Spin models for 2-site resonant tunnelling dynamics of bosons in a tilted optical lattice

Anton Buyskikh

Department of Physics and SUPA
University of Strathclyde

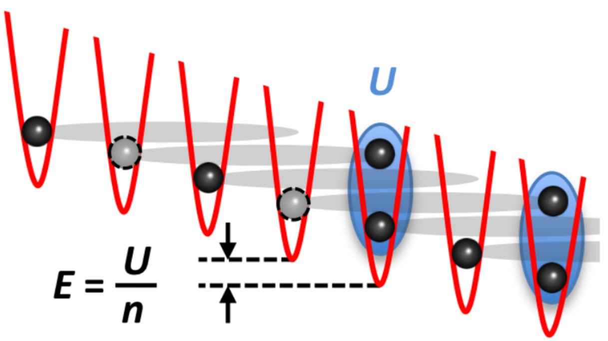
Adviser: Andrew Daley

Collaboration:
Luca Tagliacozzo
David Pekker (Pittsburgh, USA)
Chris Hooley (St. Andrew's, UK)
Dirk Schuricht (Utrecht, Netherlands)

QuProCS II | UIB, Palma, Spain | 10:10am, April 6th, 2017

Ongoing work at Strathclyde

- WP4: Dynamics in tilted optical lattices with **superlattice configuration**



Precursors: F. Meinert et al., PRL **111**, 053003 (2013)
F. Meinert et al., Science 344, 1259 (2014)

QuProCS (ongoing): A. Buyskikh et al., in preparation

  - Task 4: roadmap for dynamics in tilted system with **superlattices**
  - Task 5: Bloch oscillations and dephasing probed in an **interacting tilted lattice system**

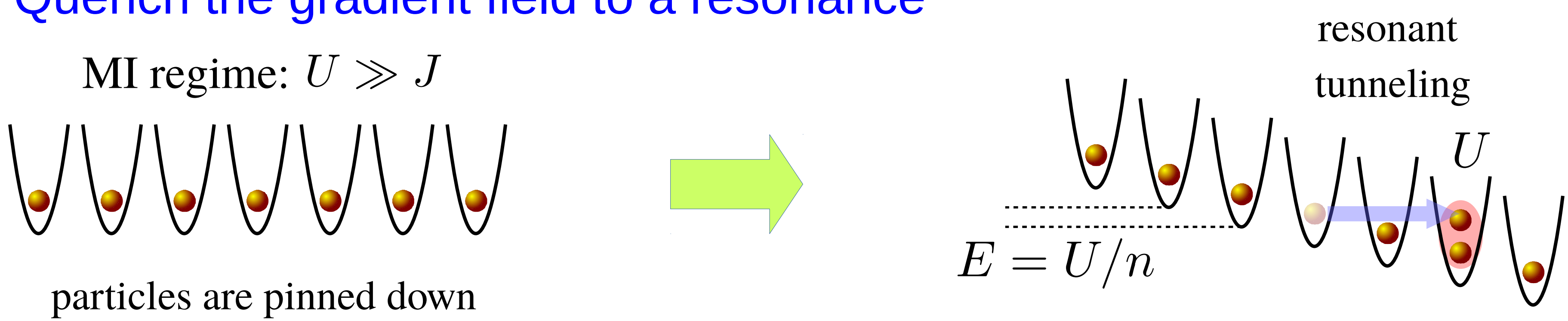
- Connections with Oxford, Freiburg Theory
- Provides a roadmap for the Strathclyde experiments

1D Bose-Hubbard model with tilt

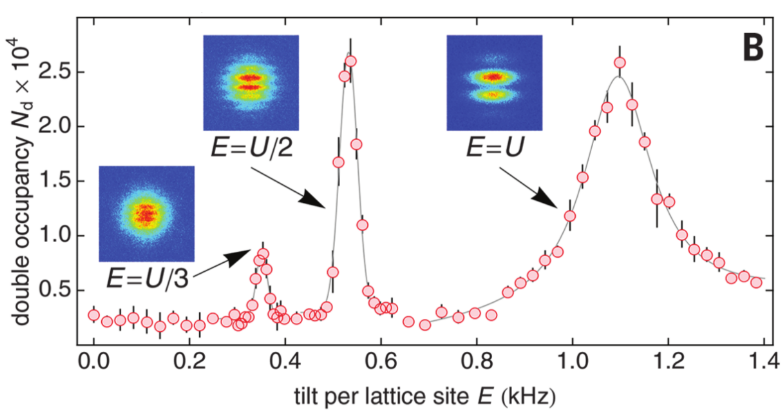
- Hamiltonian:

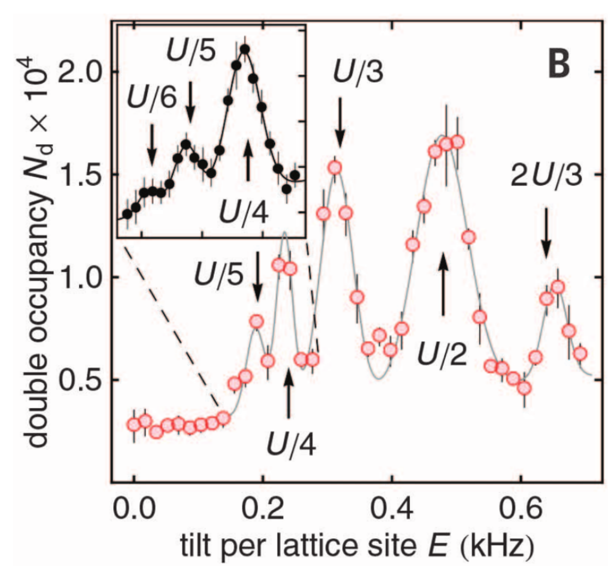
$$H = -J \sum_{\langle k,l \rangle} b_k^\dagger b_l + \frac{U}{2} \sum_k n_k (n_k - 1) - E \sum_k k n_k$$

- Quench the gradient field to a resonance



- Observations of high order **tunnelling resonances**

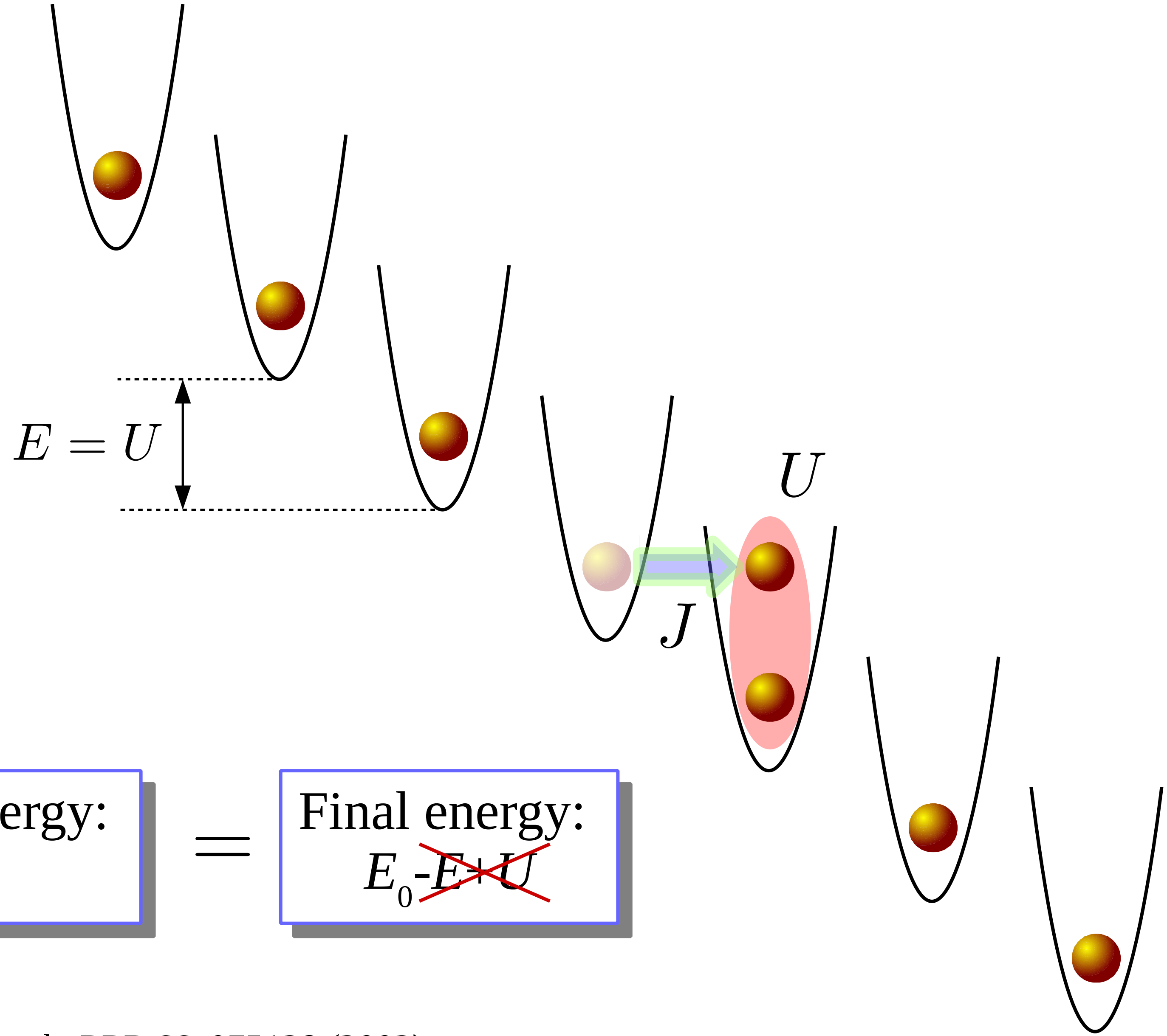




F. Meinert et al., PRL **111**, 053003 (2013)
F. Meinert et al., Science 344, 1259 (2014)

Next neighbour resonant tunnelling

- Tilt equals on-site interaction: $E = U$

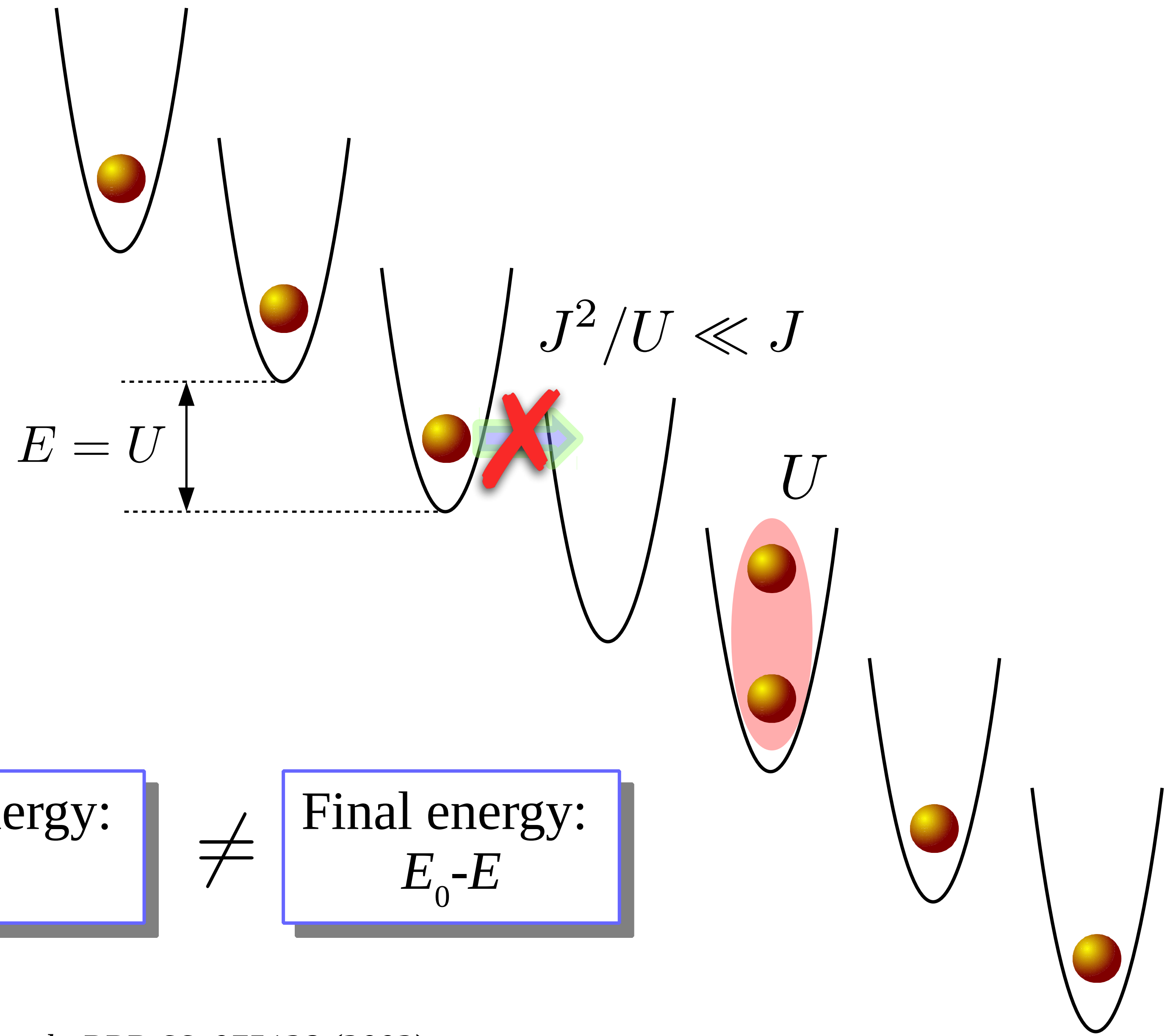


Initial energy: E_0 $=$ Final energy: $E_0 \cancel{-E+U}$

*Subir Sachdev et. al., PRB **66**, 075128 (2002)*

Next neighbour resonant tunnelling

- Off-resonant transitions are suppressed:

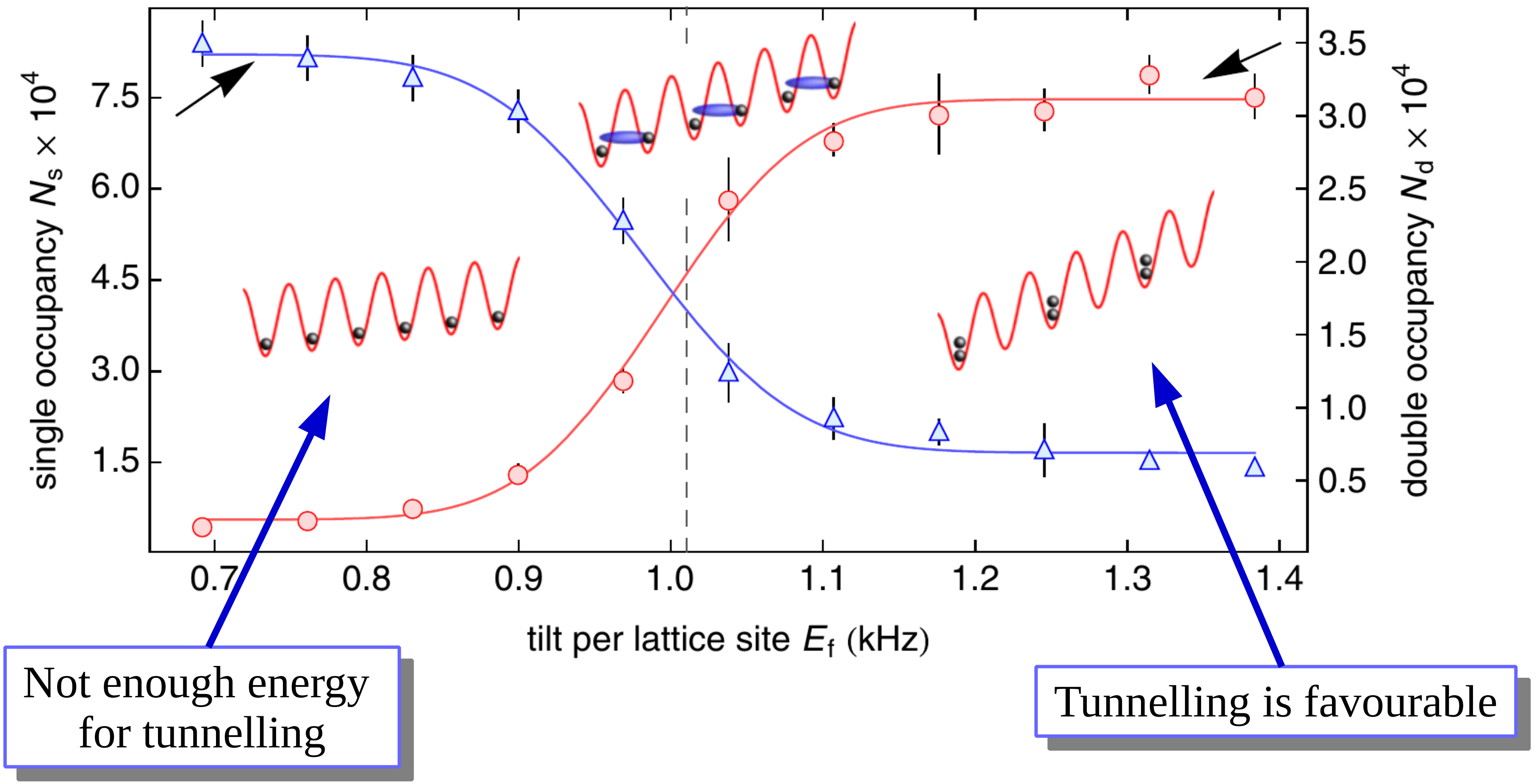


Initial energy: E_0 $\neq$ Final energy: E_0-E

Subir Sachdev et. al., PRB **66, 075128 (2002)*

Next neighbour resonant tunnelling

- Adiabatic tilt through the resonance

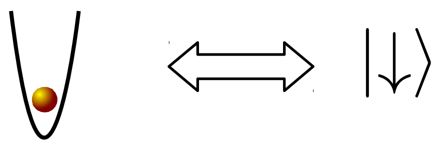


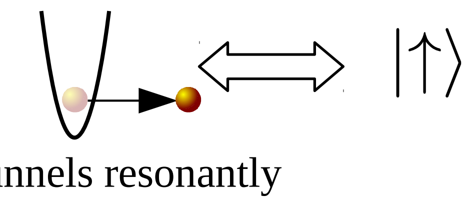
- **NB:** Due to resonance, dynamics of each atom is restricted within 2 sites

F. Meinert et al., PRL **111**, 053003 (2013)
F. Meinert et al., Science 344, 1259 (2014)

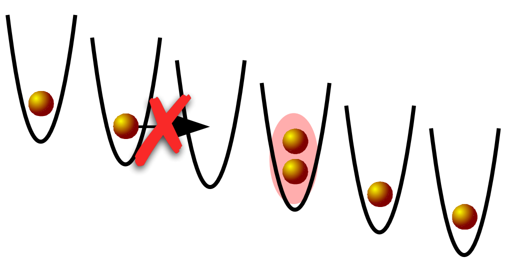
Spin-1/2 model for *E=U*

- Mapping with spins:





- Off-resonant transition are suppressed:



$$\sigma_i^{\uparrow}\sigma_{i+1}^{\uparrow} = 0$$

neighbouring spins cannot be up

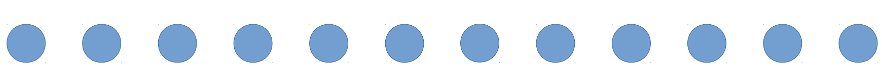
$$\sigma_i^{\uparrow} = \frac{\sigma_i^z + 1}{2}$$

- Hamiltonian: Ising chain with transverse and longitudinal fields

$$\boxed{H_U = -\sqrt{2}\sum_i \sigma_i^x + \lambda \sum_i \sigma_i^{\uparrow} + W \sum_i \sigma_i^{\uparrow}\sigma_{i+1}^{\uparrow}}$$

$$\lambda \equiv (U-E)/J$$

deviation from the resonance



$$W \to \infty$$

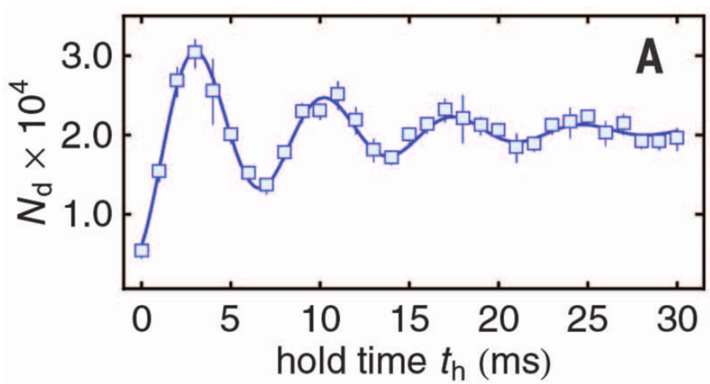
projective interaction

*Subir Sachdev et. al., PRB **66**, 075128 (2002)*

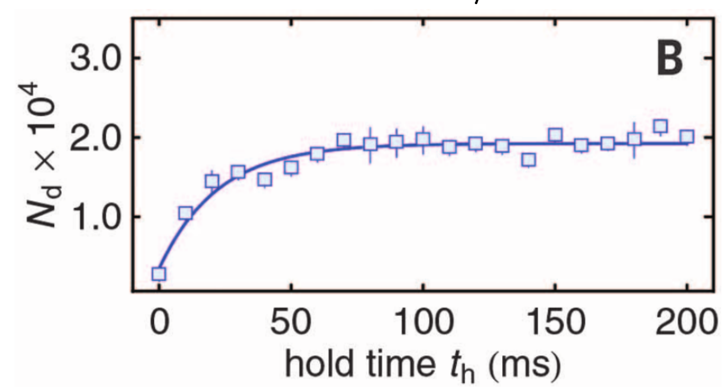
What about next-next neighbour resonance?

- **Contrasting behaviour** at resonances (double occupancies)

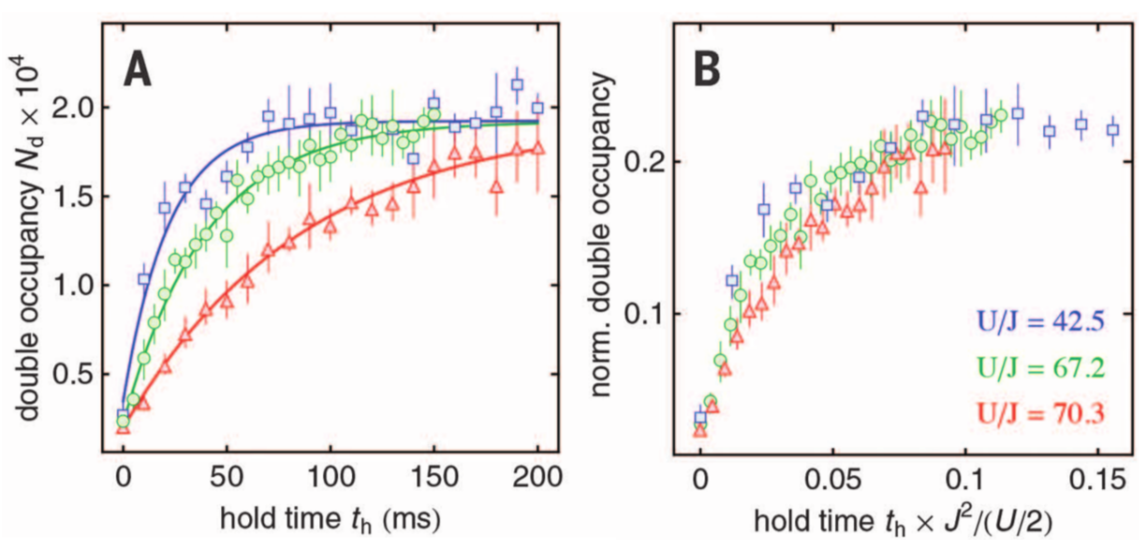
$E = U$



$E = U/2$



- Damping is due to **coherent interactions**



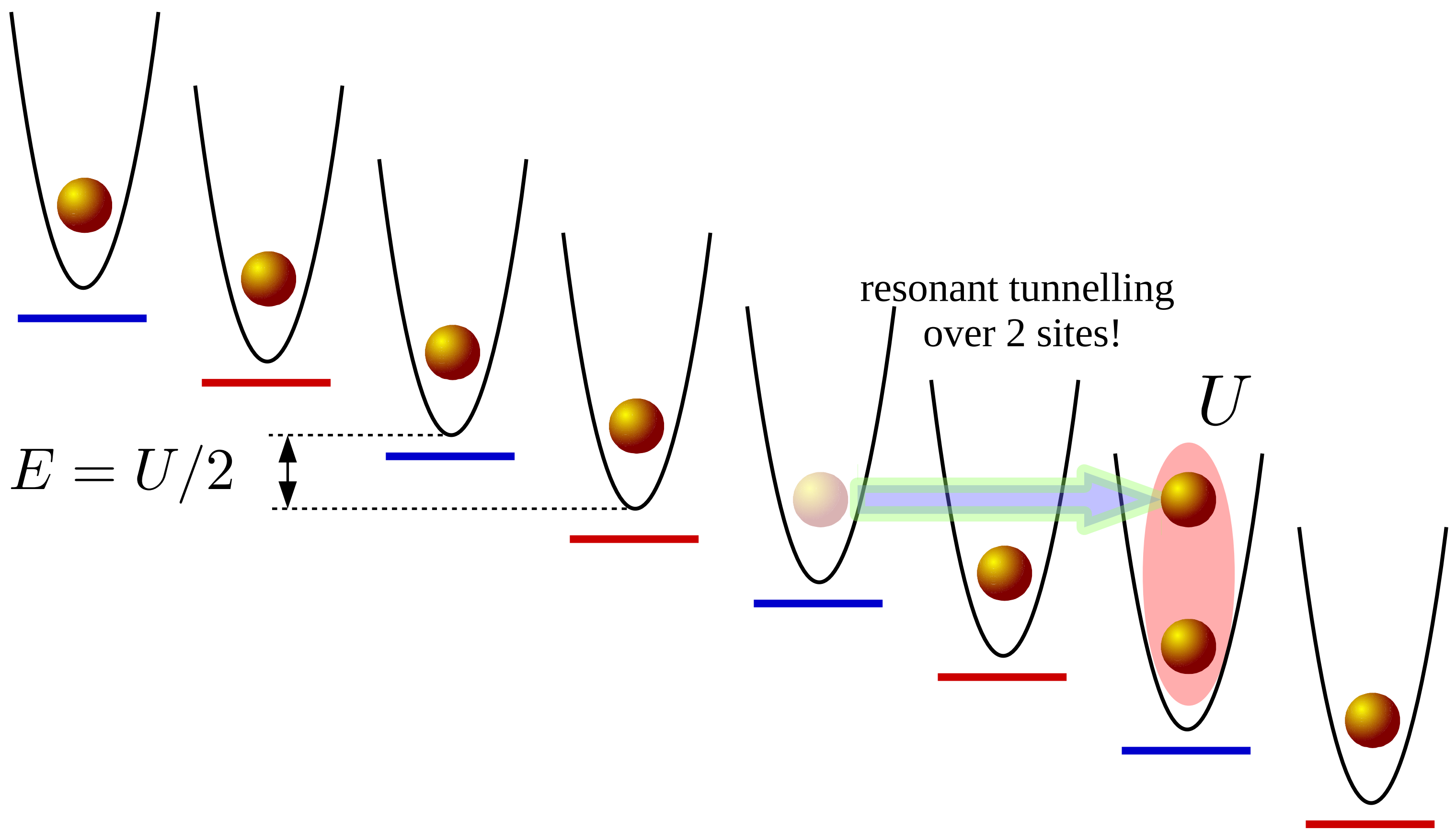
Collapse of data
after time rescaling

evidence for coherent
processes in the 2nd order
of perturbation theory

F. Meinert et al., PRL **111**, 053003 (2013)
F. Meinert et al., Science 344, 1259 (2014)

Next-next neighbour resonant tunnelling

- Tilt equals a half of on-site interaction $E = U/2$

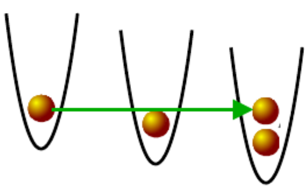


- NB: Looks like 2 independent chains with $E = U$

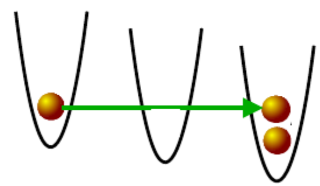
Spin model for E=U/2 resonance

- By adding a small **superlattice offset** between odd and even sites sites: bosons to tunnel only between 2 sites

- **Tunnelling** amplitude depends on near sites' occupations (XZ coupling)

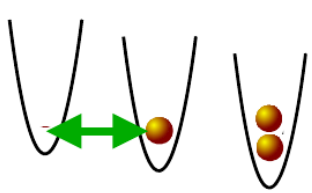


$$\propto \frac{2\sqrt{2}J^2 + \sqrt{2}J^2}{-U/2}$$

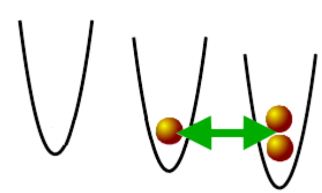


$$\propto \sqrt{2}\frac{J^2}{U/2}$$

- "**Self-energy**" terms (ZZ coupling)



$$\propto \frac{J^2}{-U/2}$$



$$\propto \frac{J^2}{-U/2}$$

- What does the spin model look like?

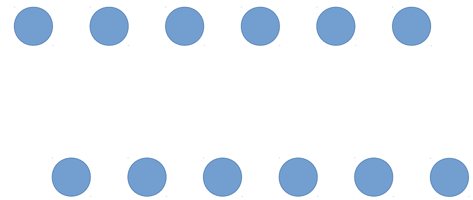
Effective coupled Ising chains

$$H_{U/2} = -\sqrt{2}\sum_i \sigma_i^x + \lambda \sum_i \sigma_i^z + W \sum_i \sigma_i^{\uparrow}\sigma_{i+2}^{\uparrow}$$

E=U subchains

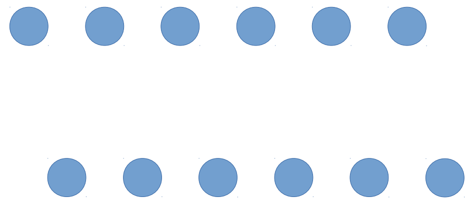
$W \to \infty$

$$\lambda = \frac{U/2 - E}{J^2/(U/2)}$$



Effective coupled Ising chains

$$H_{U/2} = -\sqrt{2}\sum_i \sigma_i^x + \lambda \sum_i \sigma_i^z + W \sum_i \sigma_i^\uparrow \sigma_{i+2}^\uparrow$$



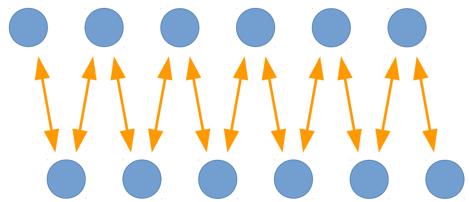
E=U subchains

$W \to \infty$

$$\lambda = \frac{U/2 - E}{J^2/(U/2)}$$

$$-2\sqrt{2}\sum_i \left(\sigma_i^x \sigma_{i+1}^z + \sigma_i^z \sigma_{i+1}^x\right)$$

XZ coupling



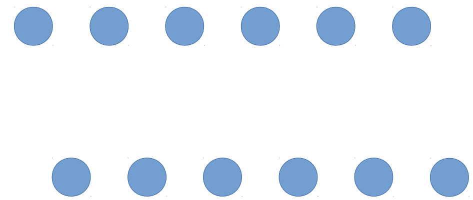
Effective coupled Ising chains

$$H_{U/2} = -\sqrt{2}\sum_i \sigma_i^x + \lambda \sum_i \sigma_i^z + W \sum_i \sigma_i^{\uparrow}\sigma_{i+2}^{\uparrow}$$

E=U subchains

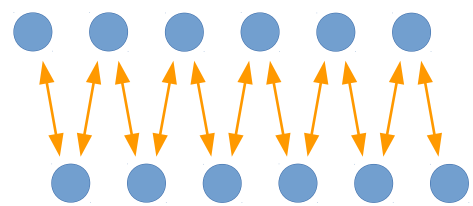
$$W \to \infty$$

$$\lambda = \frac{U/2 - E}{J^2/(U/2)}$$



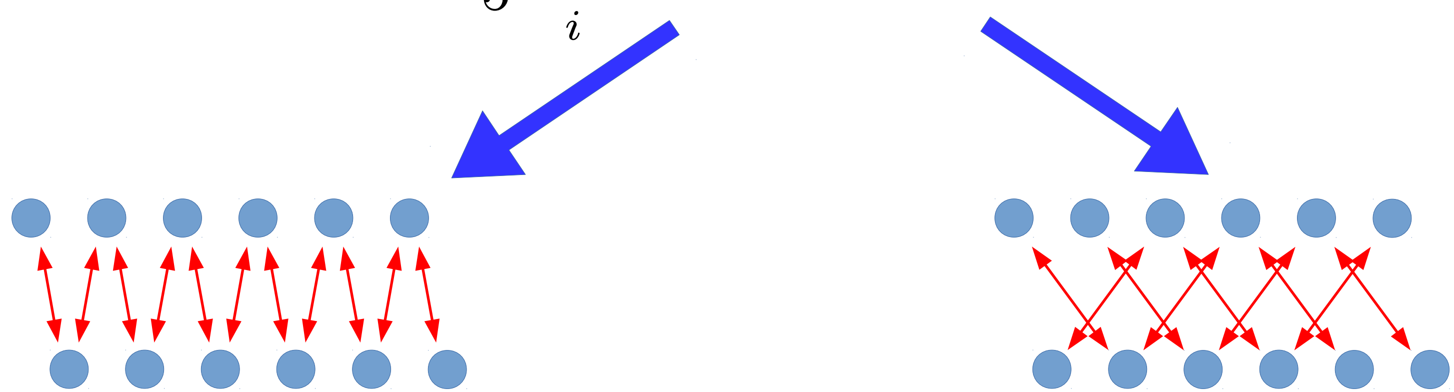
$$-2\sqrt{2}\sum_i \left(\sigma_i^x \sigma_{i+1}^z + \sigma_i^z \sigma_{i+1}^x\right)$$

XZ coupling



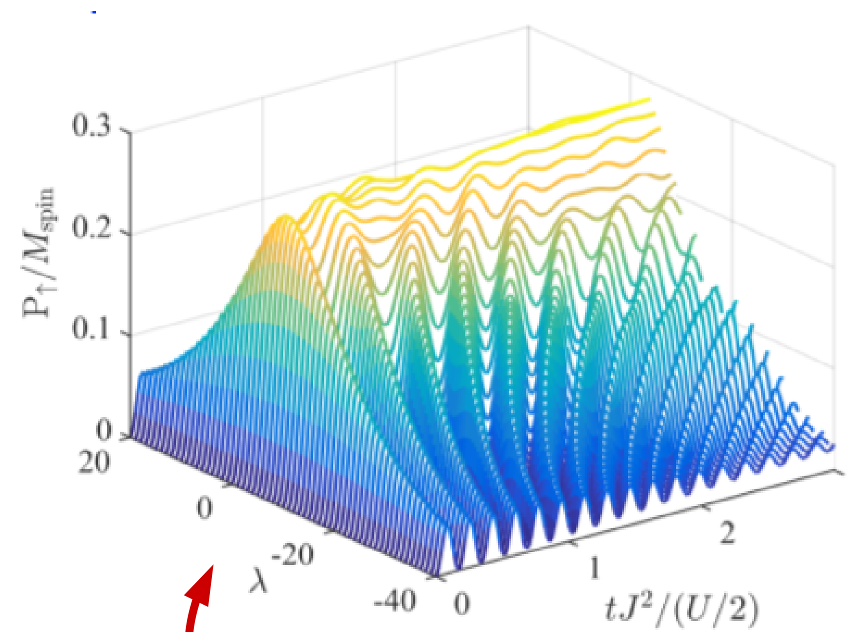
$$-\frac{8}{5}\sum_i \left(\sigma_i^z \sigma_{i+1}^z + \sigma_i^z \sigma_{i+3}^z\right)$$

ZZ coupling



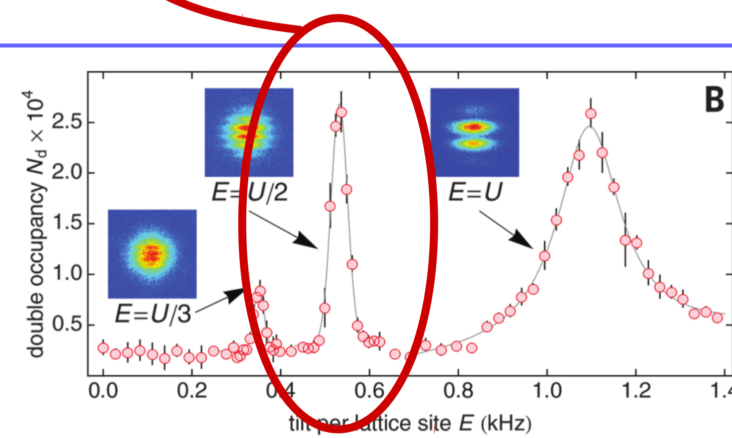
Quench dynamics

- **Bosons in tilted superlattice** geometry are simulated via **effective spin models**



- What is more: Dynamics of spins captures the essential dynamics of **bosons without superlattice**:

1. Resonance position



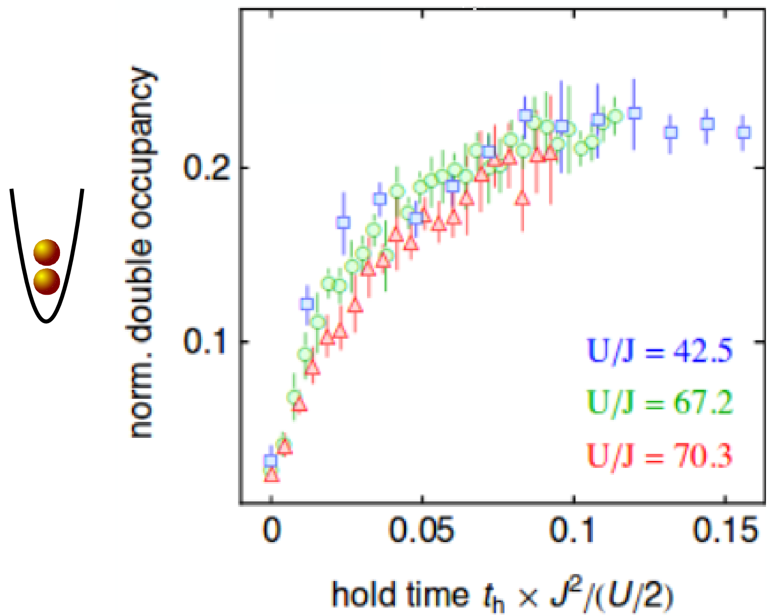
F. Meinert et al., PRL **111**, 053003 (2013)
F. Meinert et al., Science 344, 1259 (2014)

Quench dynamics

2. Resonance behaviour: Fast dumping of oscillations

Experiment with bosons **without superlattice**:

- double occupancy

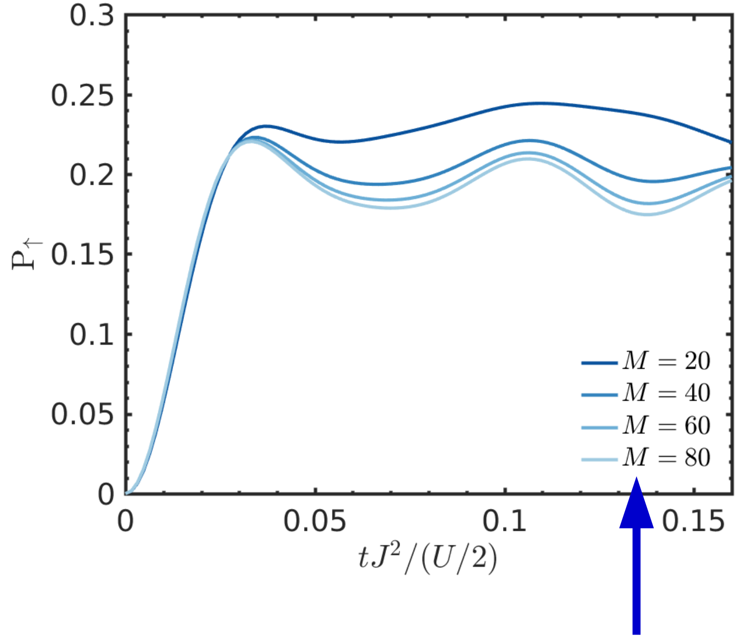


averaged system has 40 sites

F. Meinert et al., PRL **111**, 053003 (2013)
F. Meinert et al., Science 344, 1259 (2014)

Theory **with superlattice**:

- double occupancy = spin magnetization



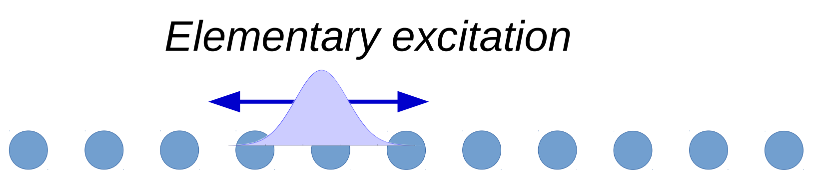
number of spins

Highlights on phase transitions

- Drastic difference between regimes <u>E=U</u> and <u>E=U/2</u>

$$E = U$$

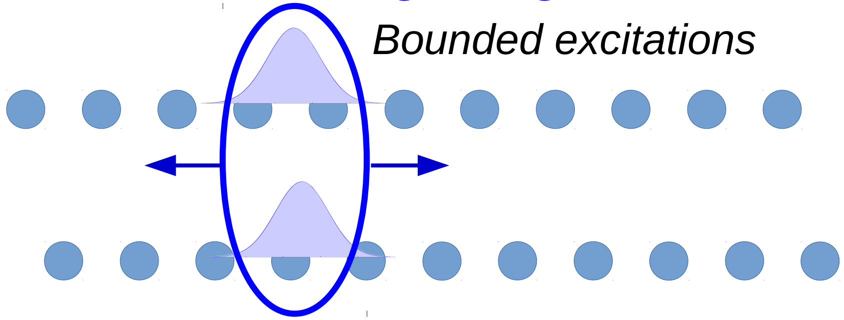
- Single Ising chain



- Standard critical exponents
- Resonance condition coincide with the 2nd order phase transition

$$E = U/2$$

- Two interacting Ising chains



- Interactions break symmetries
- Evidence for a weak 1st order transition

- Different character of the resonance behaviour

Numerical techniques

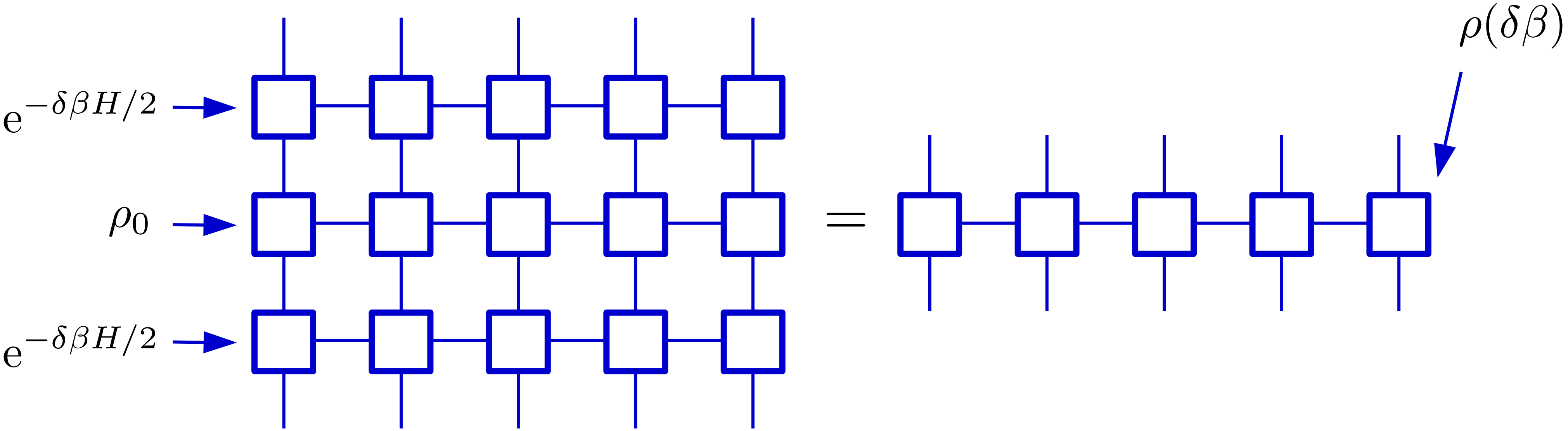
- Extensive use of **DMRG** techniques with **MPS**

- Projecting out forbidden spin configurations
  - great improvement of convergence

$$H \to P_n H P_n$$

$$\text{where } P_n = \prod_r (\mathcal{I} - \sigma_r^\uparrow \sigma_{r+n}^\uparrow)$$

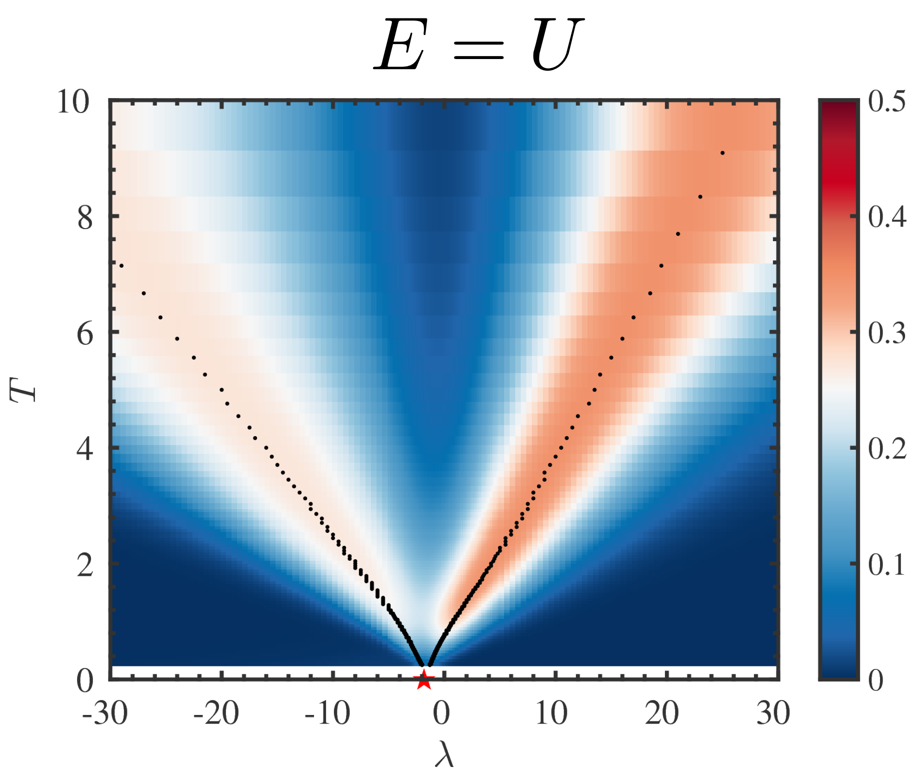
- For instance: thermal calculations: approach from infinite-*T* state + purification algorithm



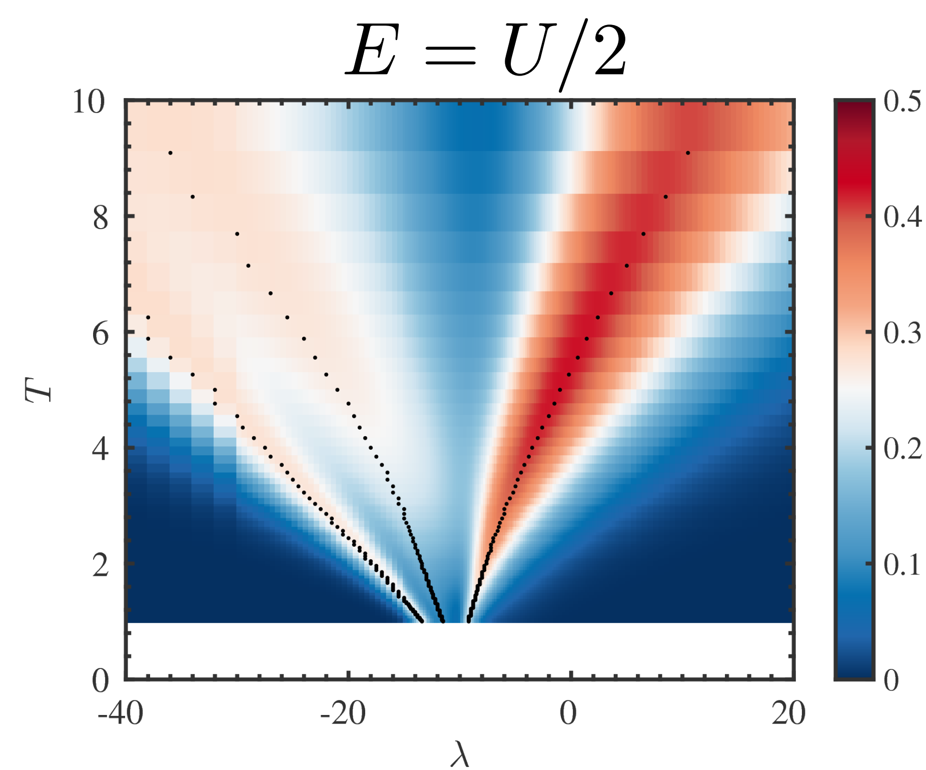
Thermal calculations

- Another probing method
  - Specific heat shows distinct difference between regimes

$$C(T,\lambda) = \frac{\beta^2}{M}\left(\left\langle H^2(\lambda)\right\rangle_T - \langle H(\lambda)\rangle_T^2\right) \qquad M \sim 100 \text{ spins}$$



very good agreement with eigenvalues calculations



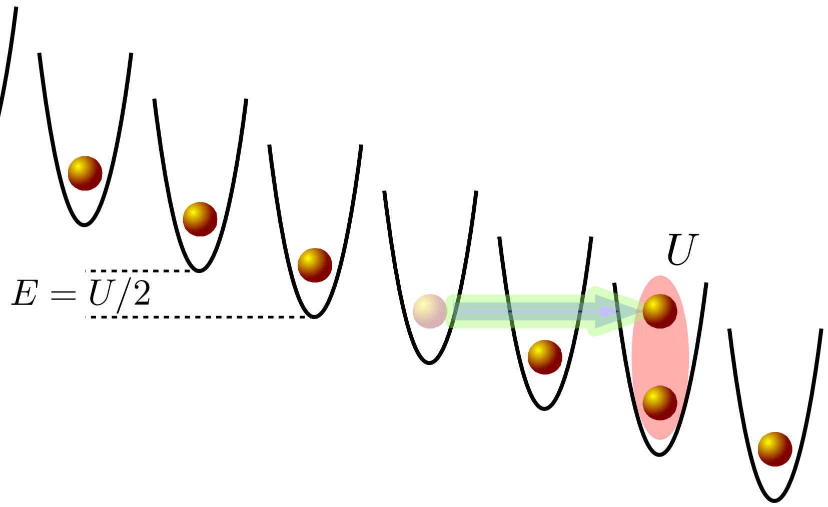
reflects complicated nature of *bounded excitations*

Summary

- **Effective spin model** for E=U/2 regime

  forms a basis of understanding of

  tilted superlattice systems



- Potential probing techniques:
  - Study effects of decoherence
  - Interaction with impurity

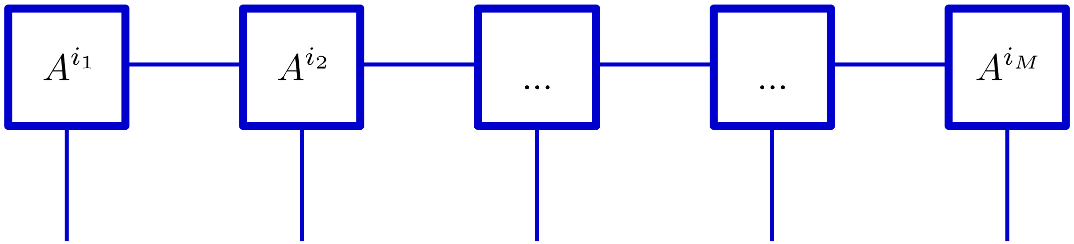
- In the context of QuProCS:
  - Roadmap for experiments: Strathclyde and Oxford

Thank you!

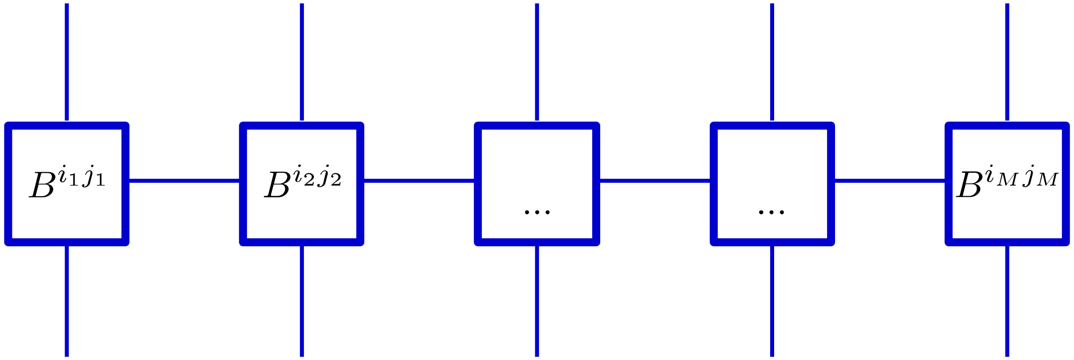
Extra slides

Numerical techniques

- MPS: $|\psi\rangle = \sum_{i_1, i_2, \ldots, i_M} A^{i_1} A^{i_2} \ldots A^{i_M} \, |i_1, i_2, \ldots, i_M\rangle$



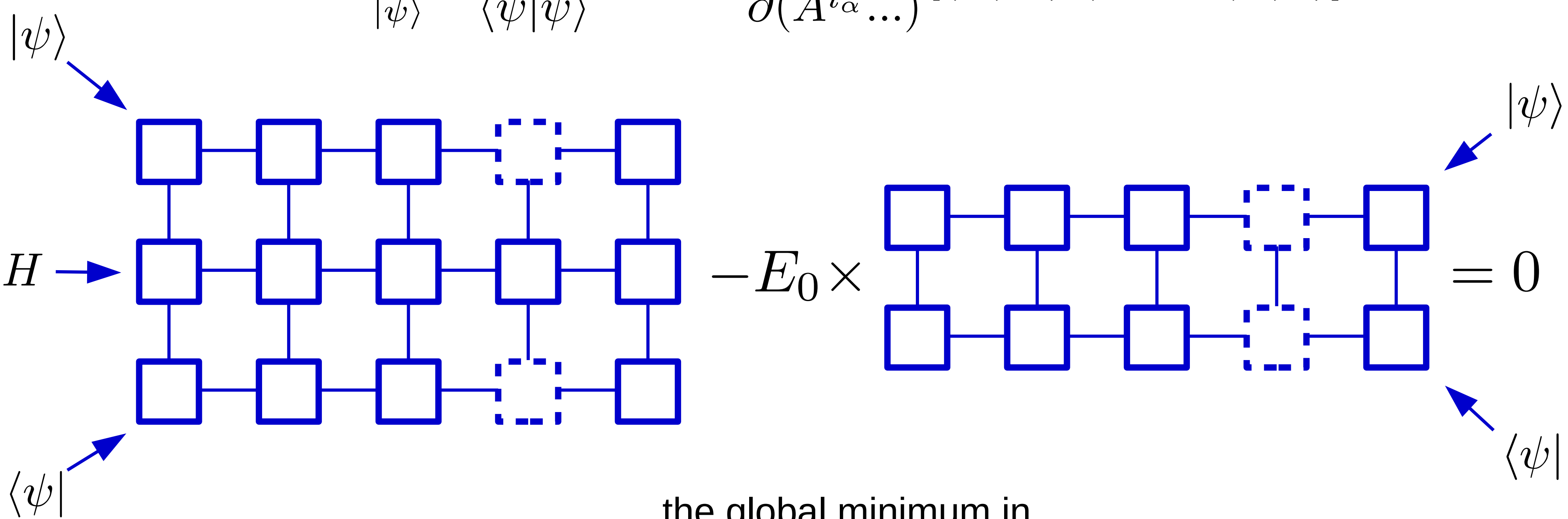
- MPO: $\hat{O} = \sum_{i_1, i_2, \ldots, i_M} \sum_{j_1, j_2, \ldots, j_M} B^{i_1 j_1} B^{i_2 j_2} \ldots B^{i_M j_M} \, |i_1, i_2, \ldots, i_M\rangle \langle j_1, j_2, \ldots, j_M|$



Numerical techniques

- Traditional ground state calculations with MPS and MPO:

$$E_0 = \min_{|\psi\rangle} \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} \quad \Leftrightarrow \quad \frac{\partial}{\partial(A^{i_\alpha}\dots)}\left[\langle\psi|H|\psi\rangle - E_0\langle\psi|\psi\rangle\right] = 0$$



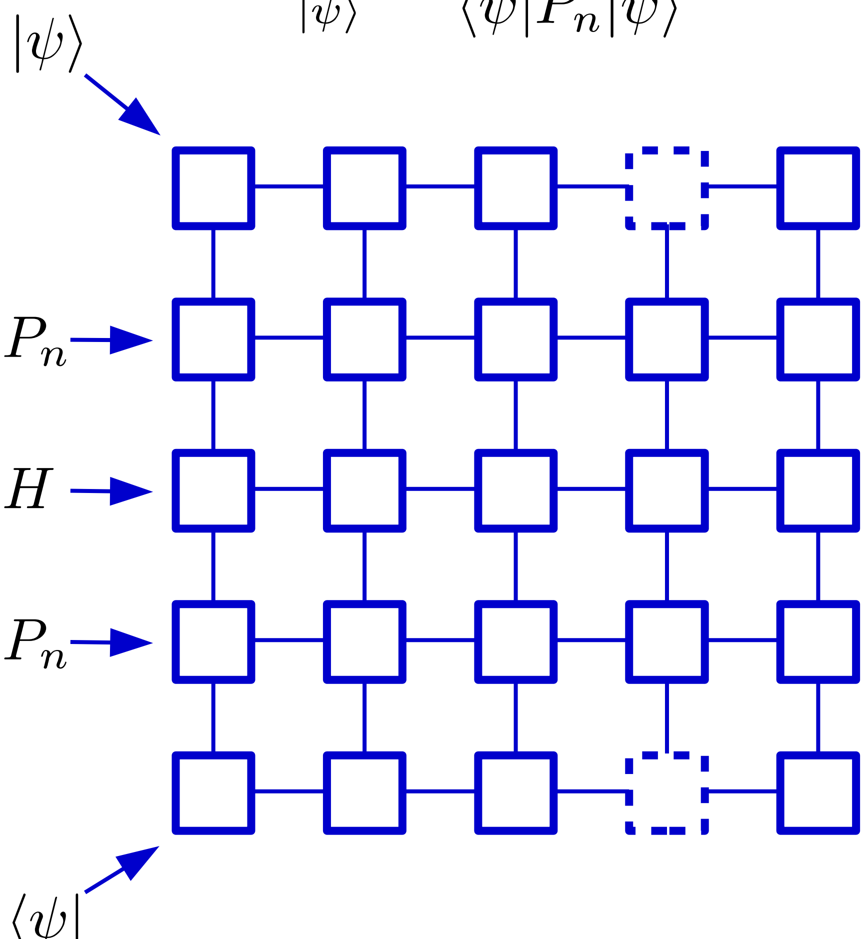
the global minimum in
the whole Hilbert space is approached
by consecutive local optimizations

Numerical techniques

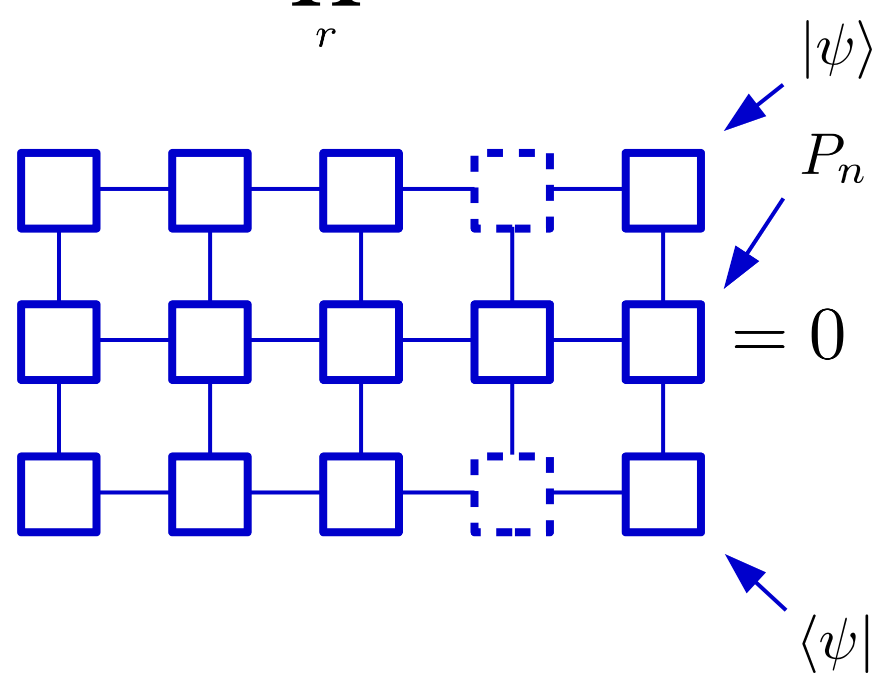
- Constraint realized via MPO projector:

$$E_0 = \min_{|\psi\rangle} \frac{\langle\psi|P_n H P_n|\psi\rangle}{\langle\psi|P_n|\psi\rangle} \quad \Leftrightarrow \quad \frac{\partial}{\partial(A^{i_\alpha}\ldots)}\left[\langle\psi|P_n H P_n|\psi\rangle - E_0\langle\psi|P_n|\psi\rangle\right] = 0$$

$$\text{where } P_n = \prod_r (\mathcal{I} - \sigma_r^\uparrow \sigma_{r+n}^\uparrow)$$



$$-E_0 \times$$

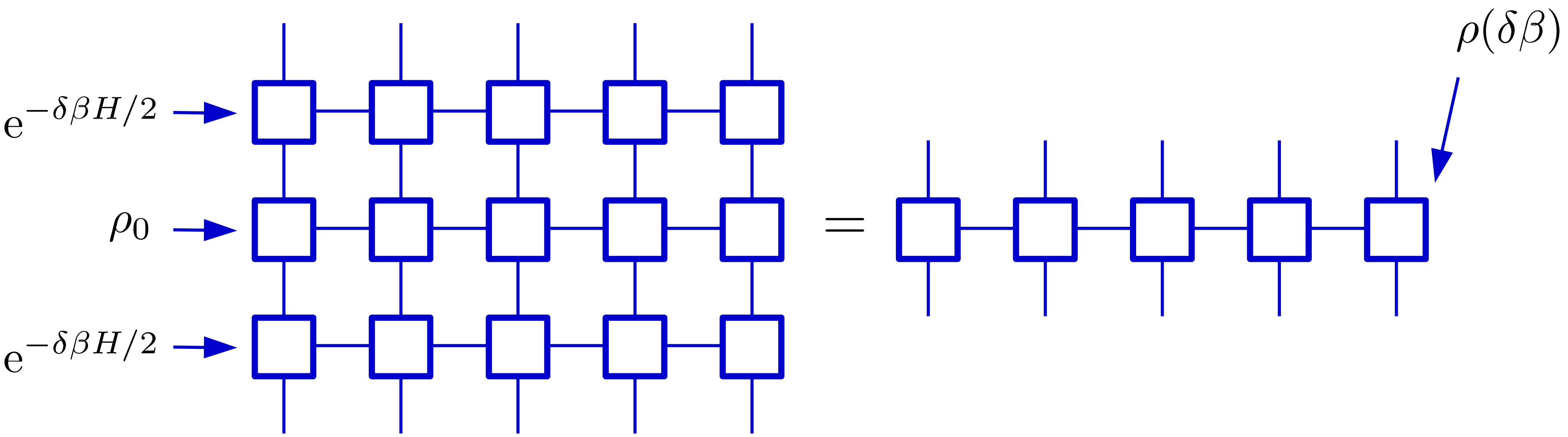


$$= 0$$

Numerical techniques

- Thermal states for specific heat calculations

$$\rho(\beta) \propto \mathrm{e}^{-\beta H} \quad \Rightarrow \quad \begin{array}{l} \rho(\beta = 0) \equiv \rho_0 \propto \mathcal{I} \\ \rho(\beta) \propto \mathrm{e}^{-\beta H/2} \rho_0 \mathrm{e}^{-\beta H/2} \end{array}$$



Numerical techniques

- To enforce positive semidefiniteness one evolves

$$\sqrt{\rho(\beta)} = \mathrm{e}^{-\beta H/2}\sqrt{\rho_0}$$

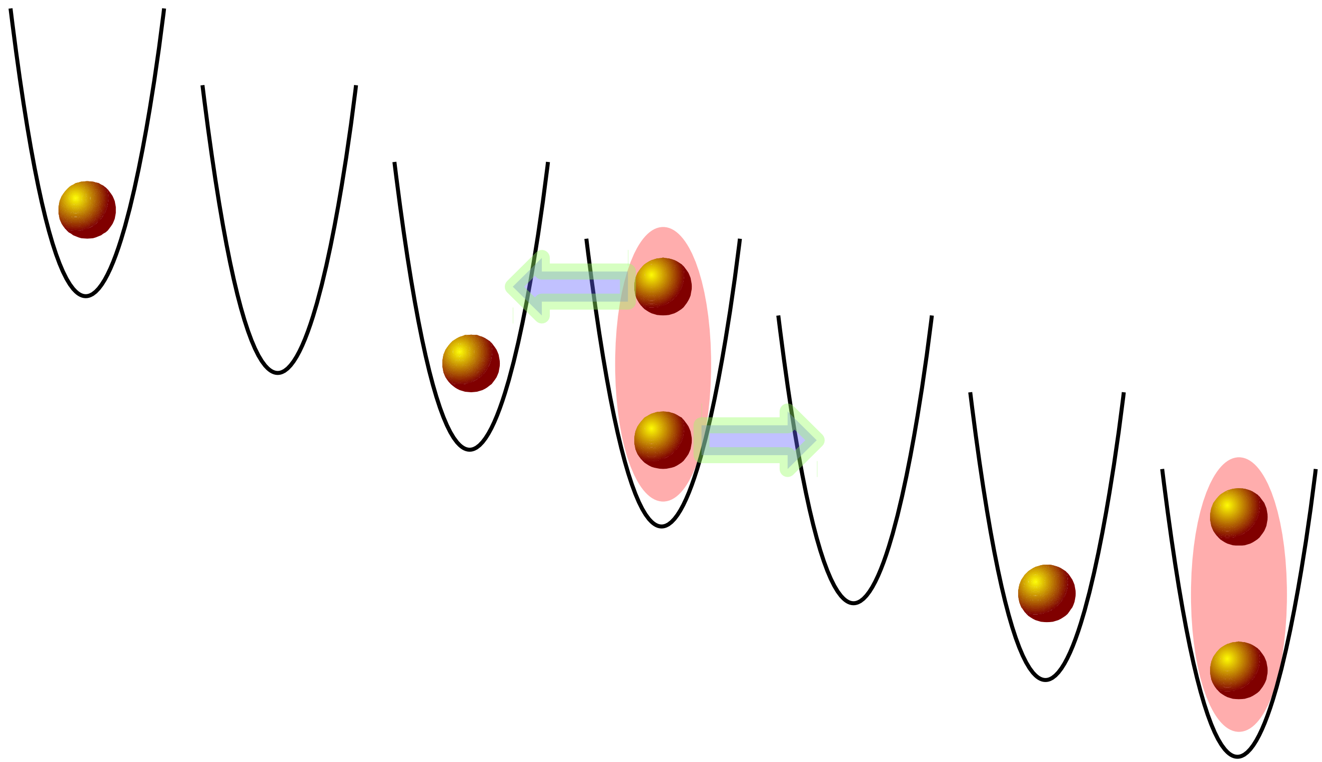
- Then, to restore the state one finds

$$\rho(\beta) = \sqrt{\rho(\beta)} \cdot \left(\sqrt{\rho(\beta)}\right)^\dagger$$

- Application $\mathrm{e}^{-\delta\beta H/2}$ is realized via TDVP methods
- With constraints: $H \to P_n H P_n$

Forbidden transitions

- There are “bad” transitions that cannot be mapped to the spins model



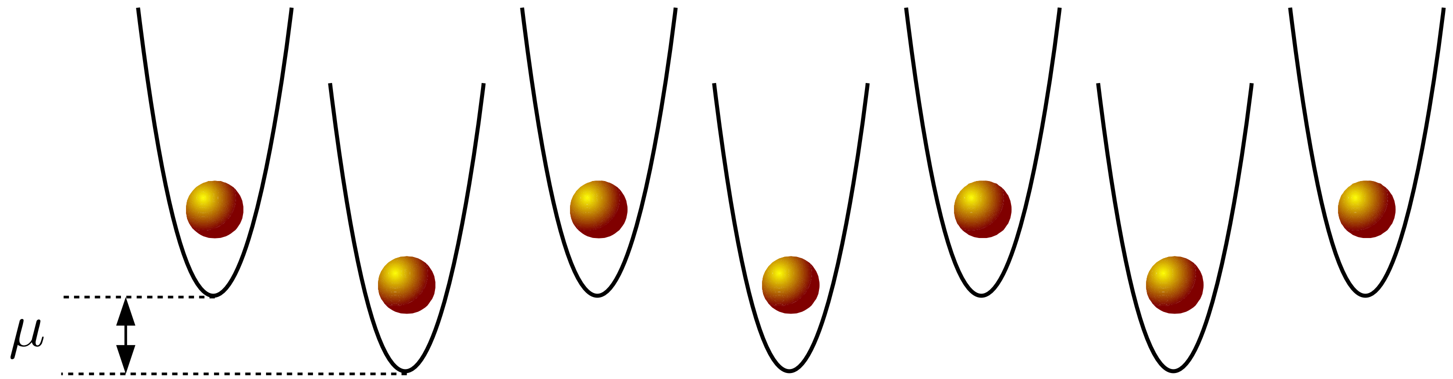
- However, there is a workaround!

Superlattice solution

- Superlattice:
  - Hamiltonian:

$$H = -J \sum_{\langle k,l \rangle} b_k^\dagger b_l + \frac{U}{2} \sum_k n_k (n_k - 1) + \frac{\mu}{2} \sum_k (-1)^k n_k$$

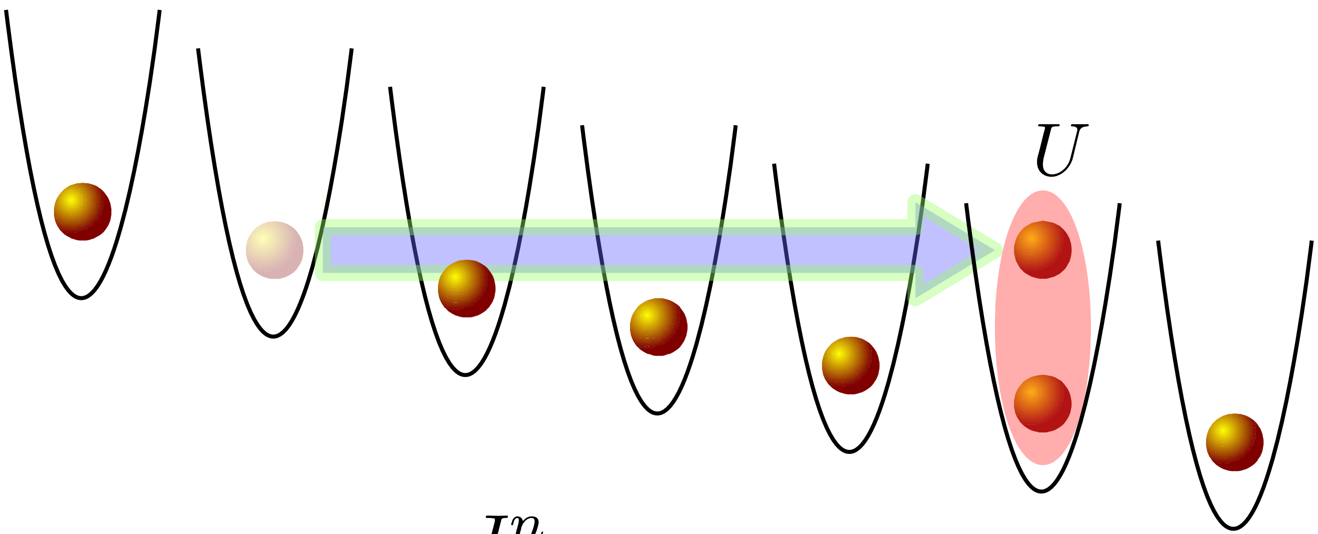
  - Initial state the same:



  - It makes “bad” transitions off-resonant

Experiments with higher order effects

- Turn on the gradient potential such that $E = U/n$

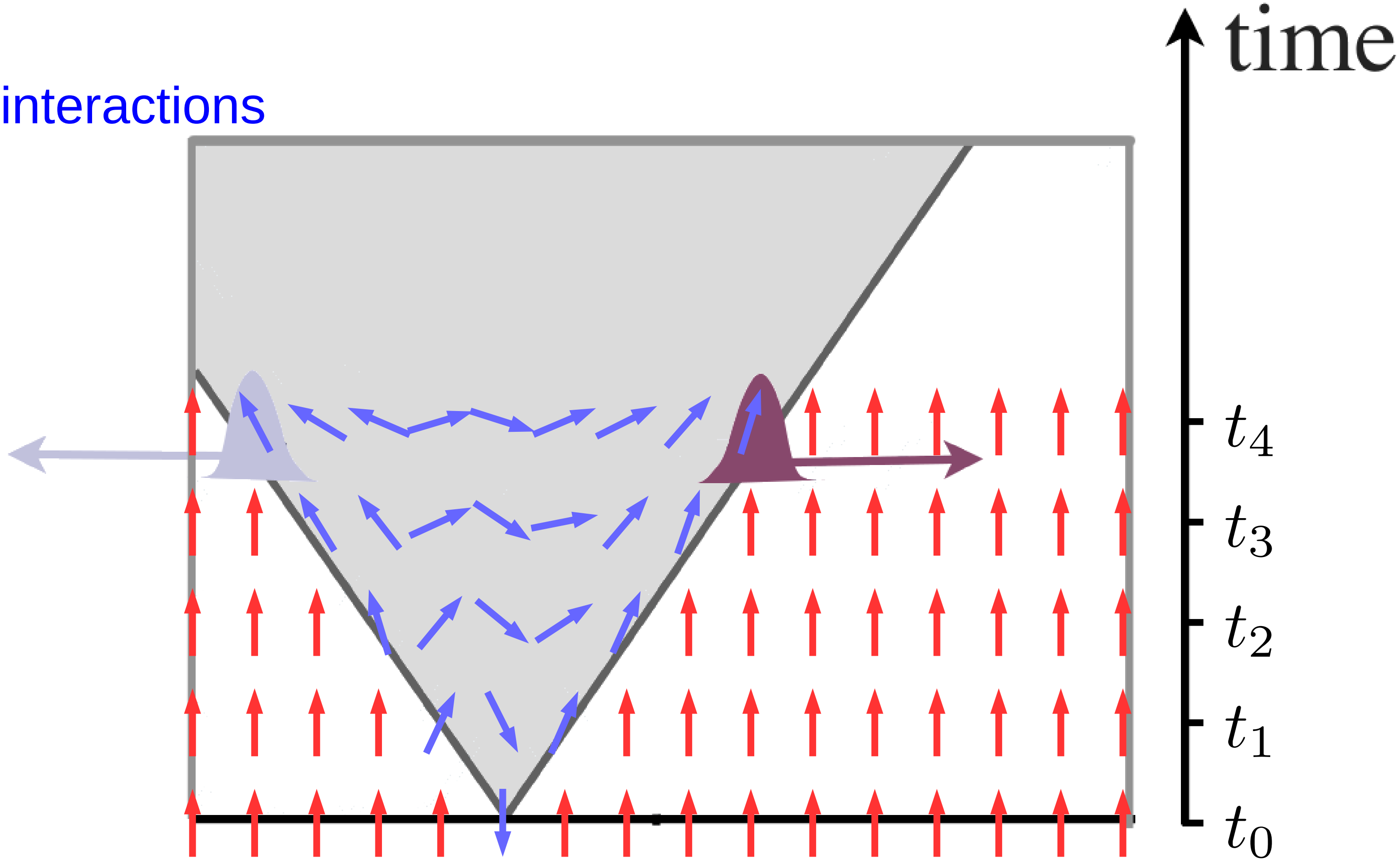


$$\propto \alpha_n \times \frac{J^n}{(U/n)^{n-1}}$$

- α_n depends on the occupation of the neighbouring sites
- However, “**self-energy**” terms $\propto J^2/(U/2)$ are always present

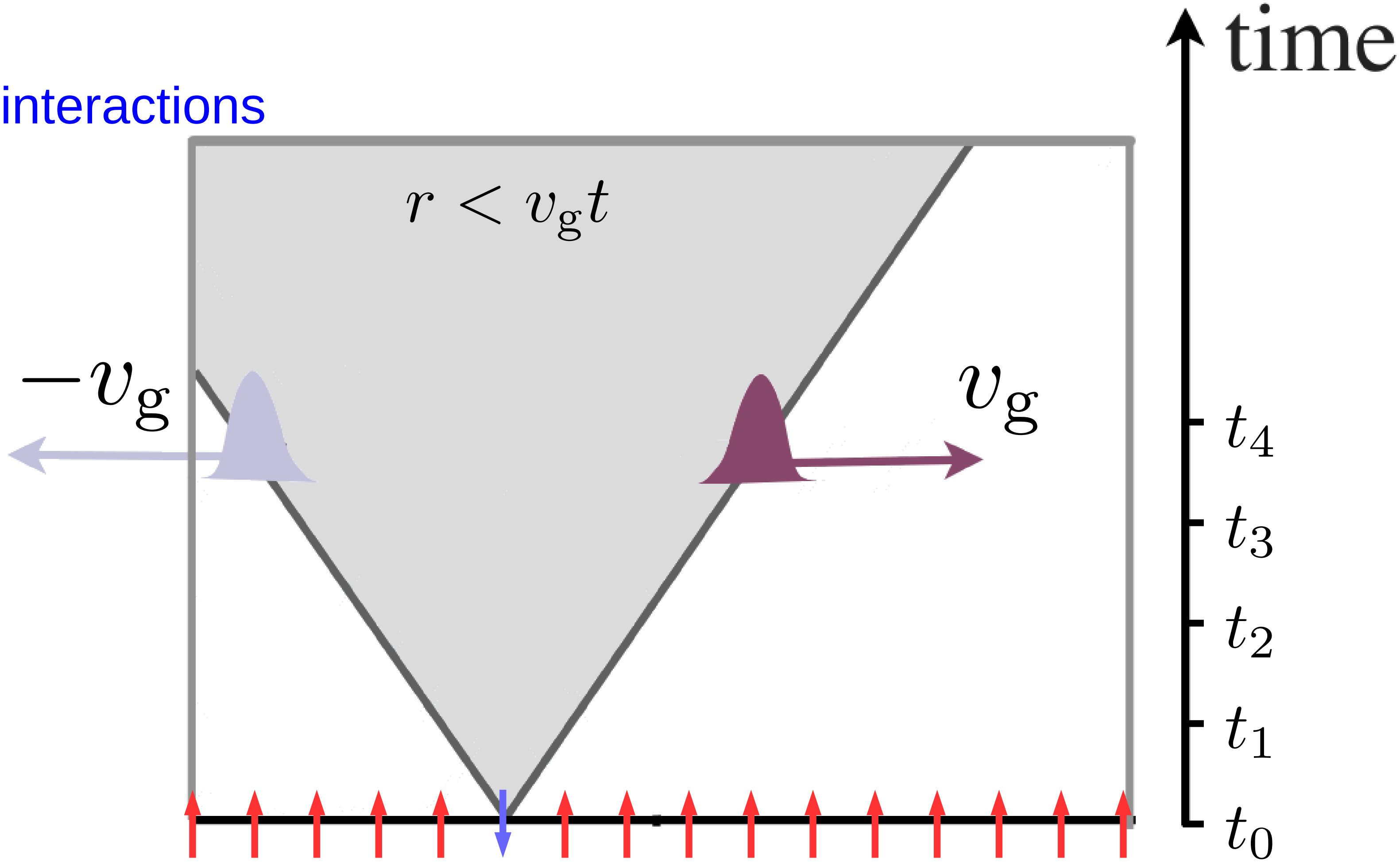
Light cone effects

- Finite-range interactions



Light cone effects

- Finite-range interactions



- Leib-Robinson bound:

$$\lim_{\substack{|t| \to \infty \\ |x| > v_{\mathrm{g}} |t|}} \left\| \left[\tau_t^{\Phi} \tau_x (A), B \right] \right\| \exp\left(\mu_{v_{\mathrm{g}}} t \right) = 0$$

E.H. Lieb and D.W. Robinson Comm. Math. Phys. 28, 251 (1972)