Characterization of qubit chains by Feynman probes

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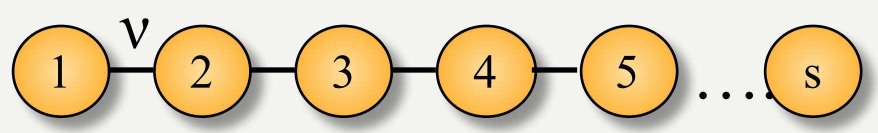
WHY?

Characterization of qubit registers or spin networks

- Quantum computers
- Quantum communication
- State transfer
- Reservoir engineering
- ...



Requirement: Fine tuning of the interaction parameters

Coupling constant may be inaccessible quantities

QET provides tools to evaluate the ultimate precision of any estimation procedure

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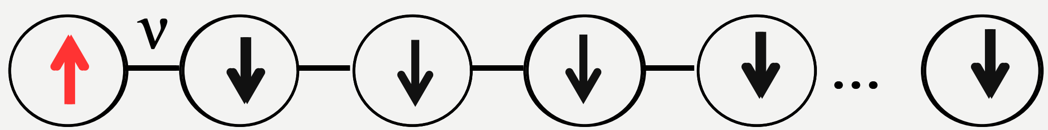
Characterization of qubit registers or spin networks

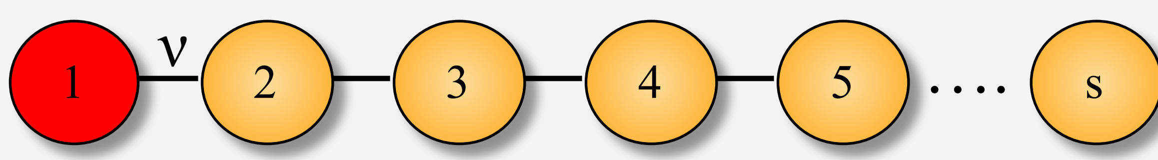


$$\mathcal{H}_0 = -\frac{\nu}{2}\sum_{j=1}^{s-1} \sigma_+^{j+1}\sigma_-^{j} + \sigma_+^{j}\sigma_-^{j+1}$$

$\sigma_\pm^j = \frac{1}{2}(\sigma_x^j \pm i\sigma_y^j)$ $\quad$ $N_z = \sum_{j=1}^{s} \frac{1}{2}(\mathbb{I} + \sigma_z^j)$

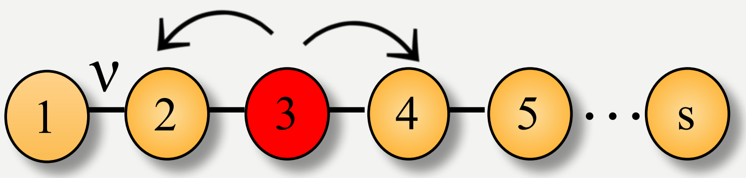
Single excitation

$$\mathcal{H}_0 = -\frac{\nu}{2}\sum_{j=1}^{s-1} |j+1\rangle\langle j| + |j\rangle\langle j+1|$$



A Quantum Walk on the Line

$$\mathcal{H}_0 = -\frac{\nu}{2}\sum_{j=1}^{s-1} |j+1\rangle\langle j| + |j\rangle\langle j+1|$$



$$e_k(\nu) = -\nu \cos\left(\frac{k\pi}{s+1}\right)$$

$$|e_k\rangle = \sqrt{\frac{2}{s+1}}\sum_{j=1}^{s} \sin\left(\frac{k\pi j}{s+1}\right)|j\rangle$$

$$\lambda = \nu\tau$$

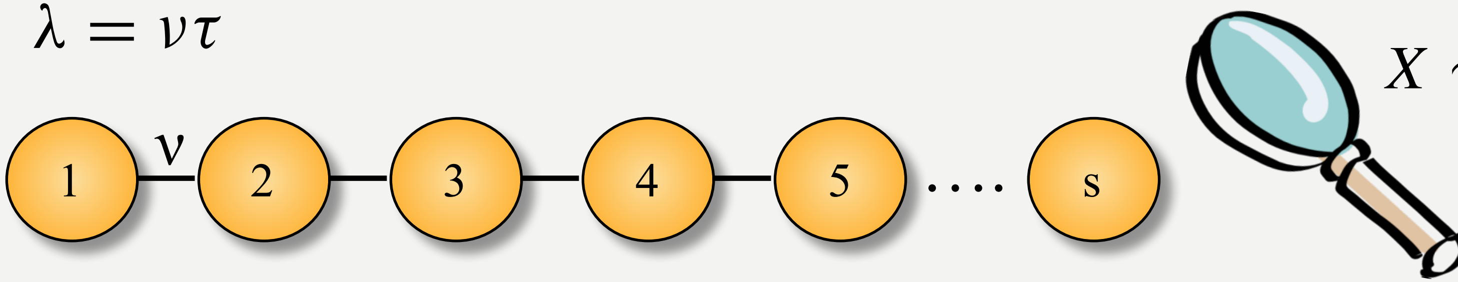
$$\mathcal{H}_0 = -\frac{\nu}{2}\sum_{j=1}^{s-1} |j+1\rangle\langle j| + |j\rangle\langle j+1|$$

$$U_\lambda = E^{-i\mathcal{H}_0\tau}$$

$$|\psi_\lambda\rangle = U_\lambda |x_0\rangle$$

Estimation schemes for the coupling constant

$$\lambda = \nu\tau$$



$$X \sim p(x|\lambda)$$

$\lambda(x_1, x_2, \ldots)$ **estimator**

$$F(\lambda) = E\left[\left(\frac{\partial}{\partial\lambda}\ln p(x|\lambda)\right)^2\right]$$ **Fisher information**

$$\mathrm{Var}\lambda \geqslant \frac{1}{M F(\lambda)}$$ **Cramèr-Rao Bound**

$$\mathrm{Var}\lambda \geqslant \frac{1}{M H(\lambda)}.$$ **Quantum CR Bound**

Unitary evolutions:

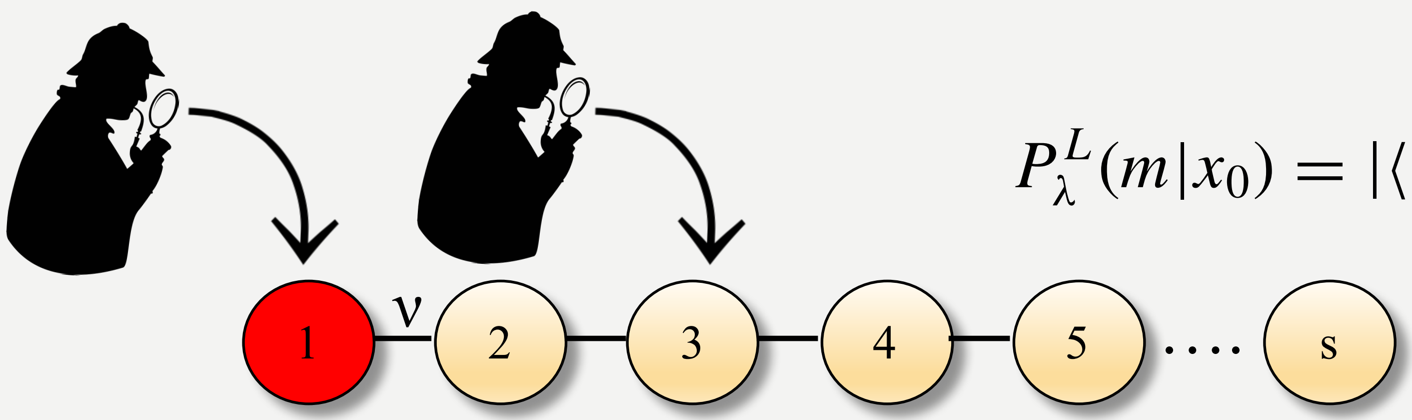
***H*=1** **x_0=1 or s**
***H*=2** **Other**

$$H = 4\langle\psi_0|G^2|\psi_0\rangle - (\langle\psi_0|G|\psi_0\rangle)^2 \qquad G = -\frac{1}{2}\sum_{j=1}^{s-1} |j+1\rangle\langle j| + |j\rangle\langle j+1|$$

Estimation scheme I: Local measurement

$$\mathcal{H}_0 = -\frac{\nu}{2}\sum_{j=1}^{s-1} |j+1\rangle\langle j| + |j\rangle\langle j+1|$$

$$\lambda = \nu\tau$$



$$P_\lambda^L(m|x_0) = |\langle\psi_\lambda|m\rangle|^2$$

$$\boxed{F_m^L(\lambda; x_0) = \frac{\left[\partial_\lambda P_\lambda^L(m|x_0)\right]^2}{P_\lambda^L(m|x_0)\left[1 - P_\lambda^L(m|x_0)\right]}}$$

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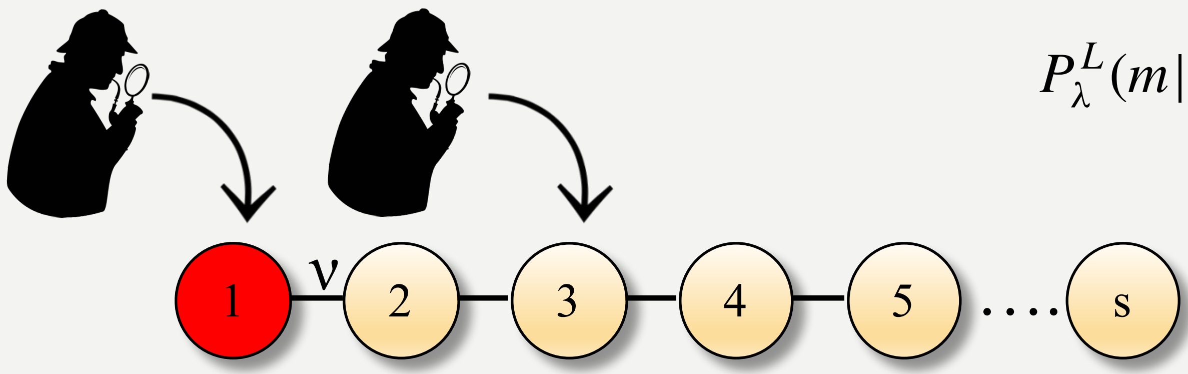
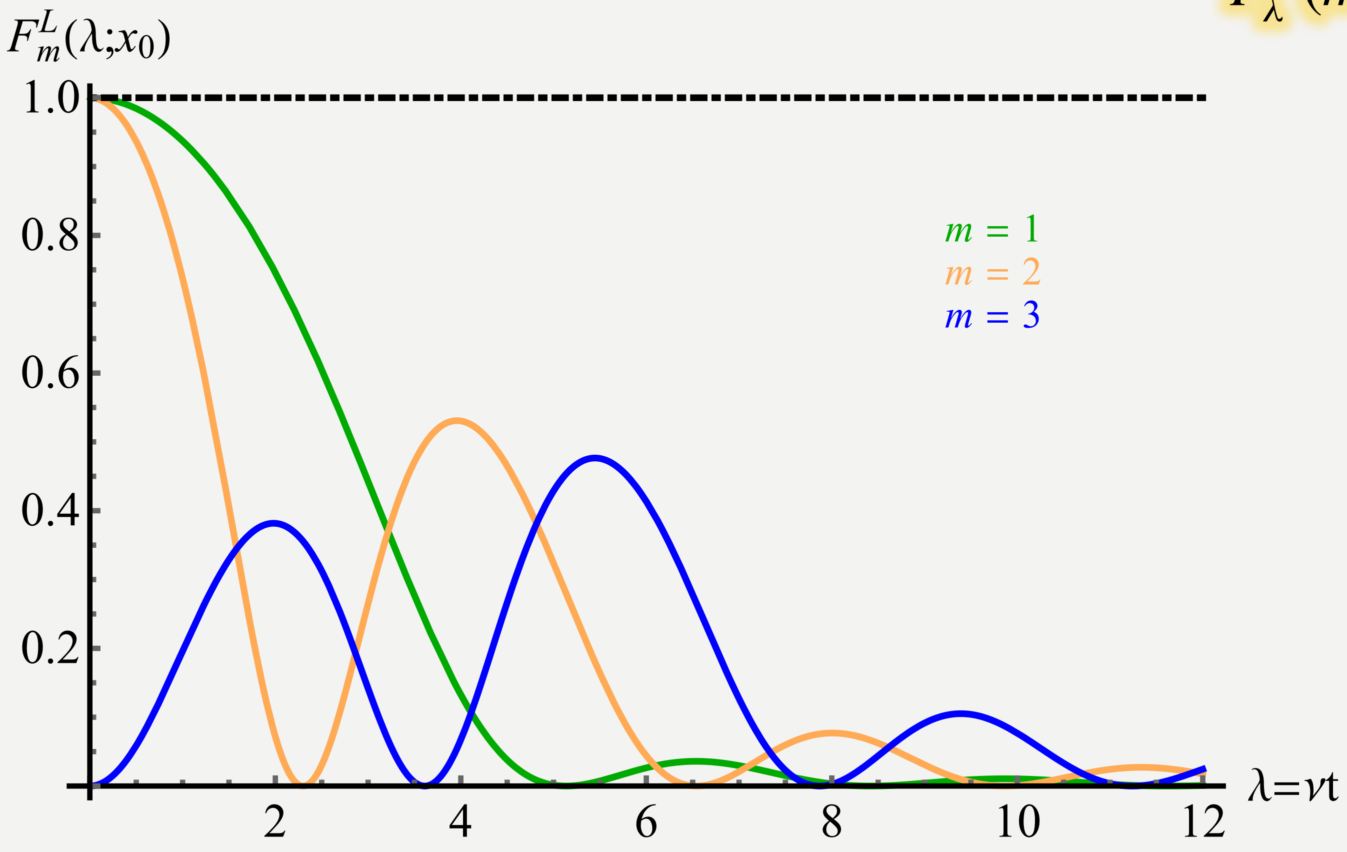
Estimation scheme I: Local measurement



$$P^L_\lambda(m|x_0) = |\langle\psi_\lambda|m\rangle|^2$$

$$F^L_m(\lambda;x_0) = \frac{\left[\partial_\lambda P^L_\lambda(m|x_0)\right]^2}{P^L_\lambda(m|x_0)\left[1 - P^L_\lambda(m|x_0)\right]}$$



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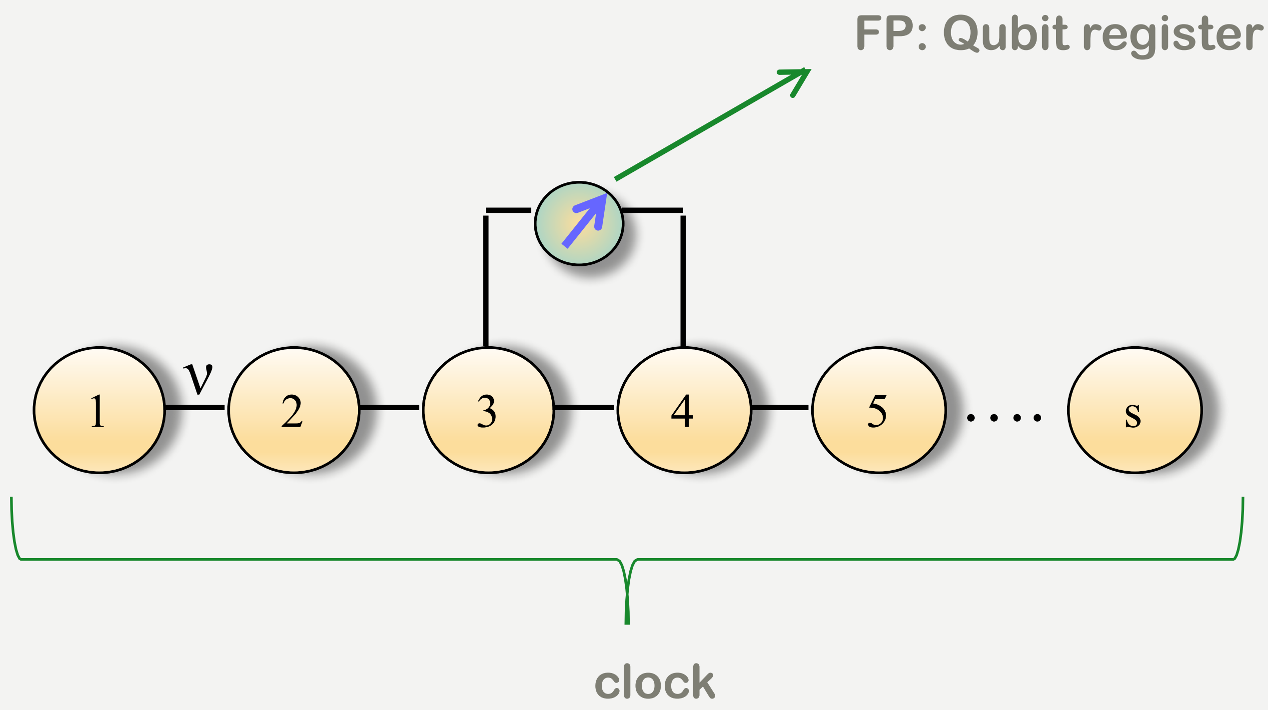
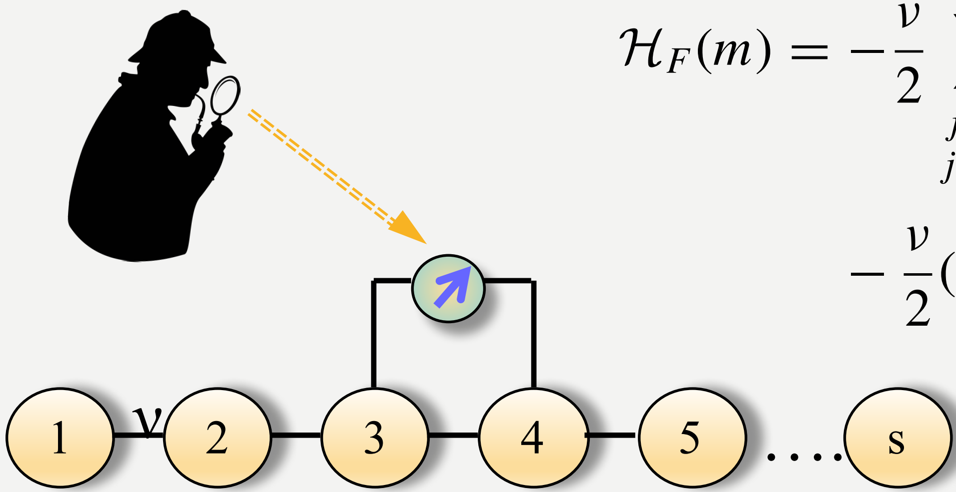
Estimation schemes II: Feynman probe



$$\mathcal{H}_F(m) = -\frac{\nu}{2}\sum_{\substack{j=1\\ j\neq m}}^{s-1} |j+1\rangle\langle j| + |j\rangle\langle j+1| - \frac{\nu}{2}\left(|m+1\rangle\langle m|\otimes\sigma_x + |m\rangle\langle m+1|\otimes\sigma_x\right)$$

Estimation schemes II: Feynman probe



$$\mathcal{H}_F(m) = -\frac{\nu}{2}\sum_{\substack{j=1\\ j\neq m}}^{s-1} |j+1\rangle\langle j| + |j\rangle\langle j+1|$$

$$-\frac{\nu}{2}(|m+1\rangle\langle m|\otimes\sigma_x + |m\rangle\langle m+1|\otimes\sigma_x).$$

$$|1,\uparrow\rangle,\ldots,|m,\uparrow\rangle,|m+1,\downarrow\rangle,\ldots,|s,\downarrow\rangle$$

$$P_\lambda(\uparrow|m,x_0) = \sum_{x=1}^{m} |\langle\psi_\lambda|x\rangle|^2.$$

$$\boxed{F_m^P(\lambda;x_0) = \frac{[\partial_\lambda P_\lambda(\uparrow|m,x_0)]^2}{P_\lambda(\uparrow|m,x_0)[1-P_\lambda(\uparrow|m,x_0)]}}$$

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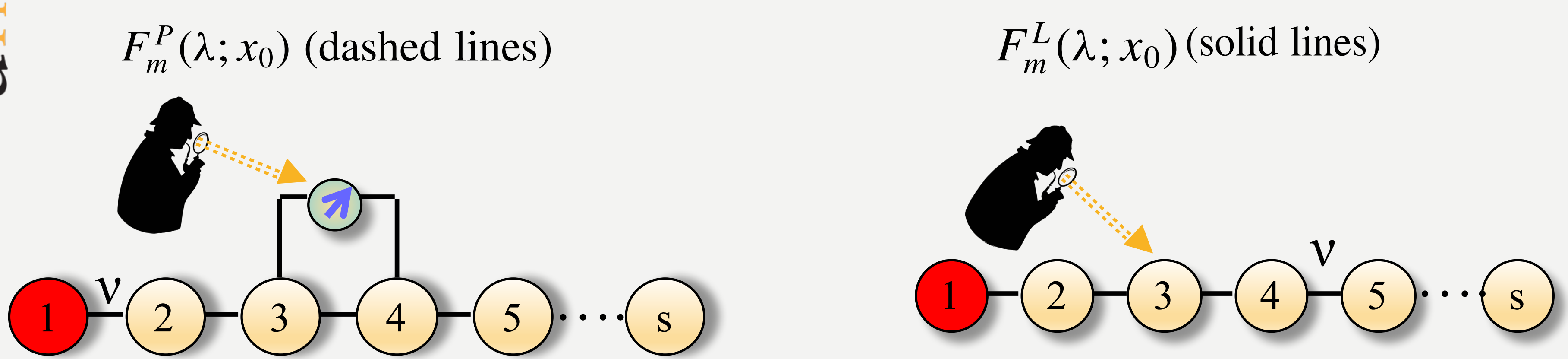
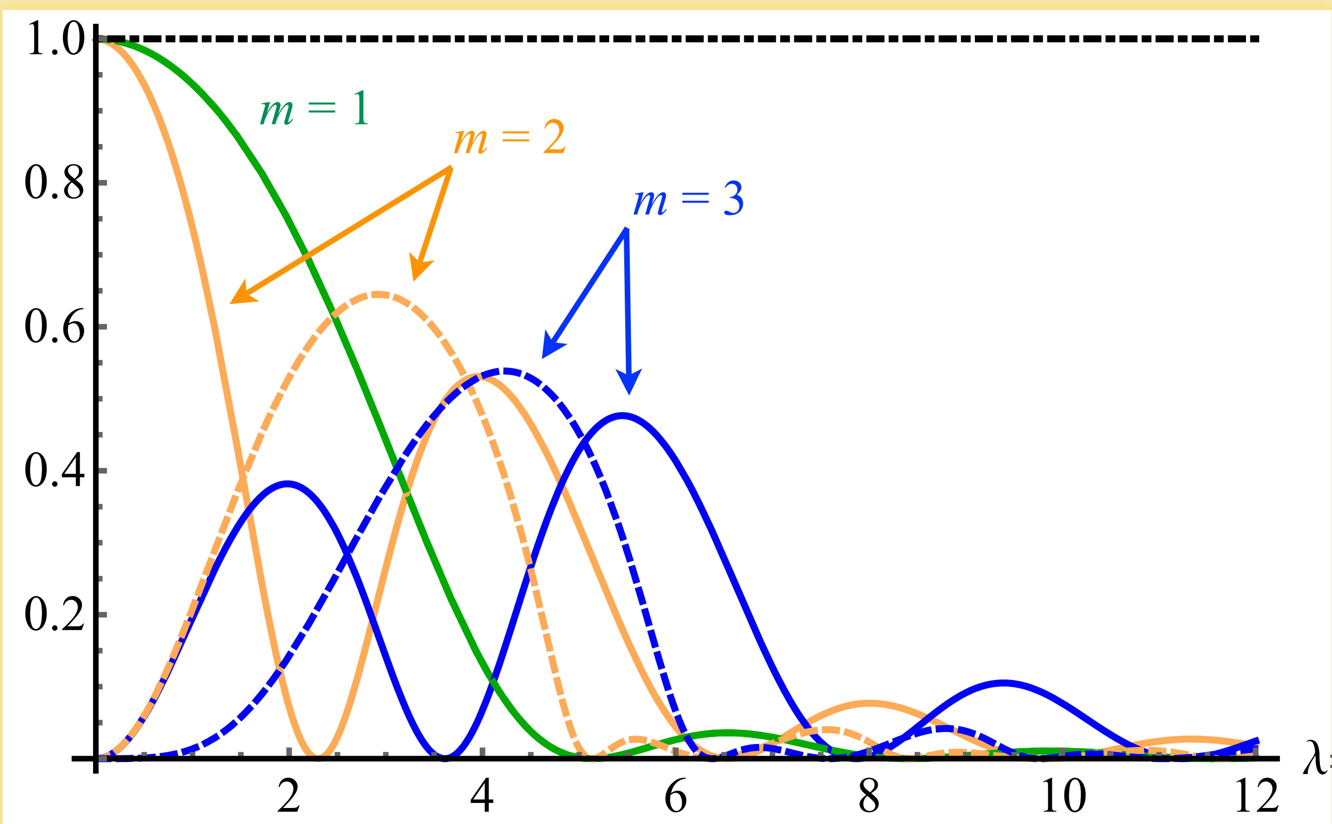
Estimation schemes: Feynman probe





(a) $x_0 = 1$

Bayesian estimator

$$P_B(\lambda|\Omega) = \frac{P(\Omega|\lambda)P(\lambda)}{\int P(\Omega|\lambda')P(\lambda')d\lambda'}$$

Bayes Theorem

$$P(\Omega|\lambda) = p_\lambda^{N_0}(1-p_\lambda)^{M-N_0}$$

Likelihood

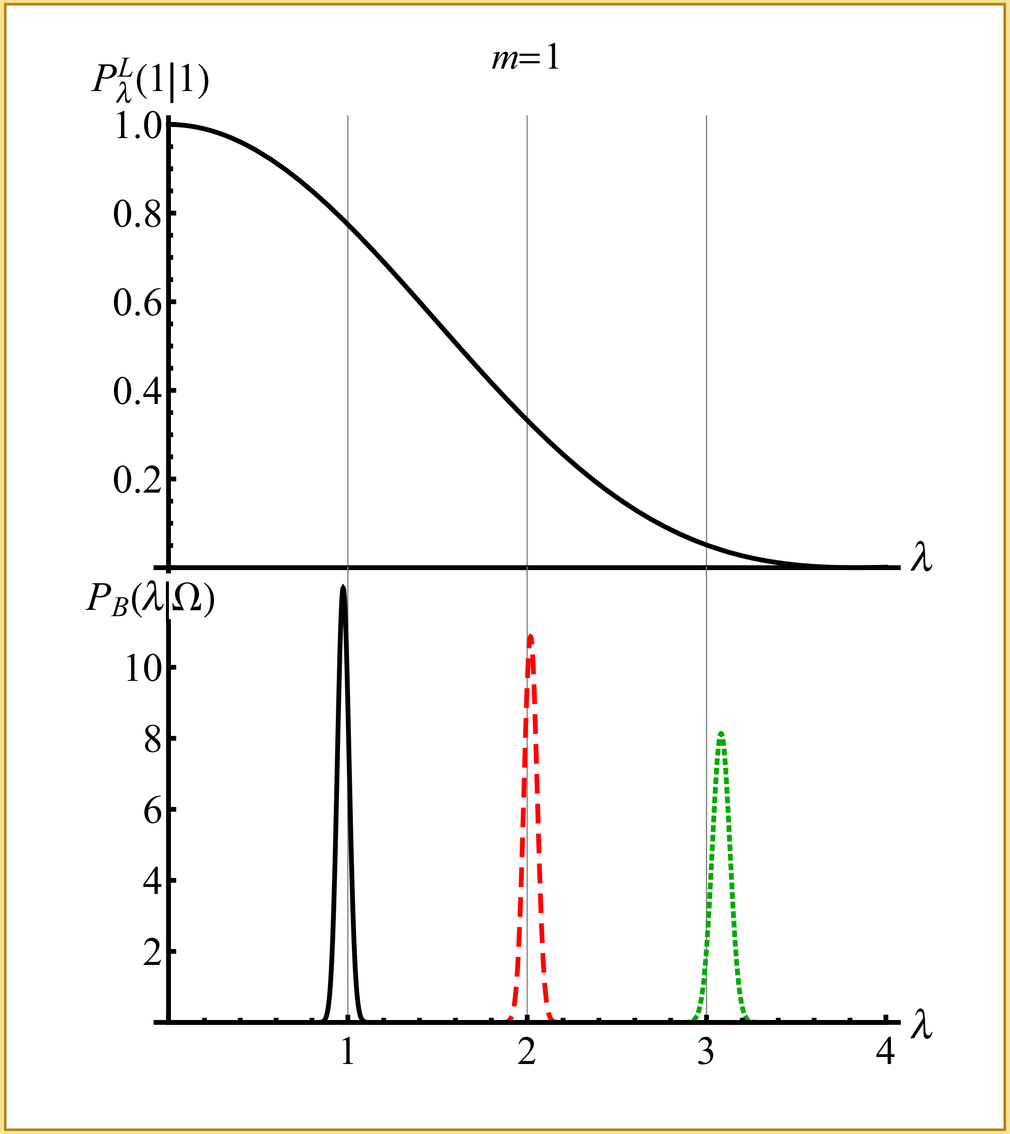
$$\hat{\lambda} = \int \lambda P_B(\lambda|\Omega)d\lambda$$

Bayesian estimator

$$\sigma^2[\hat{\lambda}] = \int [\lambda - \hat{\lambda}]^2 P_B(\lambda|\Omega)d\lambda$$

Bayesian variance

Measure on the first site: LM vs FP



$$P(\Omega|\lambda) = p_\lambda^{N_0}(1-p_\lambda)^{M-N_0}$$

$$P_B(\lambda|\Omega) = \frac{P(\Omega|\lambda)P(\lambda)}{\int P(\Omega|\lambda')P(\lambda')d\lambda'}$$

$\lambda_T = 1(\text{black}), 2(\text{red}), 3(\text{green})$

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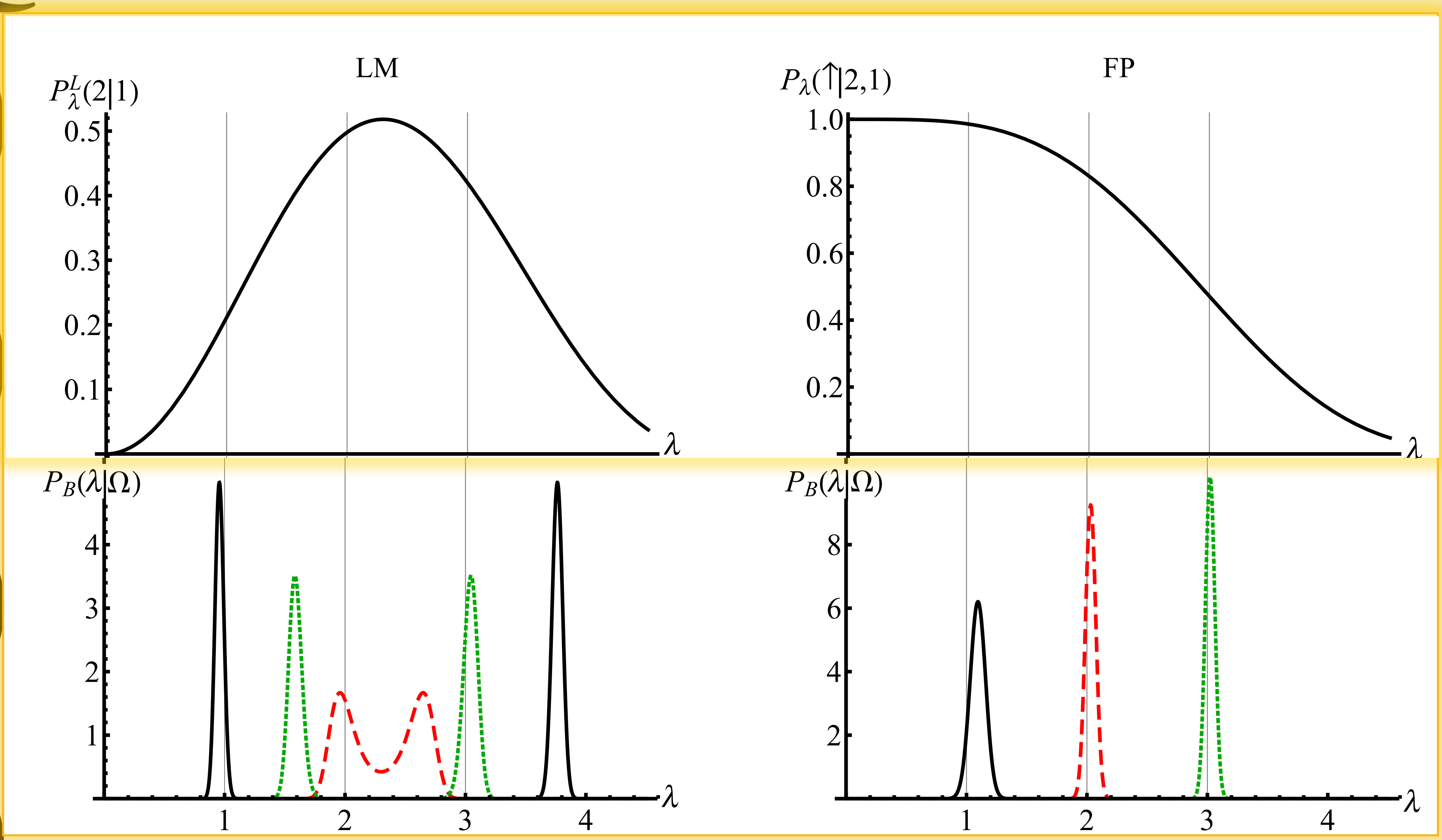
Measure on a generic site: LM vs FP



$\lambda_T = 1(\text{black}), 2(\text{red}), 3(\text{green})$

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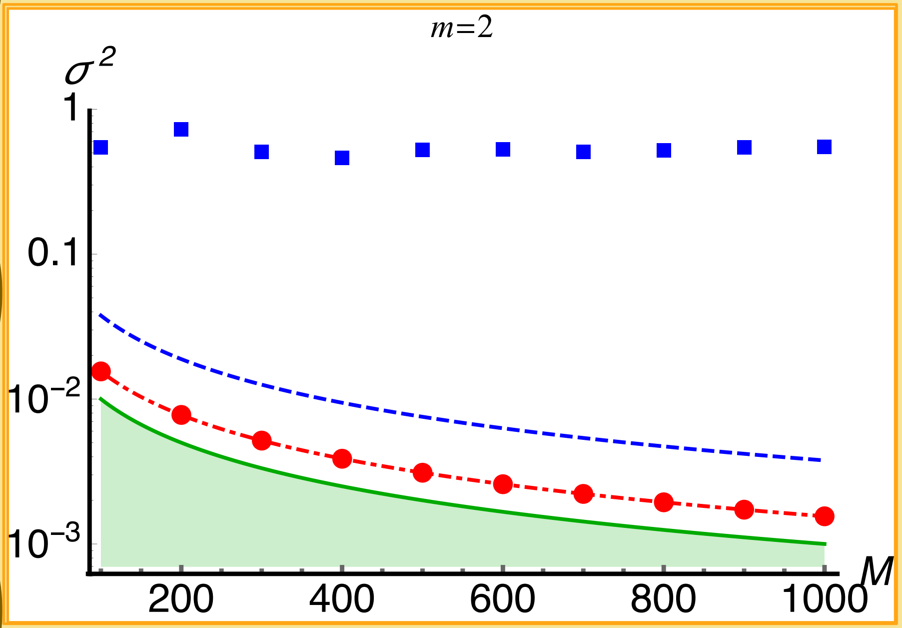
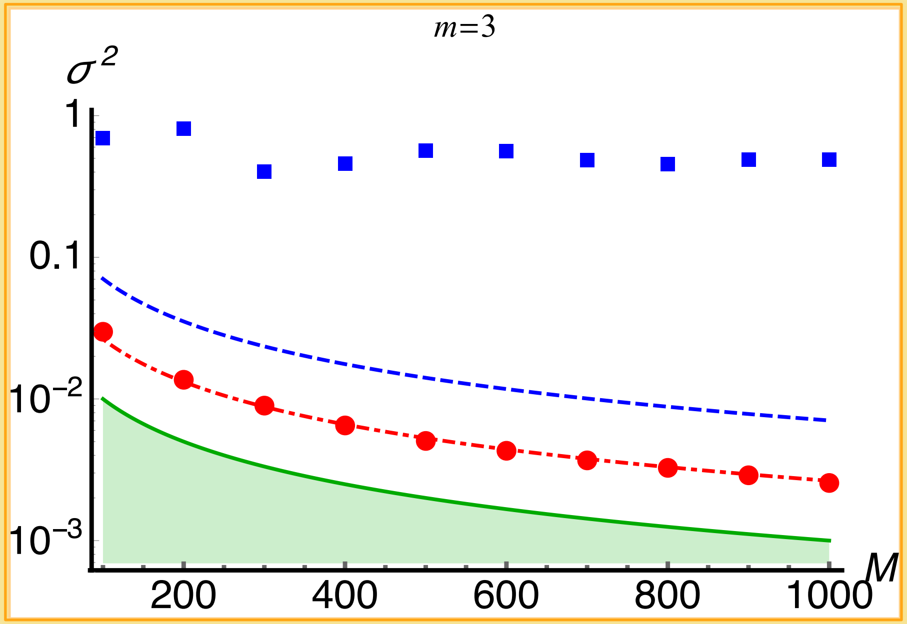
Cramèr-Rao bound: LM vs FP





$\lambda_T = 3$

LM (blue squares), FP (red circles)

QCR bound (green area)

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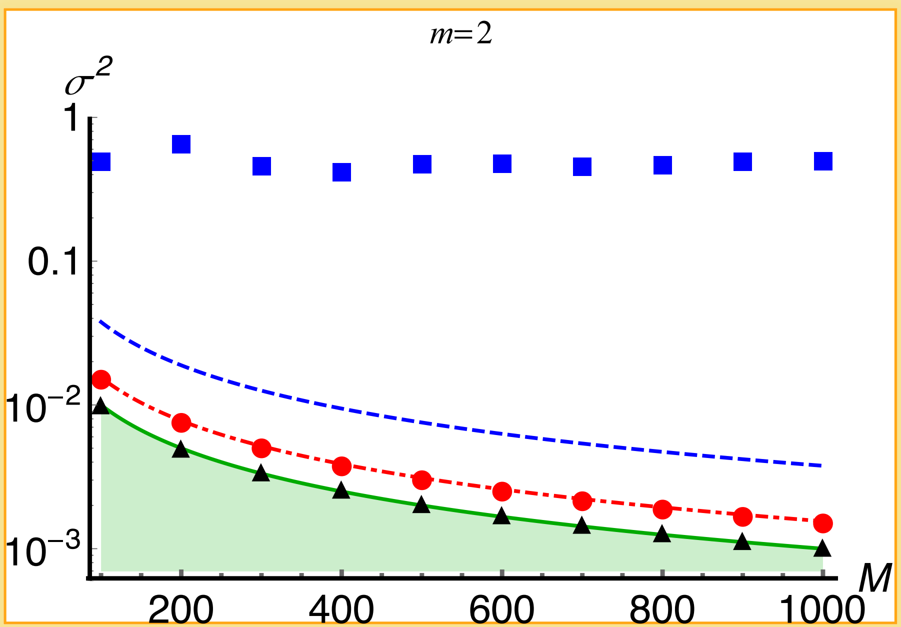
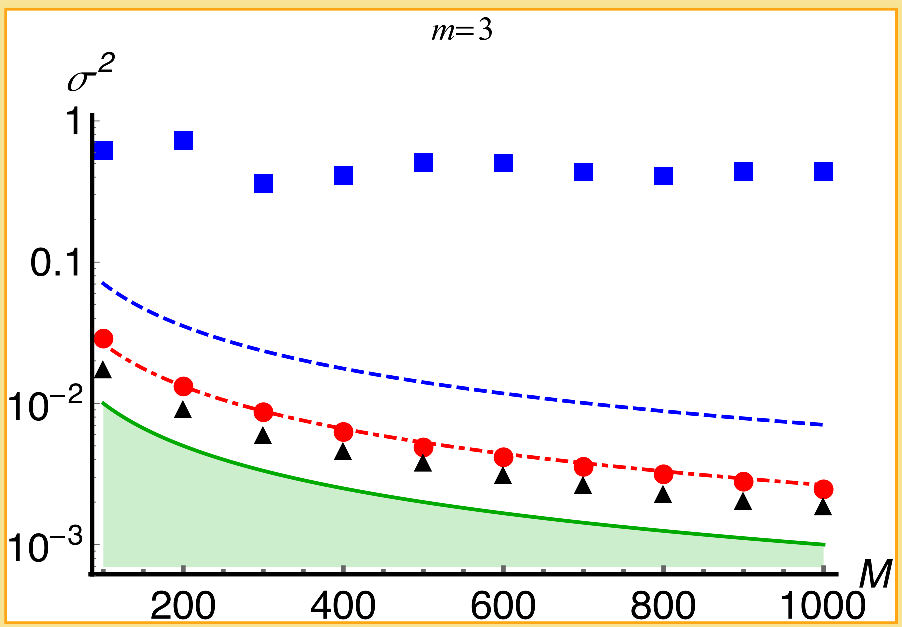
A new scheme: FP + LM

$$P_B(\lambda|\Omega) = \frac{P(\Omega|\lambda)P(\lambda)}{\int P(\Omega|\lambda')P(\lambda')d\lambda'}$$

Two-step measurement scheme:

- **FP to construct the *a posteriori* probability distribution** $P_B(\lambda|\Omega)$
- **This probability is used as a *a-priori* probability in a LM Bayesian inference**





$\lambda_T = 3$

LM (blue squares), FP (red circles), FP+LM (black triangles)

QCR bound (green area)

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Conclusions

- We compared the estimation procedures based on LM and FP for the caracterization of linear chains of qubits.
- LMs provide optimal charactrization for small values of the coupling and when the site x_0 is measured.
- For strongly coupled systems, FP provides a consistent Bayesian estimator that saturated the CR bound.
- We suggested a two-step adaptive measurement scheme FP+LM, that achieves the QCR bound.

OUTLOOK

- Complex networks
- Effects of decoherence

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Thank you!

D. Tamascelli, C. Benedetti, S. Olivares and M. G. A. Paris
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