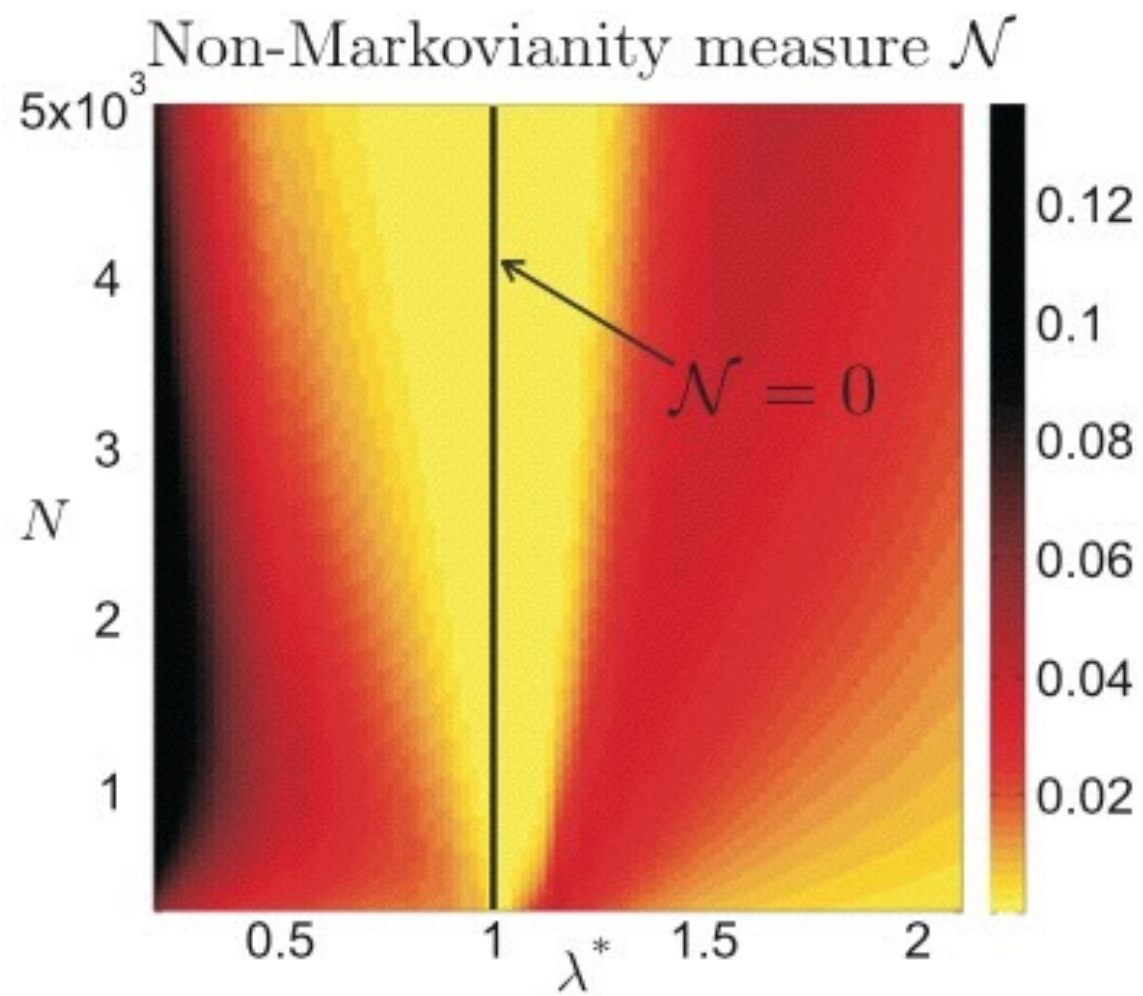
G. De Chiara, T. Calarco, S. Fishman, and G. Morigi, Phys. Rev. A **78**, 043414 (2008)

## example 2: decoherence in a transverse Ising model

$$H = -J \sum_i \sigma_i^z \sigma_{i+1}^z + \lambda \sigma_i^x + \delta |e\rangle \langle e| \sigma_i^x$$

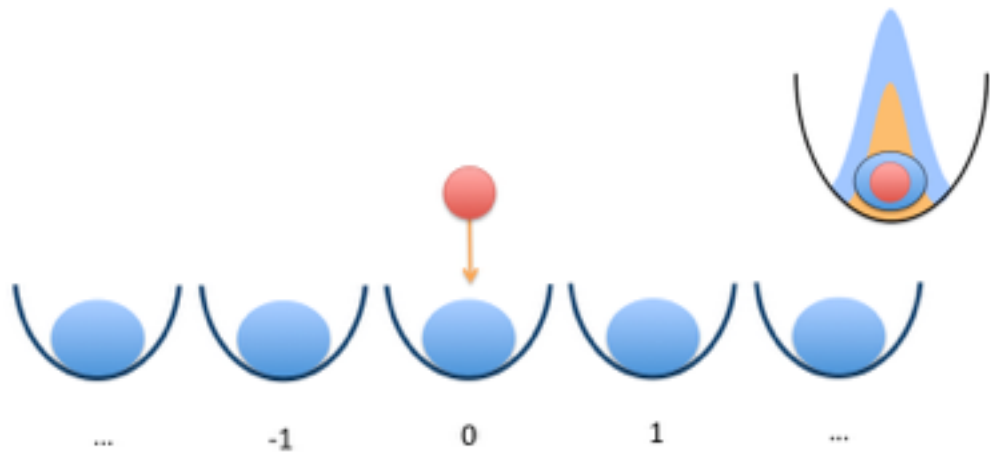
$$D_{\text{opt}}(t) = \sqrt{L(t)}$$

P. Haikka *et al.*, Phys. Rev. A **85**, 060101(R) (2012)



at criticality the dynamics is completely Markovian

## example 2: Bose-Hubbard model in ultracold atoms



- lasers creating a standing wave pattern
- an atomic impurity that does not significantly perturb the lattice

$$H_{BH} = \underbrace{-J \sum_{i=1}^{N_S} (a_{i+1}^\dagger a_i + a_i^\dagger a_{i+1})}_{\text{kinetic}} + \underbrace{\frac{U}{2} \sum_{i=1}^{N_S} n_i(n_i - 1)}_{\text{interaction}} - \mu \sum_{i=1}^{N_S} n_i$$

transition from a superfluid  $J \gg U$  to an insulating phase  $J \ll U$   
depending on the ratio  $J/U$  and filling factor  $\bar{n}(J/U, \mu/U)$

Wanna hear the results?  
Wait for the next talk :-)

- preliminary results show that a quantum probe based approach to many-body systems might be promising
- none or very little perturbation of the many-body Hamiltonian
  - the decoherence in the open system dynamics signals critical features of the many-body environment
  - some statistical knowledge of the environment can be achieved by looking at the system only

to do's next:

- higher order correlation functions
- possibility of mapping entanglement in the environment onto the state of probe/s

Thank you for your attention!

Grazie per l'attenzione!

Kiitos!

Vielen Dank für Ihre Aufmerksamkeit!

İlginiz için teşekkürler!

Dziękuję za uwagę!