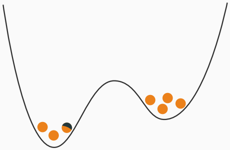
Dynamics of a bosonic Josephson junction with an impurity

QuPro CS II — IFISC (Mallorca)

Maximilian Dirkmann, Gabriel Dufour, and Andreas Buchleitner

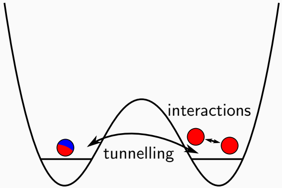
6-7/4/2017

Albert-Ludwigs-Universität Freiburg



Introduction

The Josephson junction with an impurity

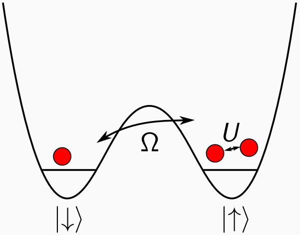


- Symmetric double well with one mode per site.
- Hopping from one mode to the other by tunnelling through the barrier.
- On-site interactions between the particles.
- All particles are indistinguishable bosons except for the impurity.

Quantum probing

Single species system

Bose-Hubbard model[1] and Schwinger representation[2]



$$\hat{J}_x \equiv (\hat{a}_R^\dagger \hat{a}_L + \hat{a}_L^\dagger \hat{a}_R)/2\,,$$
$$\hat{J}_y \equiv (\hat{a}_R^\dagger \hat{a}_L - \hat{a}_L^\dagger \hat{a}_R)/2i\,,$$
$$\hat{J}_z \equiv (\hat{a}_R^\dagger \hat{a}_R - \hat{a}_L^\dagger \hat{a}_L)/2 \ = (\hat{n}_R - \hat{n}_L)/2$$

$$\boxed{\hat{H} = -\Omega \hat{J}_x + U \hat{J}_z^2}$$

$|j, m\rangle$ as eigenvectors of:

$\hat{\mathbf{J}}^2 |j, m\rangle = j(j+1) |j, m\rangle$, spin $j = \frac{N}{2}$

$\hat{J}_z |j, m\rangle = m |j, m\rangle$, spin magnetic quantum number,

$m \in \{-j, -j+1, ..., j-1, j\} \mathrel{\widehat{=}}$ population imbalance,

$(\hat{n}_R - \hat{n}_L) |j, m\rangle = 2\hat{J}_z |j, m\rangle = 2m |j, m\rangle$

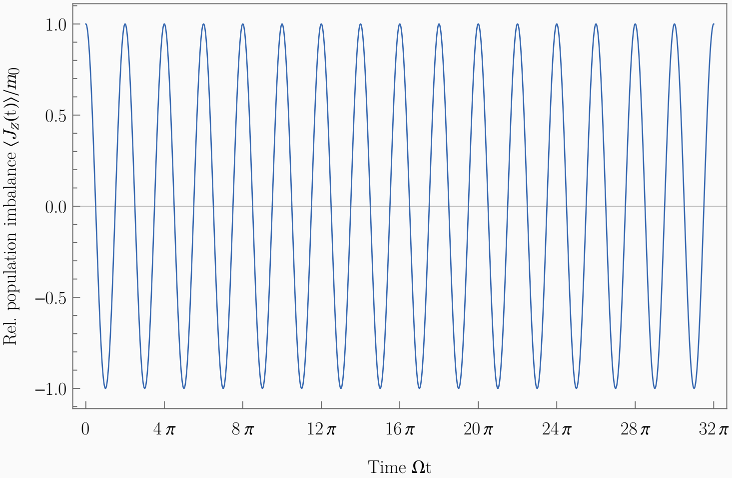
[1] G. J. Milburn et al., *Phys. Rev. A* **55**, 4318–4324 (1997).

[2] B.J. Dalton and S. Ghanbari, *J. Mod. Opt.* **59**, 287–353 (2012),
J. Schwinger, *US Atomic Energy Commission Report* No. NYO-3071, 229 (1952).

Single species system

Dynamical regimes

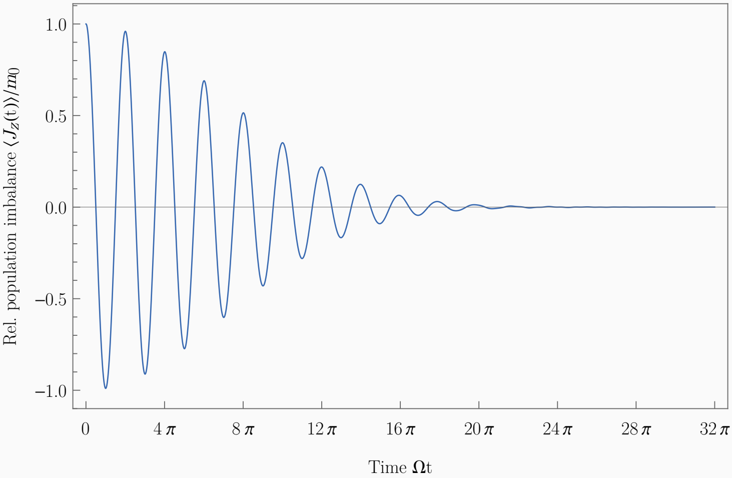
Dynamical Regimes[3]



$N = 10$ particles, initial imbalance $m_0 = 5$ and $U = 0\frac{\Omega}{N}$

[3] A. J. Leggett, *Rev. Mod. Phys.* **73**, 307–356 (2001).

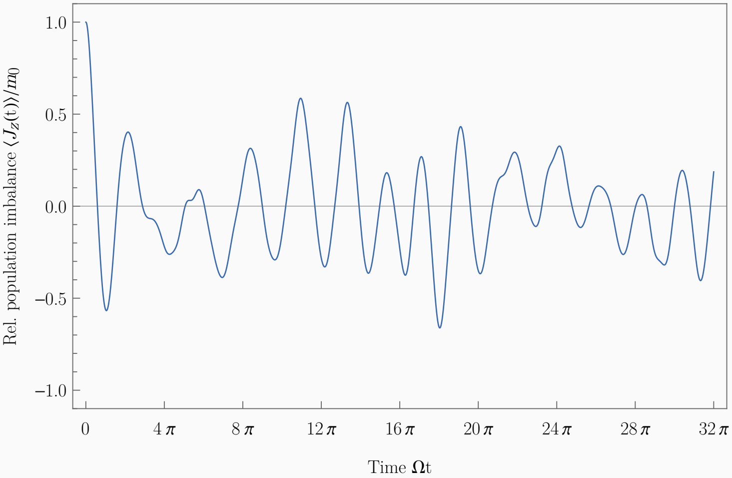
Dynamical Regimes[3]



$N = 10$ particles, initial imbalance $m_0 = 5$ and $U = 0.3\frac{\Omega}{N}$

[3] A. J. Leggett, *Rev. Mod. Phys.* **73**, 307–356 (2001).

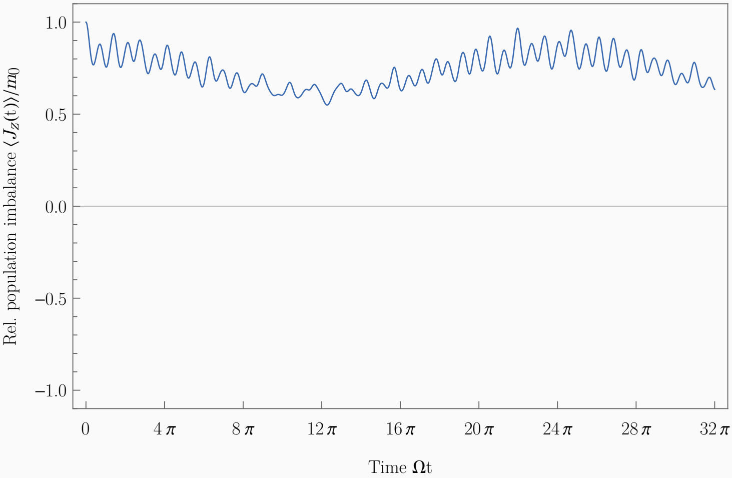
Dynamical Regimes[3]



$N = 10$ particles, initial imbalance $m_0 = 5$ and $U = 1.5\frac{\Omega}{N}$

[3] A. J. Leggett, *Rev. Mod. Phys.* **73**, 307–356 (2001).

Dynamical Regimes[3]



$N = 10$ particles, initial imbalance $m_0 = 5$ and $U = 3.5\frac{\Omega}{N}$

[3] A. J. Leggett, *Rev. Mod. Phys.* **73**, 307–356 (2001).

Two species system

Two species Hamiltonian

- Tensor product structure $\mathcal{H}_1 \otimes \mathcal{H}_2$
- New families of operators $\hat{\mathbf{J}}_1, \hat{\mathbf{J}}_2$
- States are written as $|j_1, m_1\rangle \otimes |j_2, m_2\rangle = |j_1, m_1, j_2, m_2\rangle$

$$\hat{H} = -\Omega_1 \hat{J}_{x1} - \Omega_2 \hat{J}_{x2} + U_{1,1} \hat{J}_{z1}^2 + U_{2,2} \hat{J}_{z2}^2 + 2U_{1,2} \hat{J}_{z1} \hat{J}_{z2}$$

Isospecific conditions

$$\Omega = \Omega_1 = \Omega_2 \text{ and } U = U_{1,1} = U_{2,2} = U_{1,2}$$

Two species system

Mixing behaviour of the impurity with the majority species

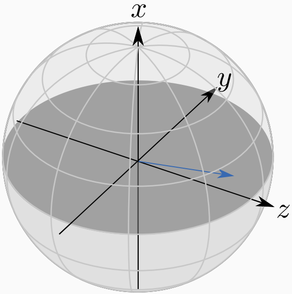
Purity of the impurity's state

- Reduced density matrix of the impurity

$$\hat{\rho}_2(t) = \mathrm{Tr}_1\left\{\hat{\rho}(t)\right\} = \frac{1}{2}\left(\mathbb{1} + \mathbf{s}(t)\cdot\boldsymbol{\sigma}\right)$$

- Purity

$$P_2(t) = \mathrm{Tr}\left\{\hat{\rho}_2(t)^2\right\} = \frac{1 + ||\mathbf{s}(t)||^2}{2}$$



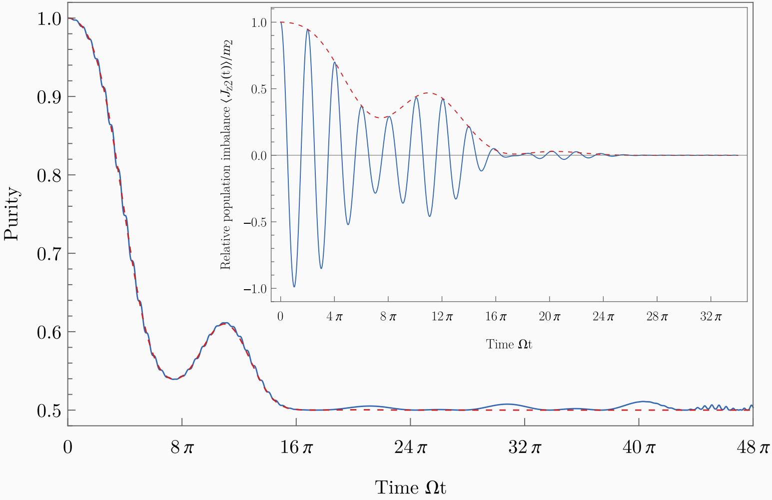
Weak interactions:

$$\mathbf{s}(t) = \frac{1}{2}||\mathbf{s}(t)||\left(\cos\left(\Omega t\right)\mathbf{e_z} + \sin\left(\Omega t\right)\mathbf{e_y}\right)$$

$$\langle \hat{J}_{z,2}(t)\rangle = \frac{1}{2}||\mathbf{s}(t)||\cos\left(\Omega t\right)$$

- **Link between envelope function and purity**

Link between envelope function and purity

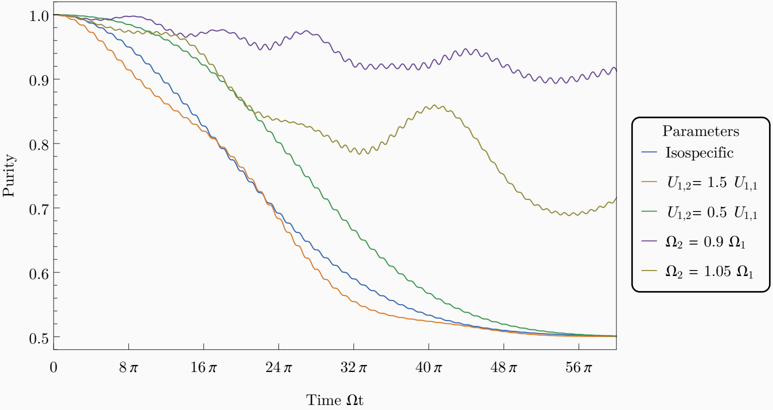


numerically exact solution

approximation through the envelope function

$N_1 = 20$ particles, $U_{1,1} = U_{1,2} = 0.4\frac{\Omega}{N}$.

Outlook: Purity for non-isospecific conditions



$N_1 = 15$, $U_{1,1} = 0.1\frac{\Omega_1}{N}$, initially all particles started in the same well.

Summary and conclusions

Summary and conclusions

- Weak interactions introduce an envelope to the unhindered Rabi oscillations
- Direct link between purity and oscillations of the impurity particle
- "Quantum probing" possible through measuring the impurity's oscillations

Questions and discussion