Department of Physics
Clarendon Laboratory

Quantum fluctuation relations with conserved quantities

Jordi Mur-Petit

with A. Relaño, R.A. Molina & D. Jaksch

CSIC
Consejo Superior de Investigaciones Científicas

Quantum fluctuation relations with conserved quantities

[WP4] Complex many-body systems
→ Impurity probes (Task 4.2)

[WP5] Non-equilibrium & emergent phenomena
→ Probing schemes based on QFRs (Task 5.1)

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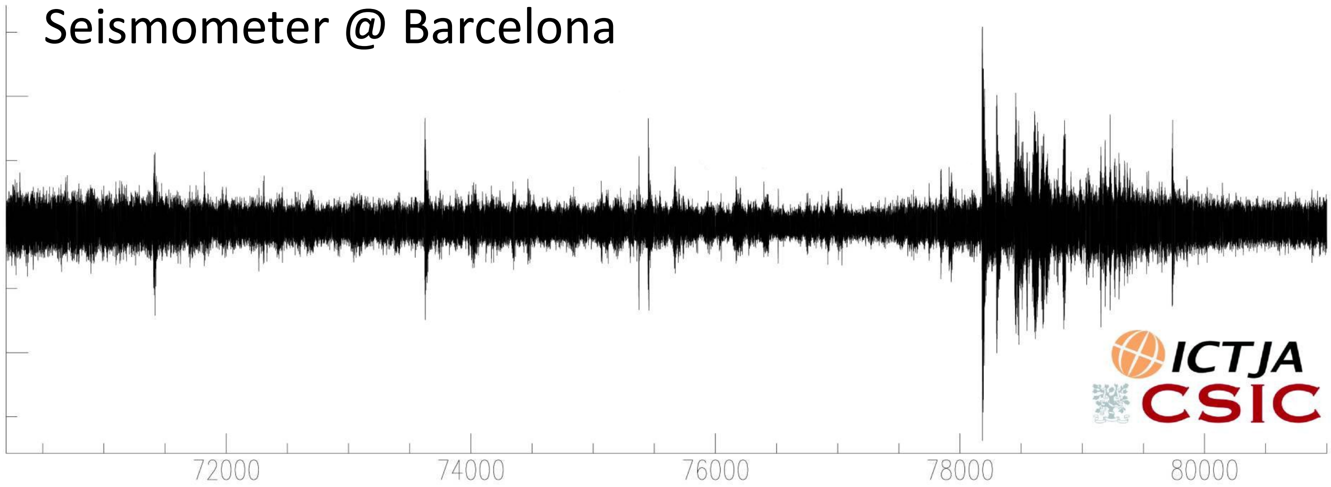
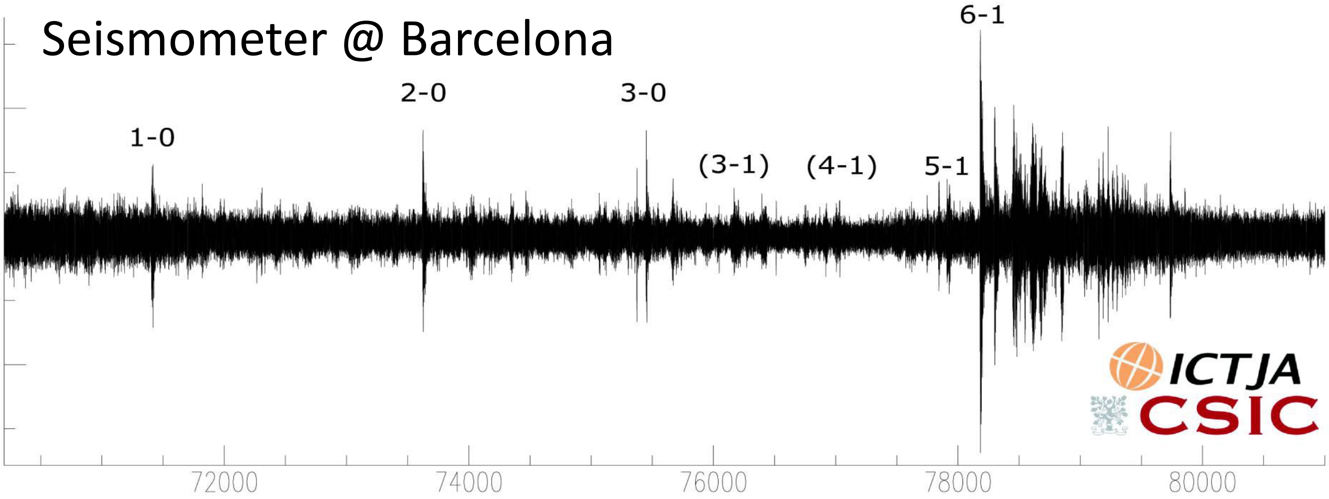
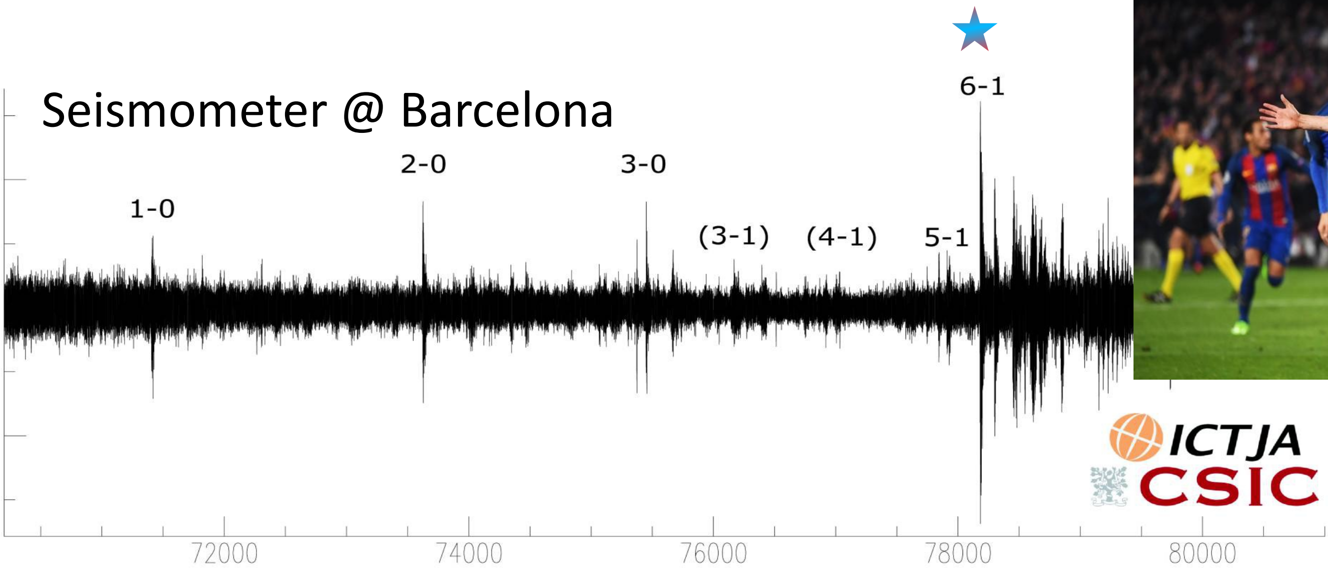
Fluctuating world



Fluctuating world



Fluctuating world



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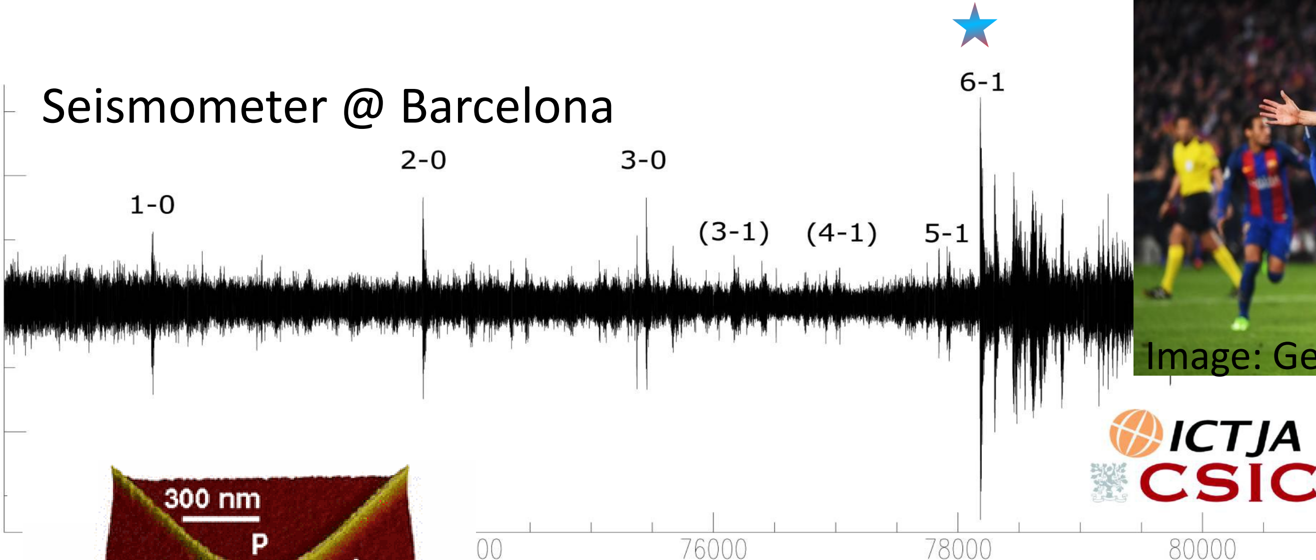
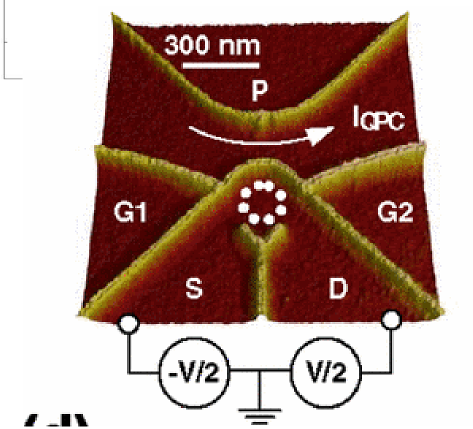
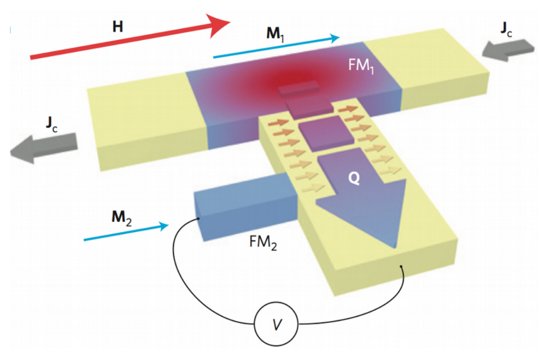


Image: Getty

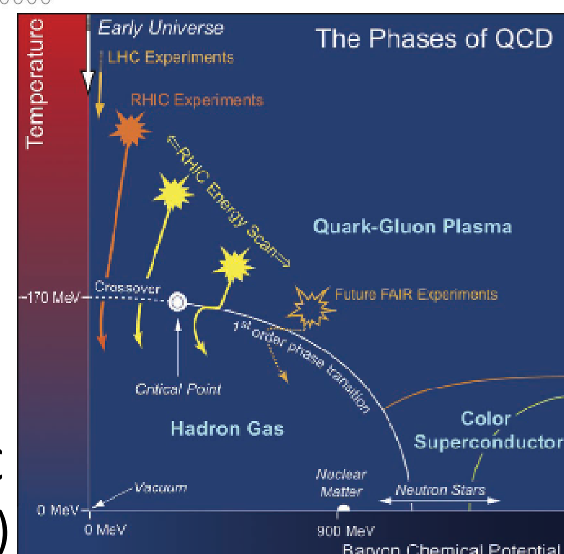


Electronic transport in nanodevices



Spin Caloritronics

RHIC (BNL)



Fluctuations?

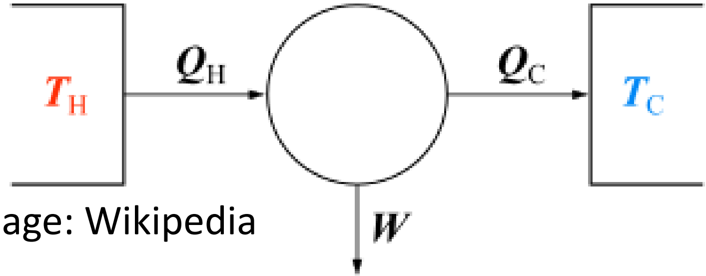


Image: Wikipedia

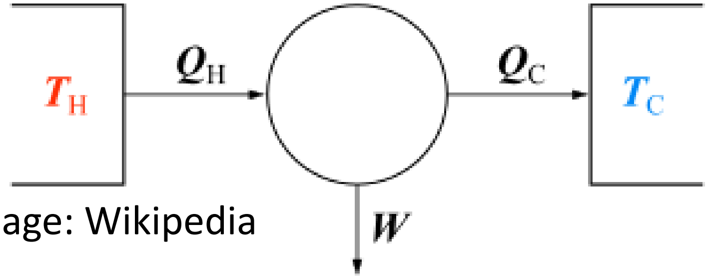
Thermodynamic Laws (1824-)

$$W \geq \Delta F\,, \quad \Delta S \geq 0$$

"41018 of Deutsche Reichsbahn climbing the Schiefe Ebene, 5 Nov 2016"

[Photo by user Chianti, Wikipedia]

Fluctuations? Bounded!



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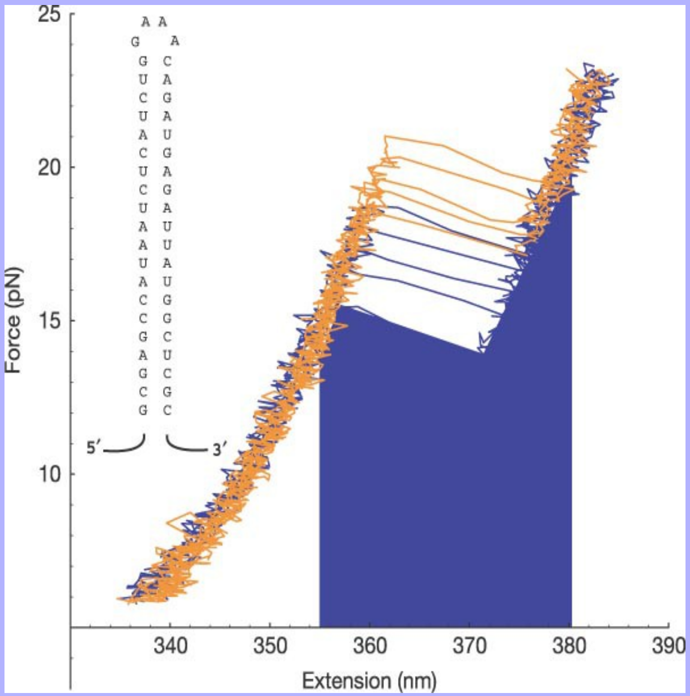


Collin et al., Nature (2005)

Fluctuation Relations (1993-)

$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$

Jarzynski equality

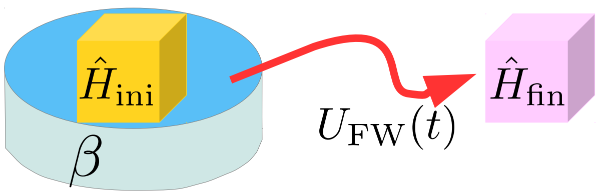
$$P_f(w) = e^{\beta(w - \Delta F)} P_b(-w)$$

Crooks relation

→ **Constrain $P(w)$:**

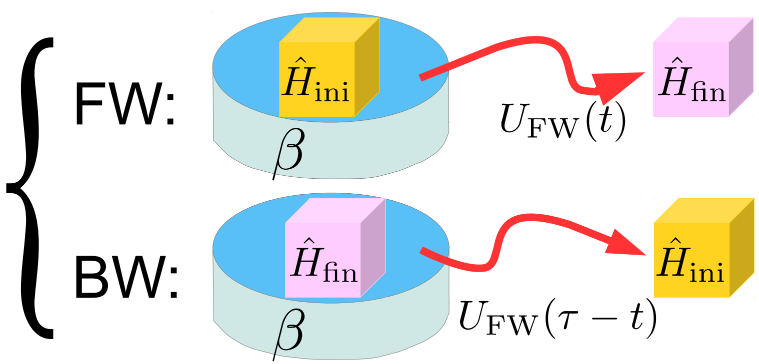
$$\langle w \rangle\,, \langle w^2 \rangle\,, \langle w^3 \rangle\,, \ldots$$

Quantum Fluctuation Relations



Quantum Jarzynski equality **(QJE)**

$$\langle e^{-\beta w}\rangle = e^{-\beta \Delta F}$$



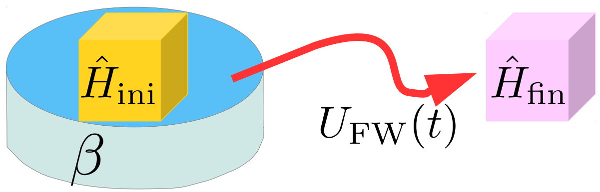
Tasaki-Crooks relation **(TCR)**

$$\frac{P_f(w)}{P_b(-w)} = e^{\beta(w-\Delta F)}$$

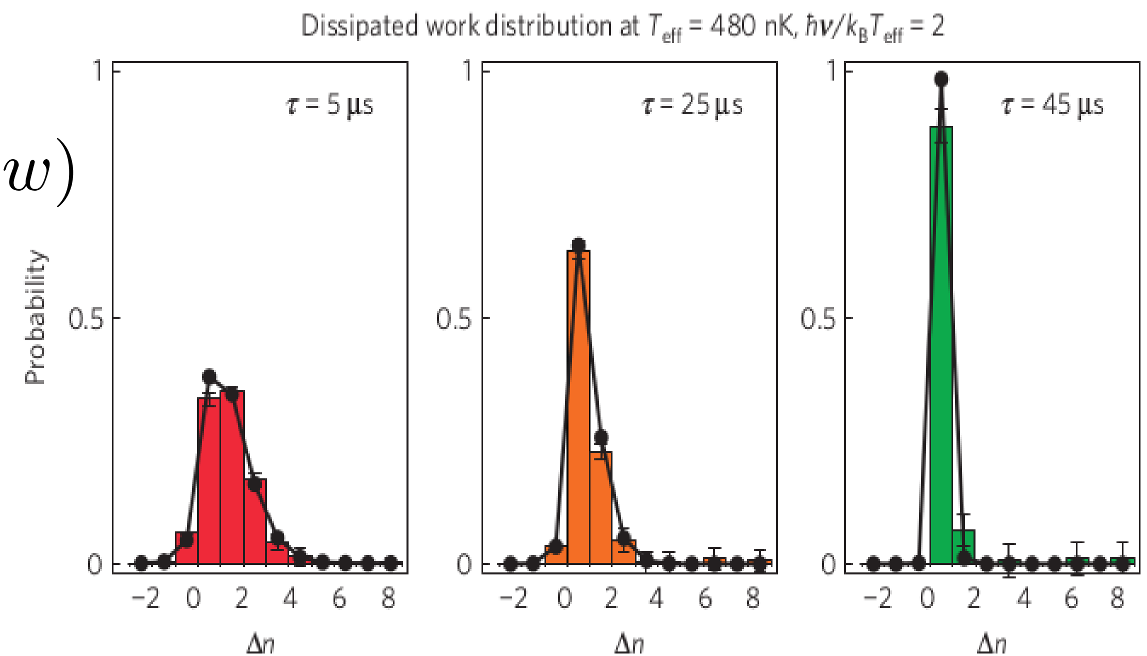
QJE: Tasaki (2000), Kurchan (2000), Yukawa (2000), Mukamel (2003), DeRoeck & Maes (2004); *TCR:* Tasaki (2000), Monnai (2005)

Testing the Quantum: QJE

$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$



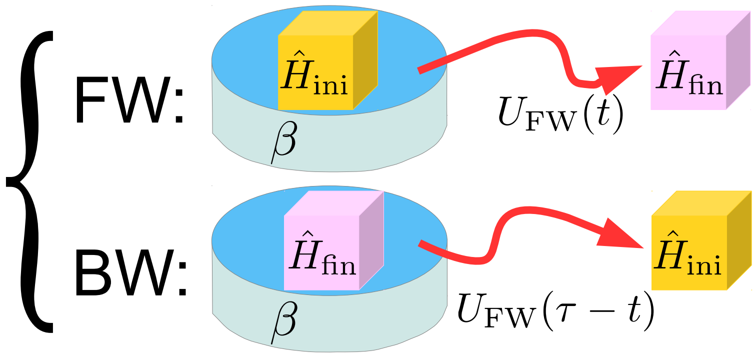
$$P(w)$$

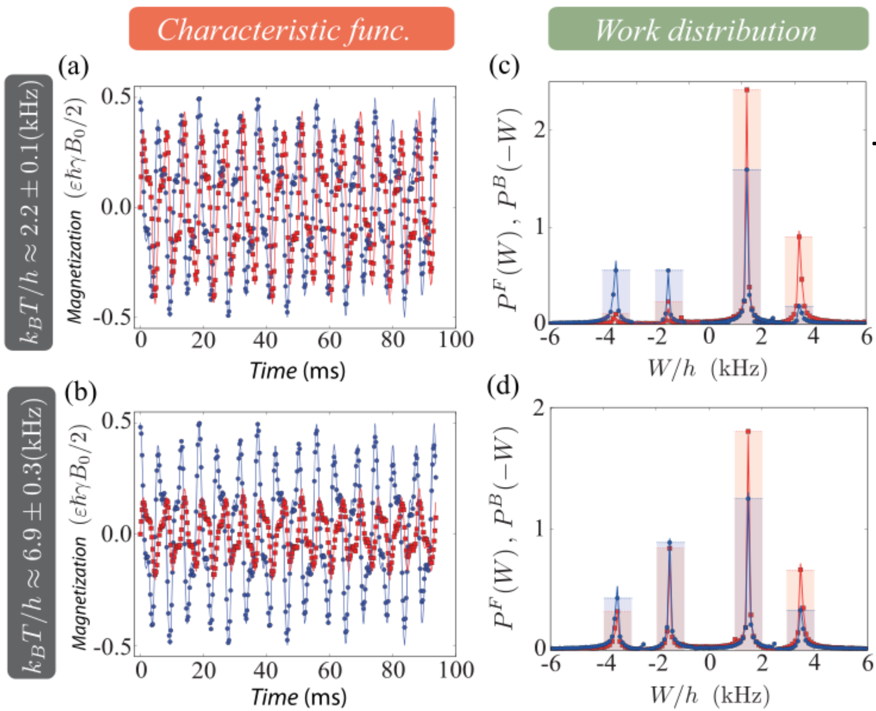


Th: Huber et al., PRL 2008
Expt: An et al., Nat. Phys. 2015 [one trapped ion]

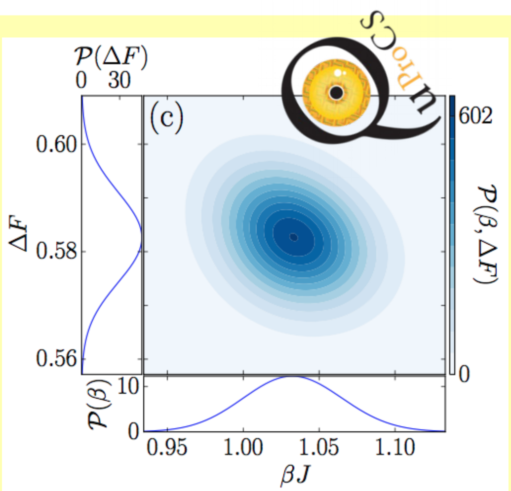
Testing the Quantum: TCR

$$\frac{P_f(w)}{P_b(-w)} = e^{\beta(w-\Delta F)}$$



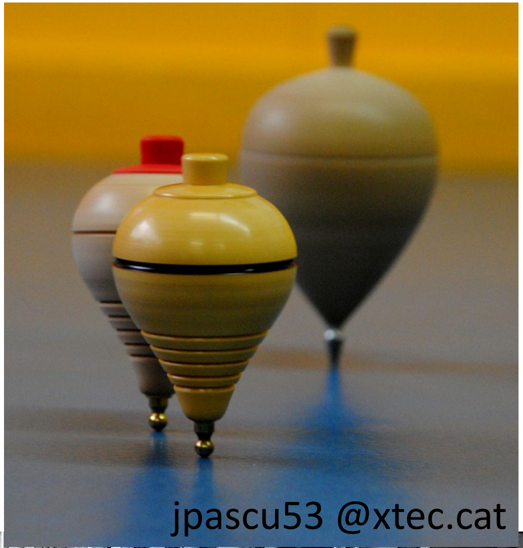


Th: Dorner et al., & Mazzola et al., PRL 2013
Expt: Batalhão et al., PRL 2014 [liquid NMR]



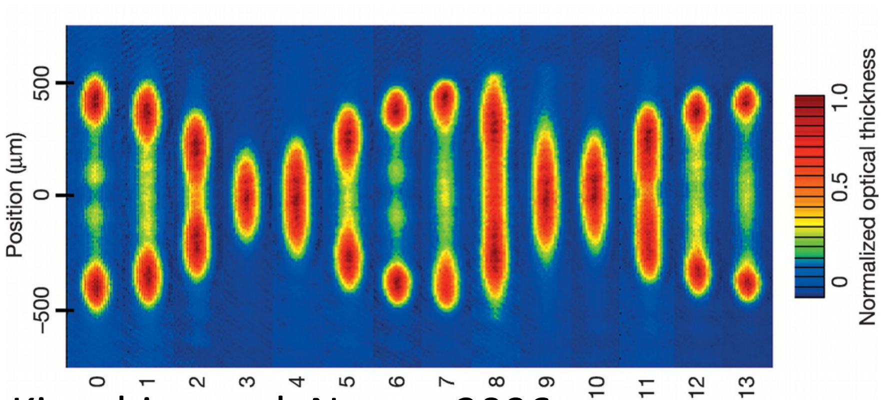
Also: TCR-based protocols for quantum probing (thermometry, correlations...):
T.H. Johnson et al., PRA 93, 053619 (2016); M. Streif et al. PRA 94, 054634 (2016); ...

Physics & conservation laws



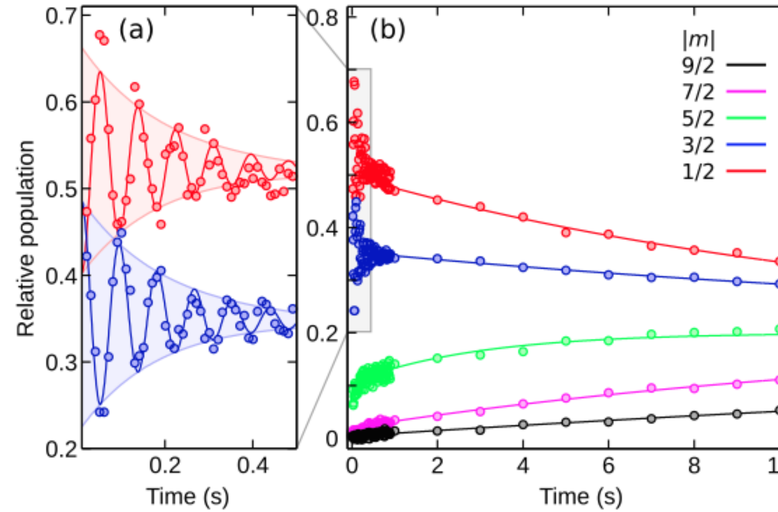
Conservation vs. Relaxation

(Quasi)particle number



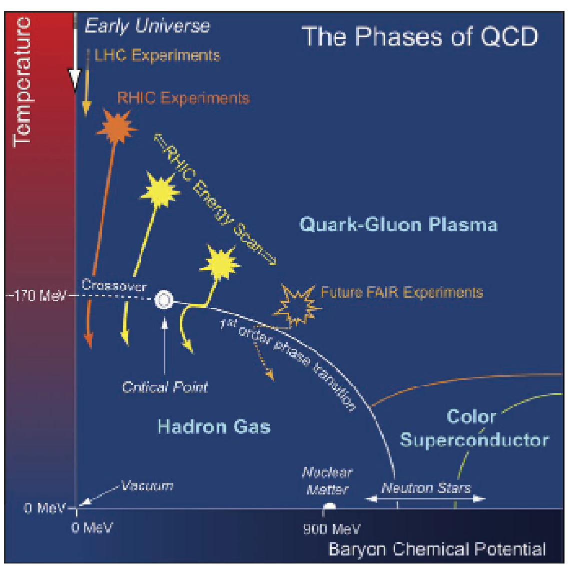
Kinoshita et al. Nature 2006

Magnetization (e.g. spinor gases)

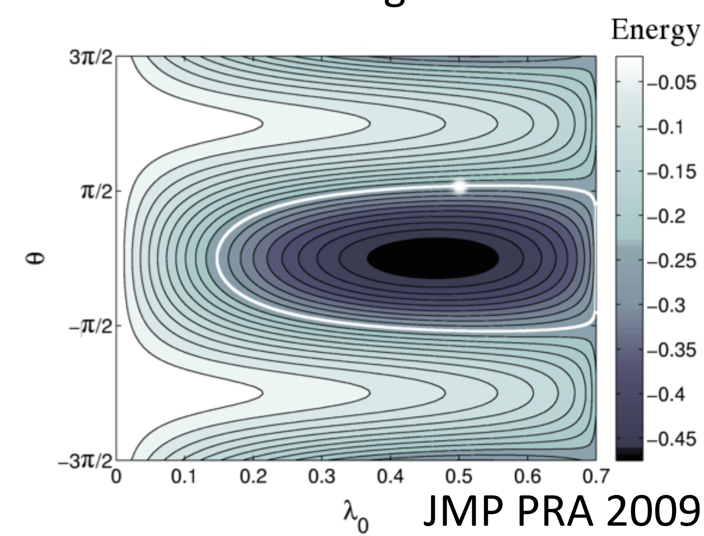


Ebling et al. PRX 2014

Charge & baryonic number



RHIC (BNL)



JMP PRA 2009

Conservation vs. Relaxation

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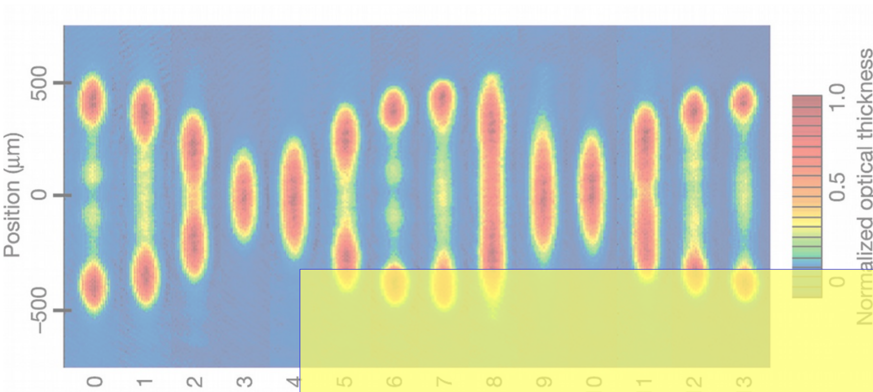

Kinoshita et al. Nature 2006

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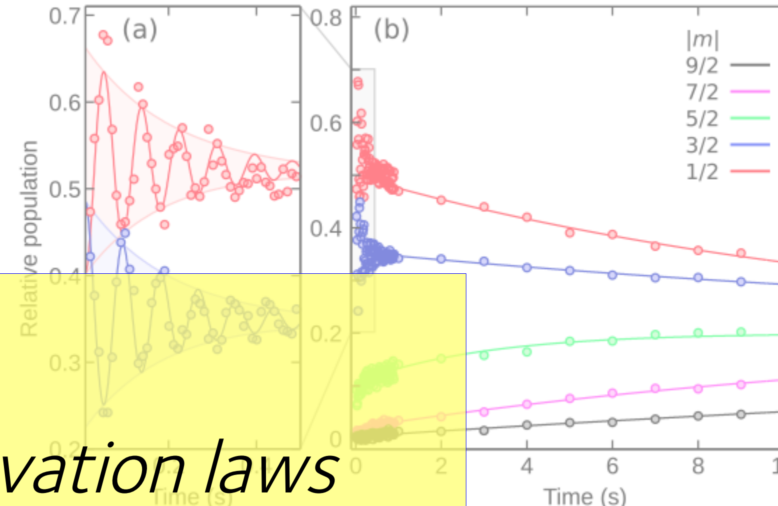
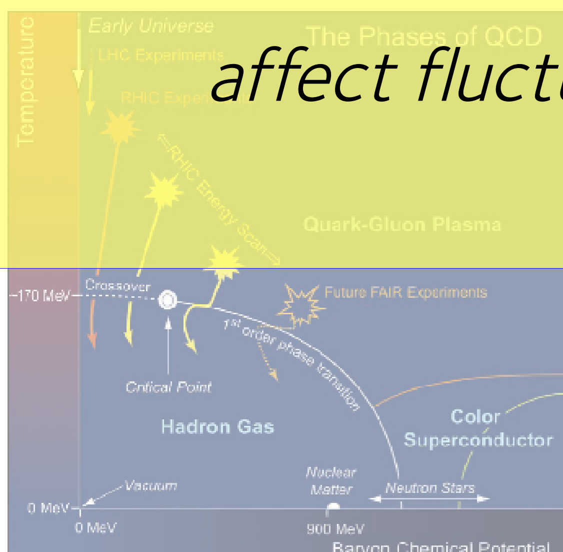


Ebling et al. PRX 2014

"How do conservation laws affect fluctuations?"

Charge & baryonic number



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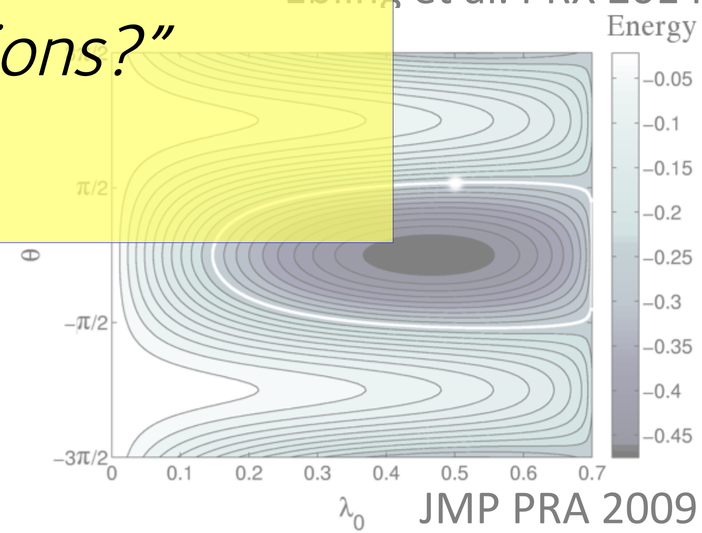
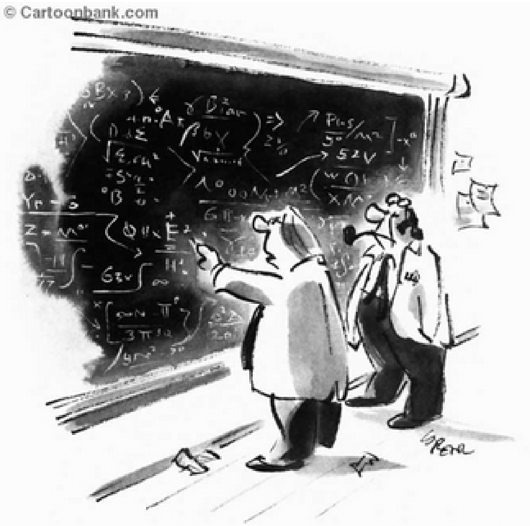


JMP PRA 2009

Back to the blackboard



"Here's your problem—you forgot the sleaze factor."

Quantum Jarzynski equality

$$\langle e^{-\beta w}\rangle = e^{-\beta \Delta F}$$

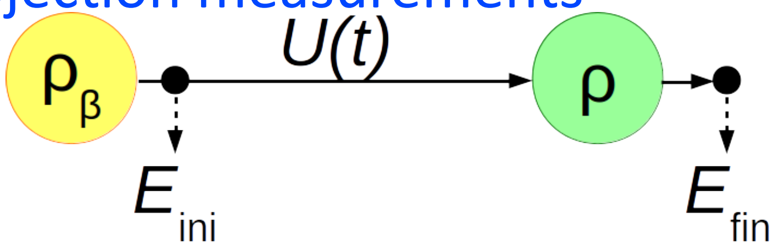
Tasaki–Crooks relation

$$\frac{P_f(w)}{P_b(-w)} = e^{\beta(w-\Delta F)}$$

Assumptions underlying their derivation:

i) Work defined via two energy-projection measurements

$$w = E_{\text{fin}} - E_{\text{ini}}$$
$$= \text{Tr}[U\rho_\beta U^{-1} H_{\text{fin}}] - \text{Tr}[\rho_\beta H_{\text{ini}}]$$



Talkner, Hänggi et al., J.Phys.A (2007), PRE (2007), PRE (2016)

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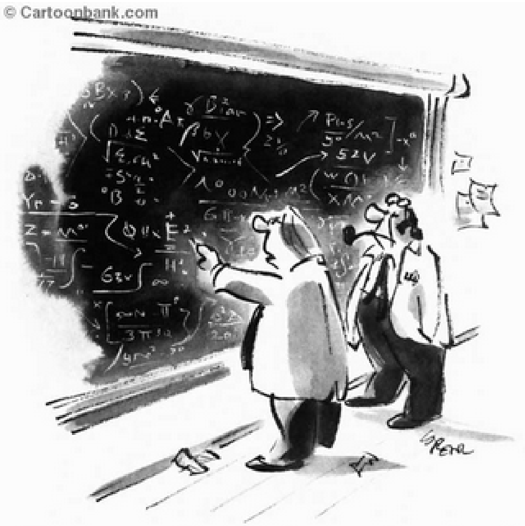

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Tasaki-Crooks relation

$$\frac{P_f(w)}{P_b(-w)} = e^{\beta(w - \Delta F)}$$

Assumptions underlying their derivation:

i) Work defined via two energy-projection measurements

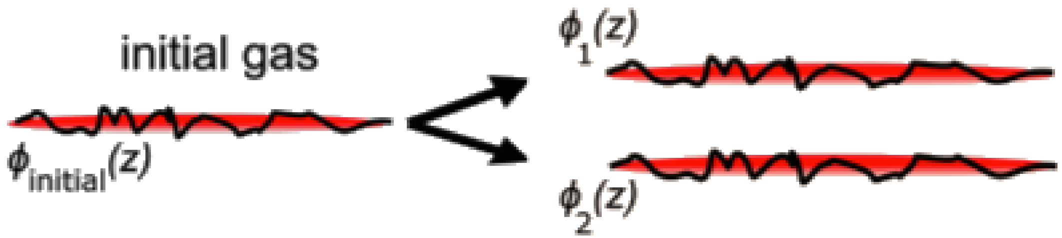
ii) Initial state: **canonical (Gibbs) equilibrium state:**

$$\rho(t=0) = \rho_\beta = \frac{1}{Z} e^{-\beta H}$$

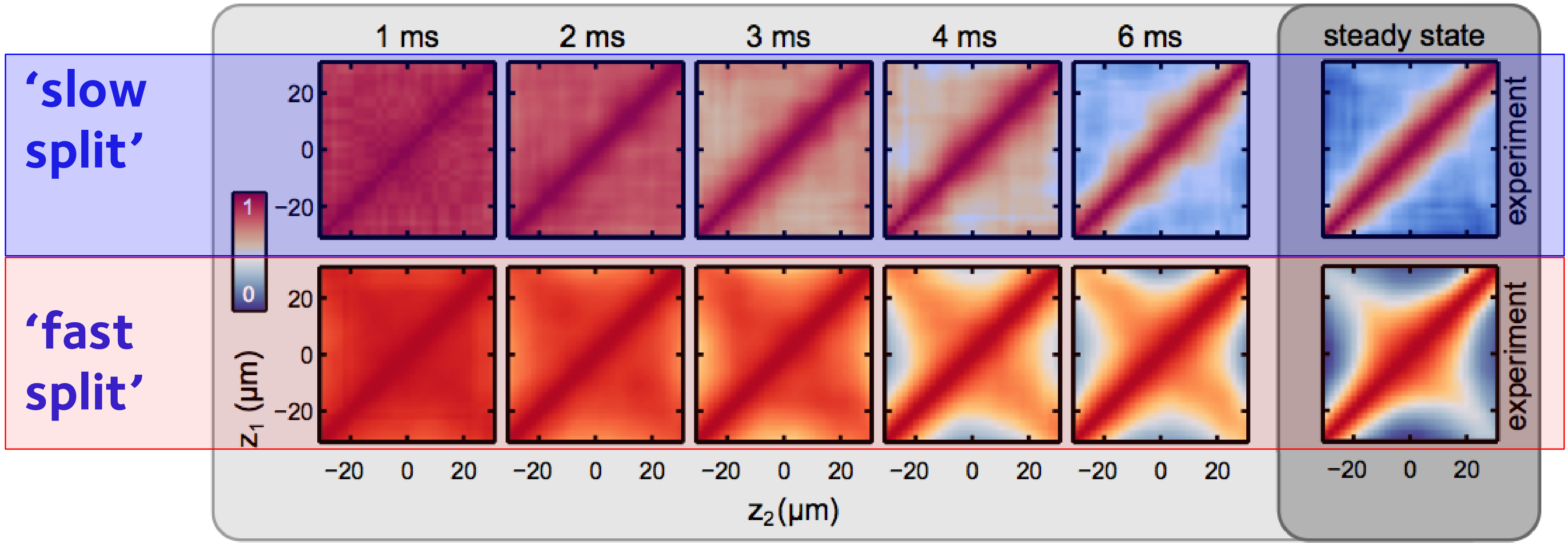
Tasaki (2000); Campisi, Hänggi, Talkner, RMP (2011)

Beyond Gibbs

Split a 1D gas non-adiabatically *[Langen et al., Science 2015]*

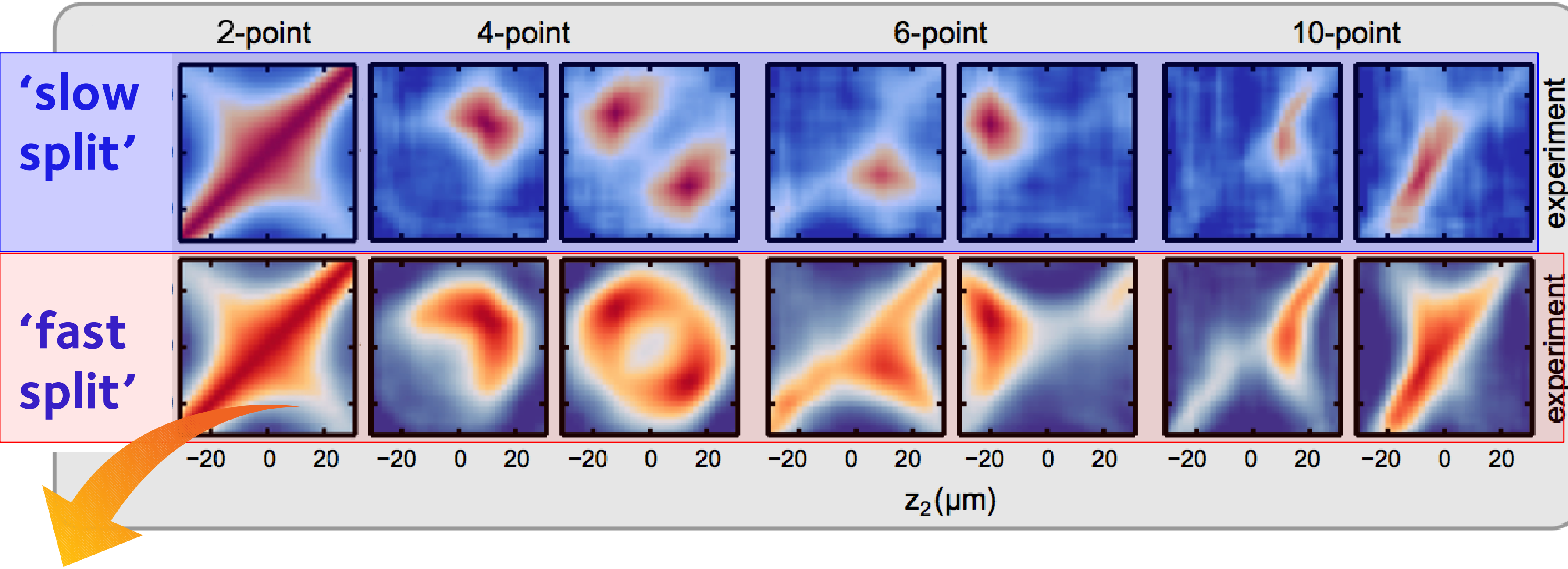


Two-point correlation function vs. splitting rate:



Beyond Gibbs

$$C(z_1, \ldots, z_N) = \langle \Psi_1(z_1)\Psi_2^\dagger(z_1)\Psi_1^\dagger(z_N)\Psi_2(z_N)\rangle$$



Fast split: Need up to 10 different 'temperatures' to fit!

=> 'Memory of conserved quantities': **generalized Gibbs ensemble**

$$\boxed{\rho_{\mathrm{GGE}} = \exp\left(-\sum \beta_k Q_k\right)/Z}$$

Langen et al., Science 2015

QFRs for GGEs

"How do conservation laws affect fluctuations?"

- *"Can we derive QFRs if initial state = GGE?"*
- *"Which conserved quantities are 'relevant' in non-equilibrium dynamics (thermalization, relaxation)?"*

$$\rho_\beta = \frac{e^{-\beta H}}{Z} \to \rho_{\mathsf{GGE}} = \frac{e^{-\beta H - \sum_k \beta_k Q_k}}{Z}, \quad [Q_k, H] = 0 \; \forall k$$

"Can we derive QFRs for GGEs?"

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$$\beta W := \beta(E' - E) + \sum_k \beta_k (Q_k' - Q_k)$$

No need [H,Q]=0 during protocol

$$\mathcal{F}_{\mathsf{GGE}} = -\beta^{-1} \ln \mathrm{Tr}[\exp(-\beta \hat{H} - \sum_k \beta_k \hat{Q}_k)]$$

$$\mathcal{P}(x) = \sum_{i,f} \frac{e^{-\beta E_i - \sum_k \beta_k Q_{k;i}}}{\mathcal{Z}} p_{i \to f} \delta\left[\beta x - \{\beta \Delta E + \sum_k \beta_k \Delta Q_k\}\right]$$

See also Hickey & Genway, PRE 2014; Guryanova et al., Nat. Comms 2016; Yunger-Halpern et al., Nat. Comms 2016; Lostaglio et al., arXiv:1511.04420 ...

“Can we derive QFRs for GGEs?”

$$\beta W := \beta(E' - E) + \sum_k \beta_k (Q'_k - Q_k)$$

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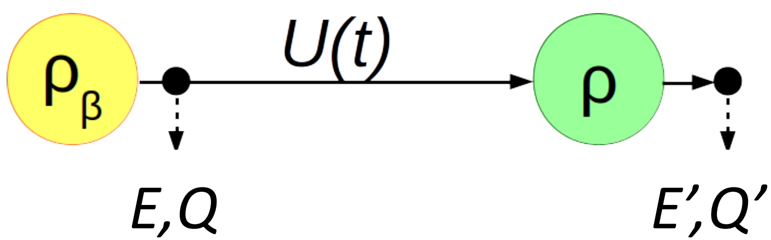
$$\langle e^{-\beta W} \rangle = e^{-\beta \Delta \mathcal{F}_{\mathsf{GGE}}}$$ Generalised QJE

$$e^{-\beta \Delta \mathcal{F}_{\mathsf{GGE}}} = e^{-\beta W} \frac{\mathcal{P}_f(W)}{\mathcal{P}_b(-W)}$$ Generalised TCR



$\Delta E = E' - E$, $\Delta Q_k = Q_k' - Q_k$

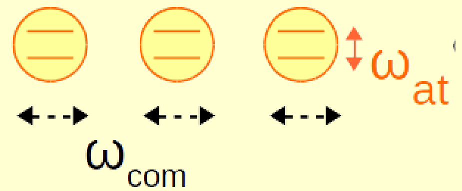
See also Hickey & Genway, PRE 2014; Guryanova et al., Nat. Comms 2016; Yunger-Halpern et al., Nat. Comms 2016; Lostaglio et al., arXiv:1511.04420 ...

Testing ground: Dicke model

$$H = \hbar\omega_{\text{com}} a^\dagger a + \hbar\omega_{\text{at}} J_z + H_{\text{int}}\ , \quad J_{x,y,z} = \sum_{j=1}^{N} \frac{1}{2}\sigma_{x,y,z}^{(j)}$$

$$H_{\text{int}} = \frac{2g}{\sqrt{N}}\left[(1-\alpha)(J_+ a + J_- a^\dagger) + \alpha(J_+ a^\dagger + J_- a)\right]$$

Parameters: $\omega_{\text{com}};\ \omega_{\text{at}};\ g, \alpha$



For $\boldsymbol{\alpha}$**={0, 1}** there's **one** additional conserved charge:

$$Q = J + J_z + a^\dagger a$$

num. phonons

num. excited atoms

num. atoms

Testing ground: Dicke model

$$H = \hbar\omega_{\text{com}} a^\dagger a + \hbar\omega_{\text{at}} J_z + H_{\text{int}}\ , \quad J_{x,y,z} = \sum_{j=1}^{N} \frac{1}{2}\sigma_{x,y,z}^{(j)}$$

$$H_{\text{int}} = \frac{2g}{\sqrt{N}}\left[(1-\alpha)(J_+ a + J_- a^\dagger) + \alpha(J_+ a^\dagger + J_- a)\right]$$

$$g = (\Omega_{\text{rsb}} + \Omega_{\text{bsb}})/2$$

$$\alpha = \Omega_{\text{bsb}}/(\Omega_{\text{rsb}} + \Omega_{\text{bsb}})$$

For ***α*={0, 1}** there's **one** additional conserved charge:

$$Q = J + J_z + a^\dagger a$$

num. phonons

num. excited atoms

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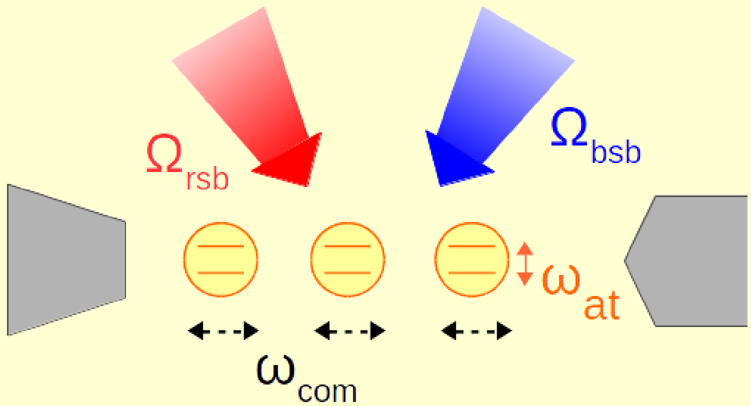


$$N = 7 \text{ ions } ({}^{43}\text{Ca}^+)$$

$$\omega_{\text{at}} = 2\pi \times 10 \text{ MHz}$$

$$\omega_{\text{com}} = 2\pi \times 3 \text{ MHz}$$

$$g = 2\pi \times (1-3) \text{ MHz}$$

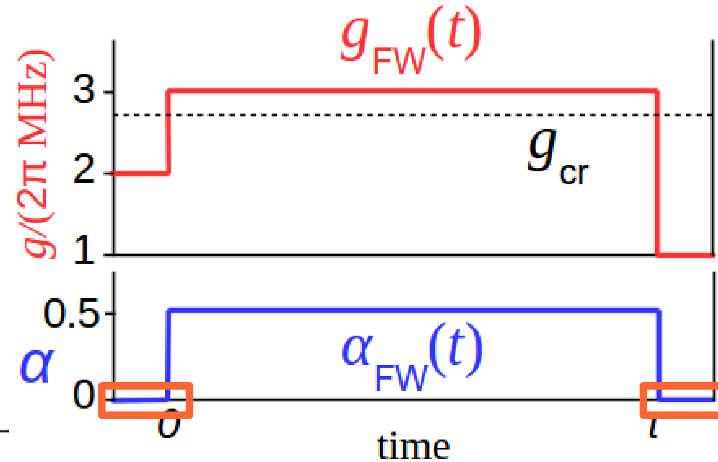
$$\Omega_{\text{rsb,bsb}} \sim 2\pi \times (0-3) \text{ MHz}$$

Results: Generalised Jarzynski

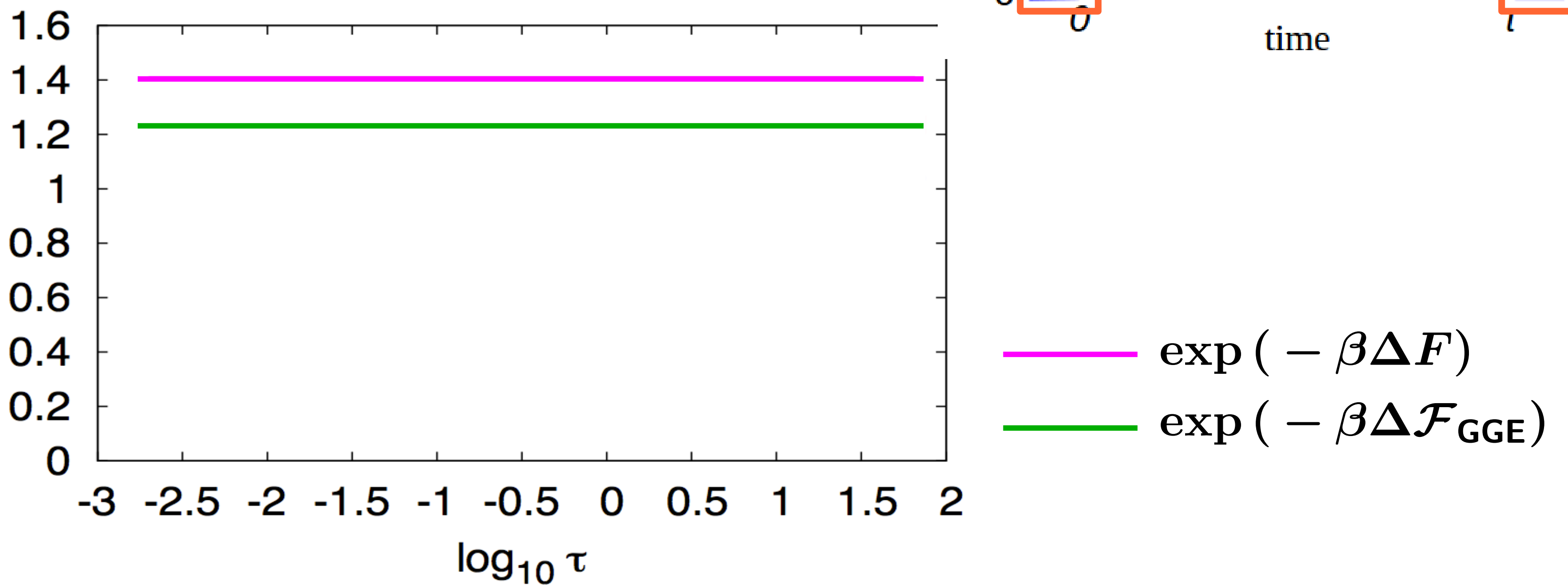
$$\rho(t=0) = \rho_{GGE}(\beta = 0.1, \beta_Q = 0.2)$$

$$\langle e^{-\beta W} \rangle = e^{-\beta \Delta \mathcal{F}_{\mathsf{GGE}}}$$

$$\beta W := \beta w + \beta_Q (Q' - Q)$$



QJE with varying protocol duration τ:



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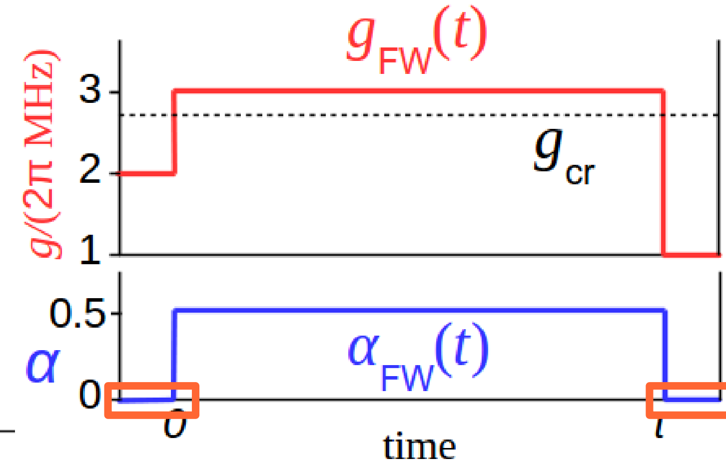

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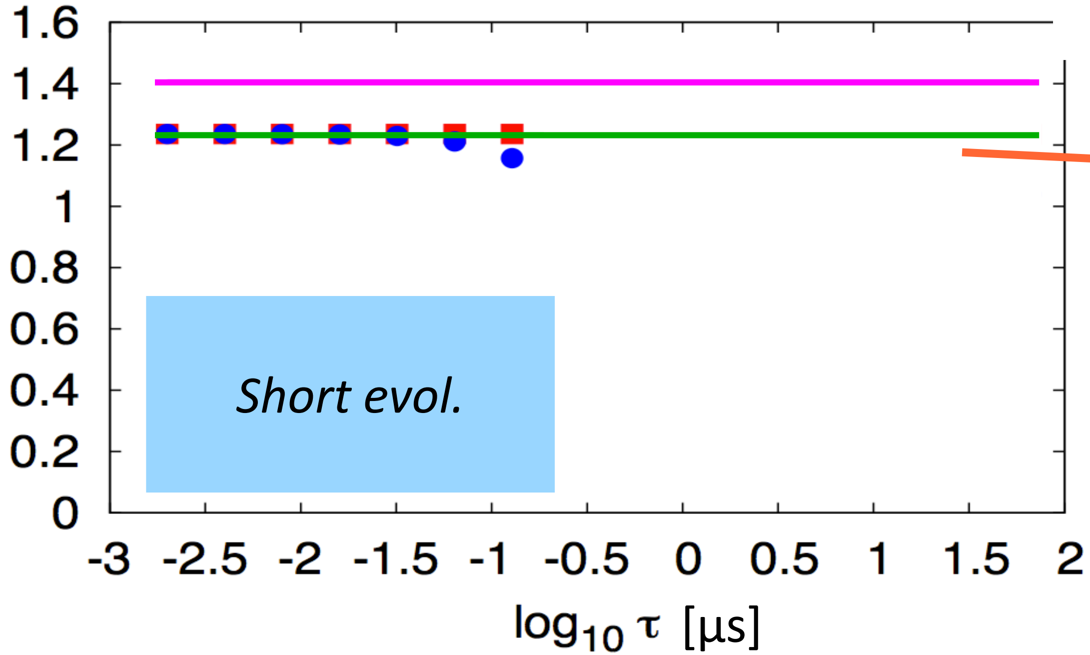

Even if $Q(t)=Q$, they make a difference

$\exp(-\beta \Delta F)$

$\exp(-\beta \Delta \mathcal{F}_{\text{GGE}})$

$\langle \exp(-\beta W) \rangle$

$\langle \exp(-\beta w) \rangle$

Results: Generalised Jarzynski

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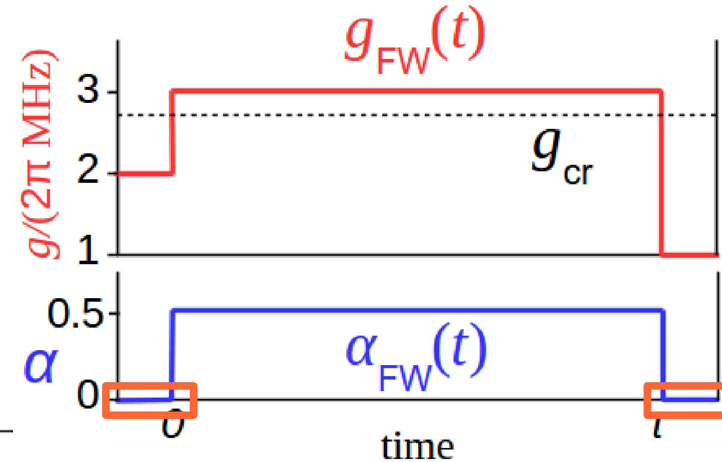

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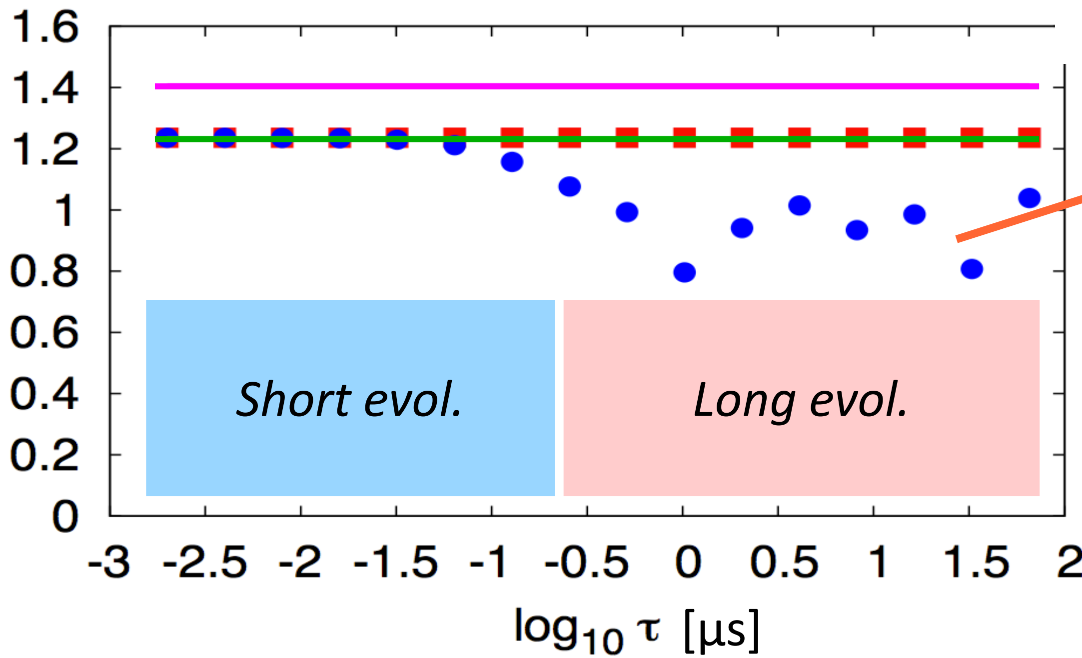

gQJE -> witness to identify all Q's relevant to dynamics

$\exp(-\beta \Delta F)$

$\exp(-\beta \Delta \mathcal{F}_{\text{GGE}})$

$\langle \exp(-\beta W) \rangle$

$\langle \exp(-\beta w) \rangle$

Results: Generalised Jarzynski

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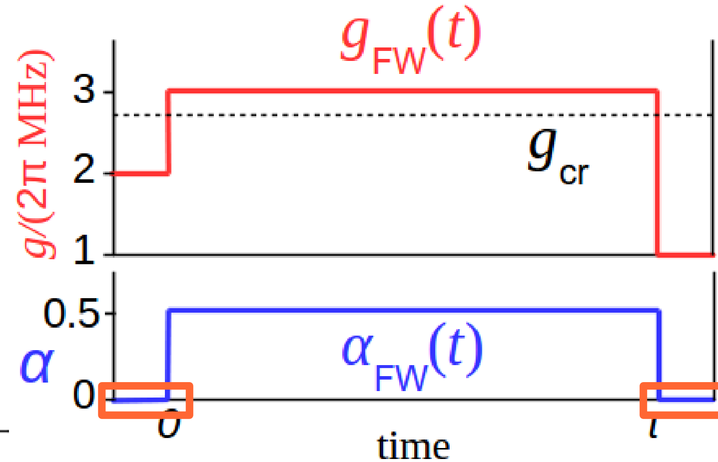

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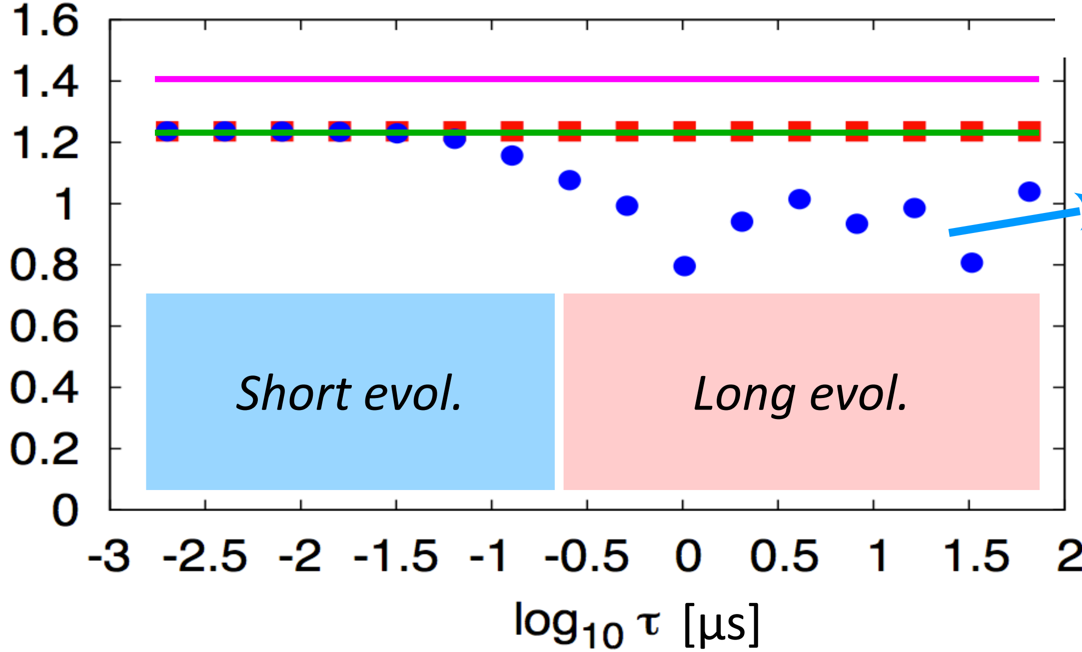


$\langle w \rangle > \Delta \mathcal{F}_{\text{GGE}}$
increased energy dissipation

$\exp(-\beta \Delta F)$

$\exp(-\beta \Delta \mathcal{F}_{\text{GGE}})$

$\langle \exp(-\beta W) \rangle$

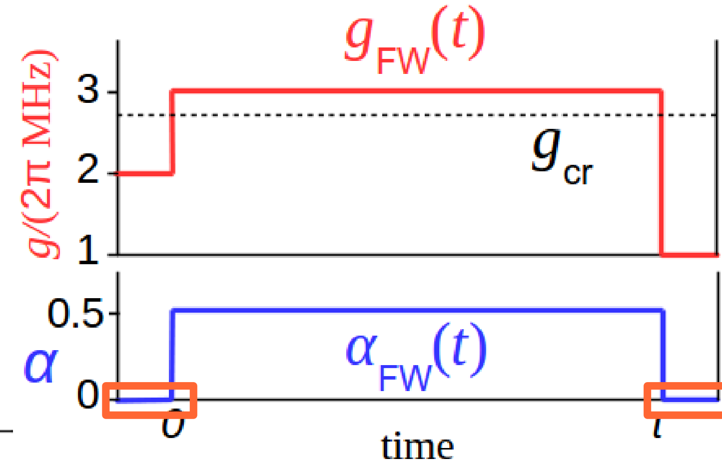
$\langle \exp(-\beta w) \rangle$

Results: Generalised Jarzynski

$$\rho(t=0) = \rho_{GGE}(\beta = 0.1, \beta_Q = -0.2)$$

$$\langle e^{-\beta W} \rangle = e^{-\beta \Delta \mathcal{F}_{\mathrm{GGE}}}$$

$$\beta W := \beta w + \beta_Q (Q' - Q)$$



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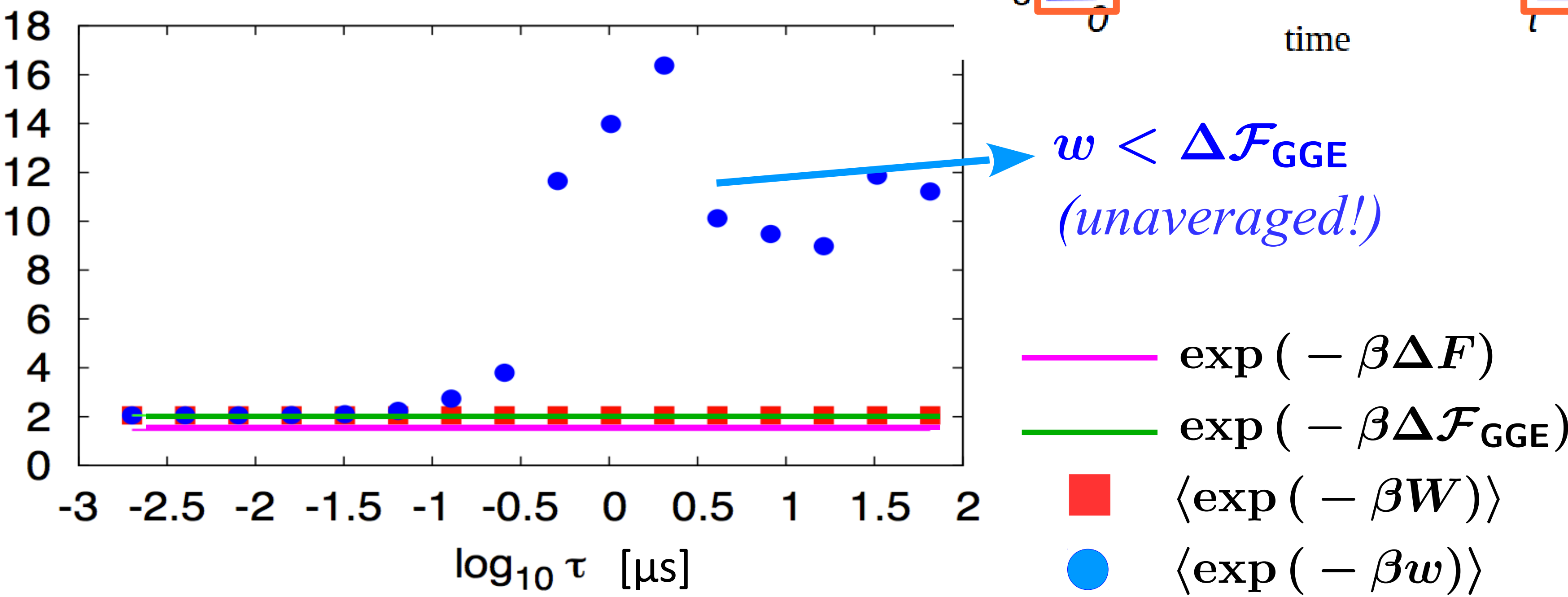


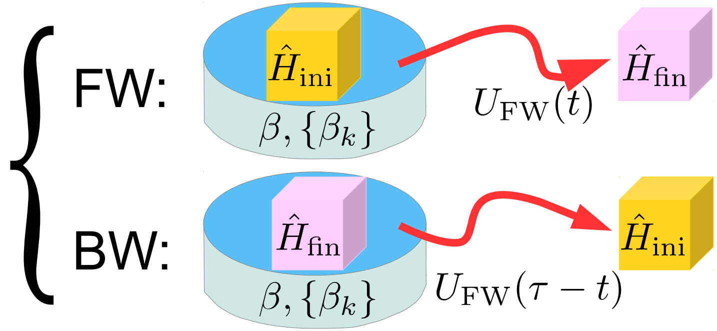
$w < \Delta\mathcal{F}_{\mathrm{GGE}}$

(unaveraged!)

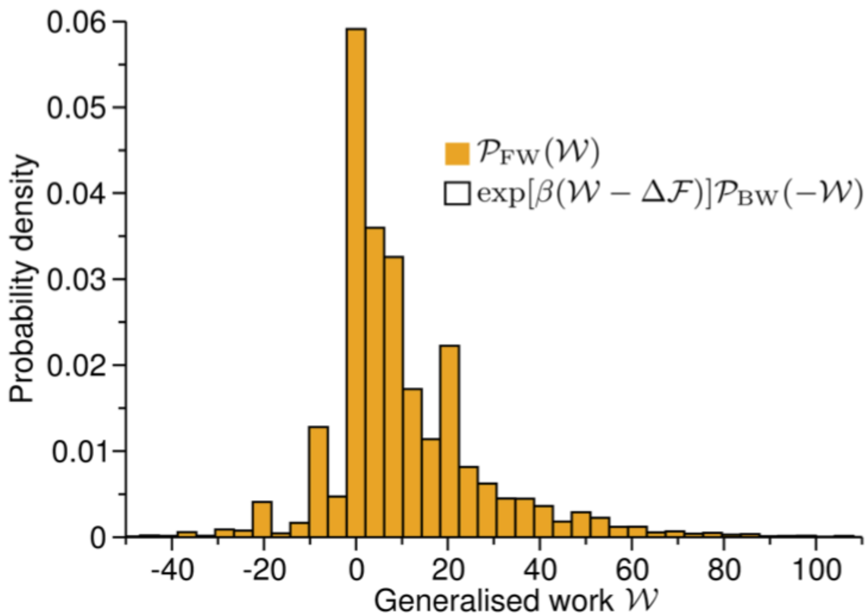
Results: Generalised Tasaki-Crooks

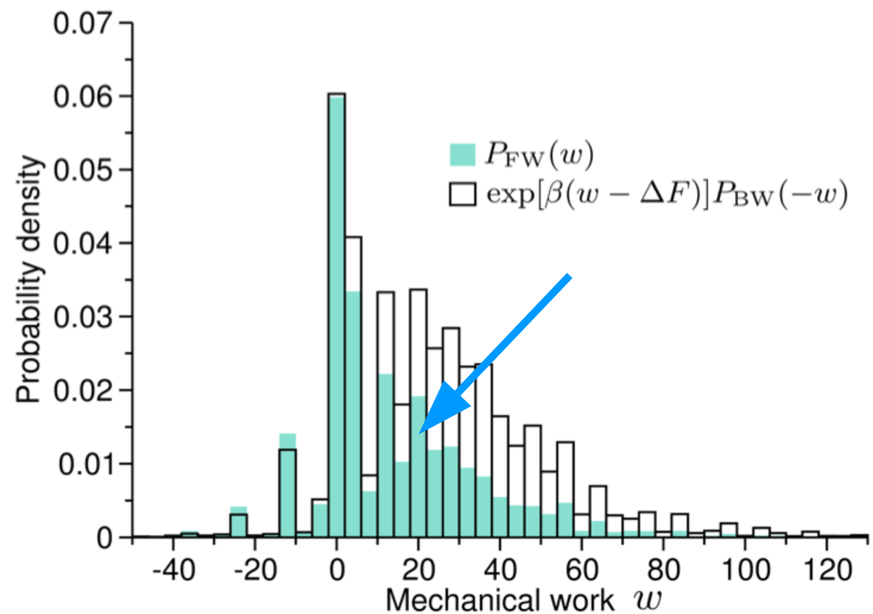
$$\mathcal{P}_{FW}(W) = e^{\beta(W-\Delta\mathcal{F})}\mathcal{P}_{BW}(-W)$$

$$(*)\ P_{FW}(w) = e^{\beta(w-\Delta F)}P_{BW}(-w)$$



Work PDFs for τ≈1 μs (midly long evolution):





$$\rho_{ini} = \rho_{GGE}(\beta = 0.1, \beta_Q = -0.1)\ , \quad \tau = 1.024\ \mu s$$

Summary

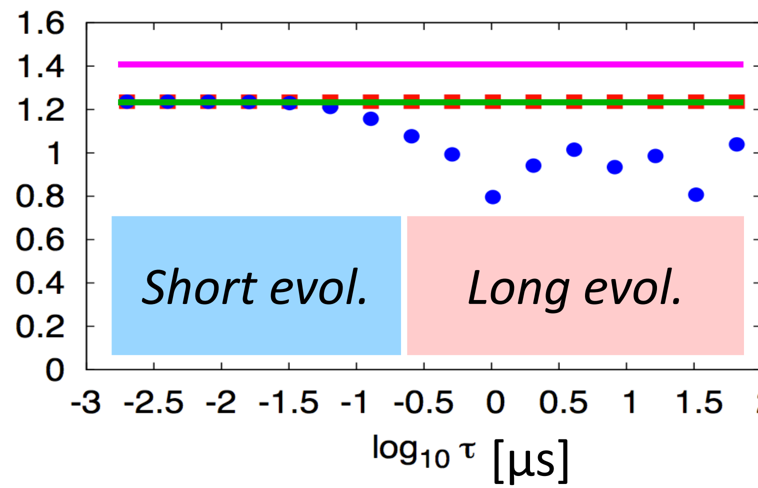
Ramon Llull's
Staircase of Understanding
(Josep Ma. Subirachs)
[Image: aagustine410 @Wordpress]

Summary

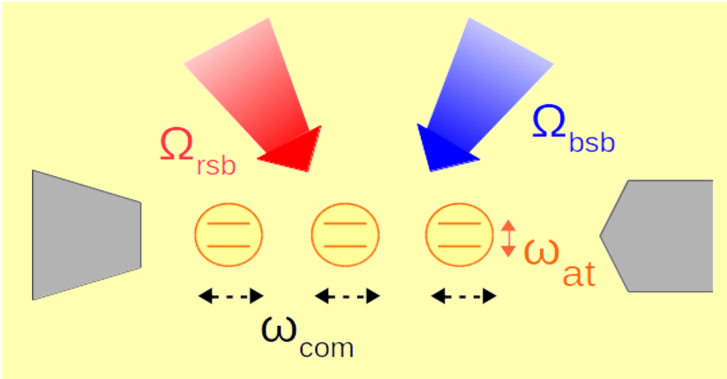
- Generalised QFRs for GGEs
  - → Non-equilibrium QTD with GGEs
  - → *Identify* conserved charges relevant to dynamics
  - → Applicable to trapped ions expts (Kim@Tsinghua, Home@ETH)

$$\langle e^{-\beta W} \rangle = e^{-\beta \Delta \mathcal{F}}$$

$$\mathcal{P}_f(W) = e^{\beta(W - \Delta \mathcal{F})} \mathcal{P}_b(-W)$$



- Outlook
  - → Applications to probe many-body systems w/ cons.quantities
    - Measuring β_k? Thermometry?
    - >1 conserved charges?
  - → Relaxation vs. integrability ?



Gràcies!

Armando Relaño

Rafael Molina

Dieter Jaksch

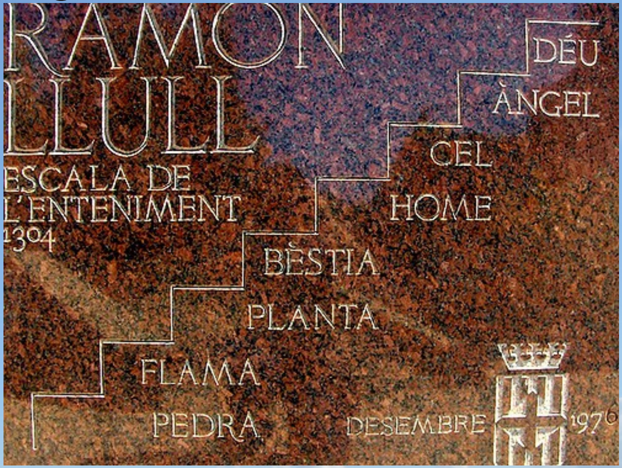
www.quprocs.eu

Discussions with J. Dukelsky, D. Jennings, D. Lucas

Contact me at **jordi.murpetit@physics.ox.ac.uk**

Summary

[Image: Elena Annunziata @flickr]

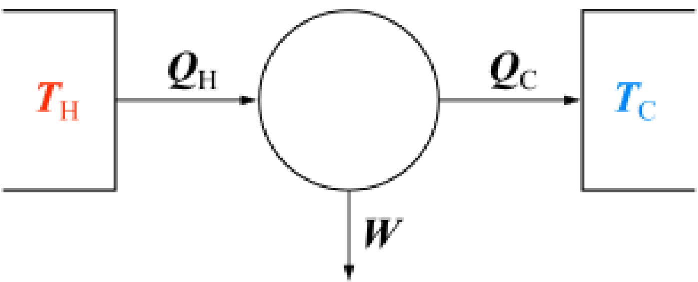


Ramon Llull's
Staircase of Understanding
Josep Ma. Subirachs (1976)
[Image: aagustine410 @Wordpress]

FRs: Better than a bound

'Textbook' laws

2nd law [1824/1851/1854/...]



$$W \geq \Delta F$$

$$\eta = 1 - \frac{T_C}{T_H}$$

$$\Delta S \geq 0$$

$$\oint \frac{\delta Q}{T} \leq 0$$

Fluctuation Relations

Jarzynski equality [1997]

$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$

Crooks relation [1998]

$$\frac{P_f(w)}{P_b(-w)} = e^{\beta(w - \Delta F)}$$

→ **Constrain *P*(*w*):**

$$\langle w \rangle, \langle w^2 \rangle, \langle w^3 \rangle, \ldots$$

Testing ground: Dicke model

$$H = \hbar\omega_{com} + \hbar\omega_{at} + H_{int}$$

$$H_{int} = \frac{2g}{\sqrt{N}}\left[(1-\alpha)(J_+ a + J_- a^\dagger) + \alpha(J_+ a^\dagger + J_- a)\right]$$

For **α={0, 1}**: exists additional conserved charge:

$$Q = J + J_z \pm a^\dagger a$$

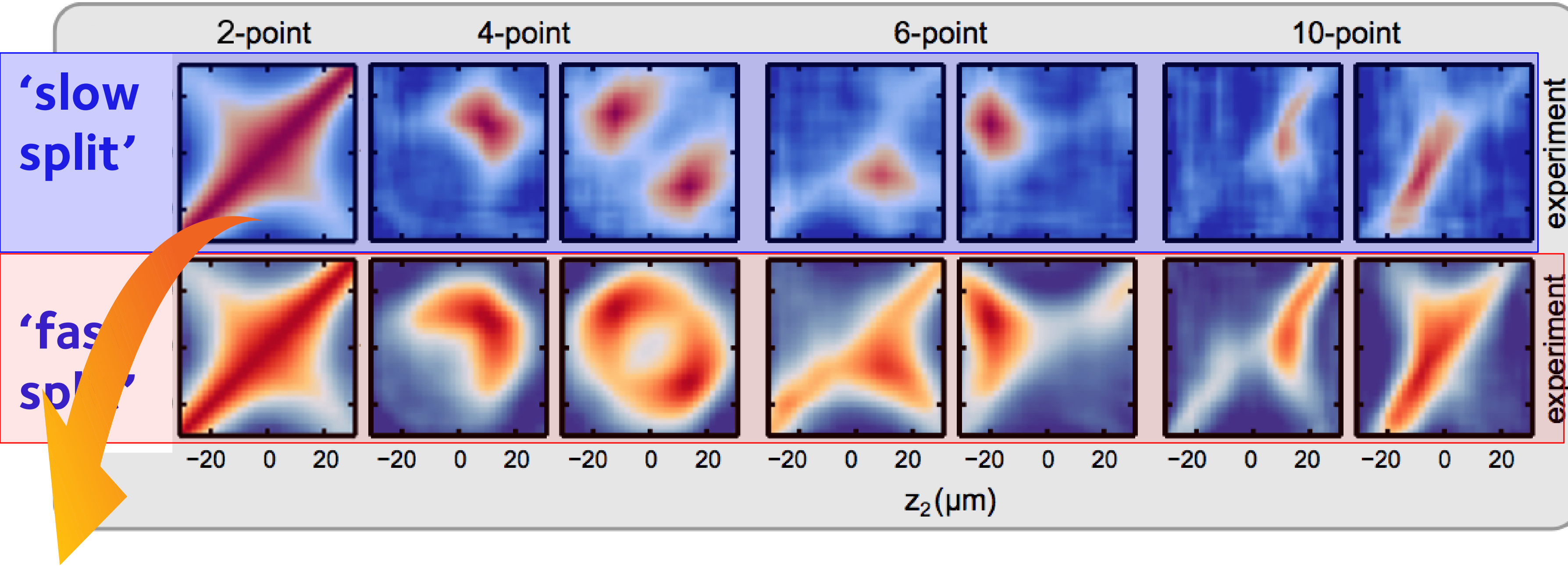
no. phonons

no. excited atoms

no. atoms

Beyond Gibbs

$$C(z_1, \ldots, z_N) = \langle \Psi_1(z_1)\Psi_2^\dagger(z_1)\Psi_1^\dagger(z_N)\Psi_2(z_N)\rangle$$



Slow split: Correlations well described with Gibbs distribution with... T_{eff} independent of initial T: Pre-thermalization

$$\boxed{\rho = \exp\left(-\beta_{\text{eff}} H\right)/Z}$$

Gring et al., Science 2012; Langen et al., Science 2015

Testing ground: Dicke model

$$H = \hbar\omega_{com} + \hbar\omega_{at} + H_{int}$$

$$H_{int} = \frac{2g}{\sqrt{N}}\left[(1-\alpha)(J_+ a + J_- a^\dagger) + \alpha(J_+ a^\dagger + J_- a)\right]$$

$$g, \alpha = \text{functions of } \Omega_{rsb}, \Omega_{bsb}$$

For **α={0, 1}**: exists additional conserved charge

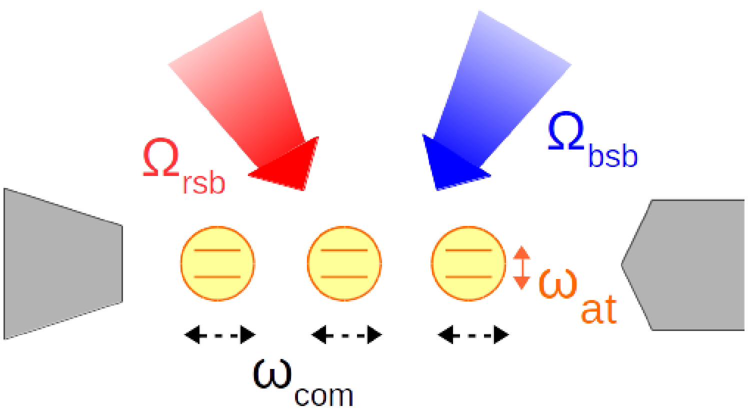
$$\hat{J}_{x,y,z} = \sum_{r=1}^{N} j_{x,y,z}^{(r)}$$

$$Q = J + J_z \pm a^\dagger a$$

no. phonons

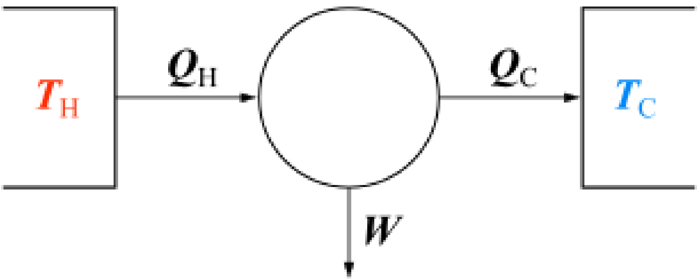
no. excited atoms

no. atoms



QFRs: Better than a bound

2nd law
[1824/1851/1854/…]



$$W \geq \Delta F$$

$$\eta = 1 - \frac{T_C}{T_H}$$

$$\Delta S \geq 0$$

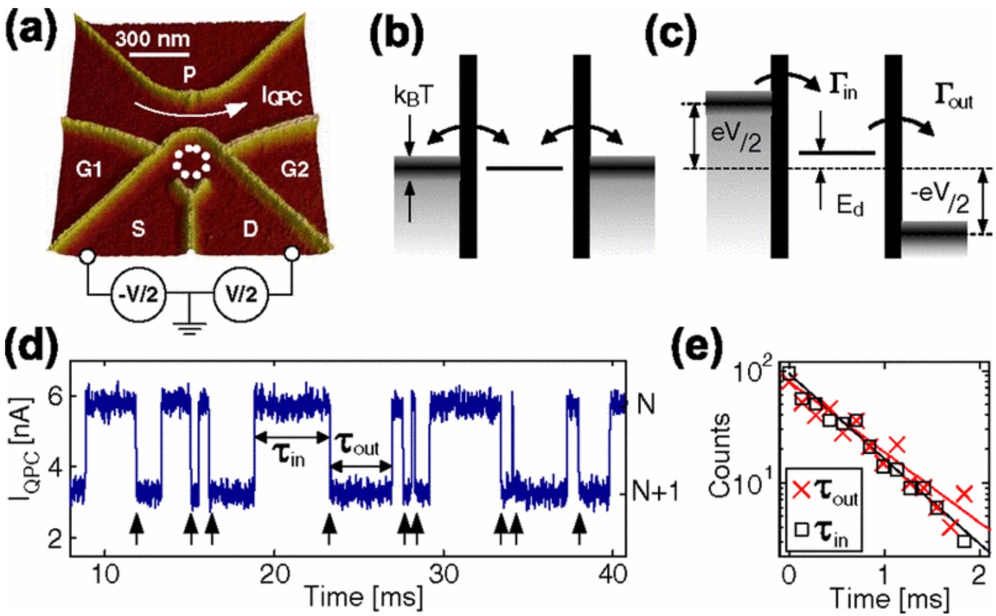
$$\oint \frac{\delta Q}{T} \leq 0$$

Evans et al. [1993]

Gallavoti-Cohen [1995]

$$\frac{P(\Delta S)}{P(-\Delta S)} = e^{\Delta S / k_B}$$

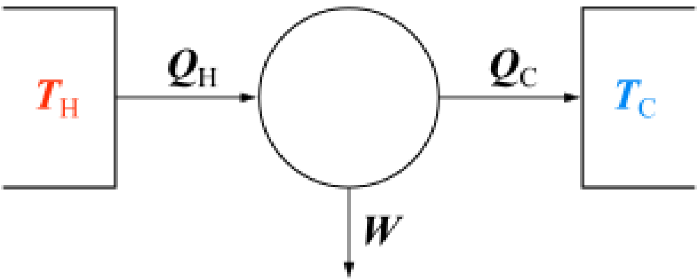
$$S_{\dot{Q}} = 2 k_B T^2 G_{\text{th}} \; [\text{heat FDT}]$$



Gustavsson et al. PRL [2006] 96 076605

QFRs: Better than a bound

2nd law
[1824/1851/1854/…]



$$W \geq \Delta F$$

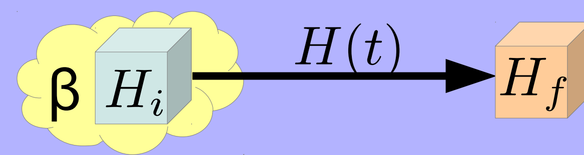
$$\eta = 1 - \frac{T_C}{T_H}$$

$$\Delta S \geq 0$$

$$\oint \frac{\delta Q}{T} \leq 0$$

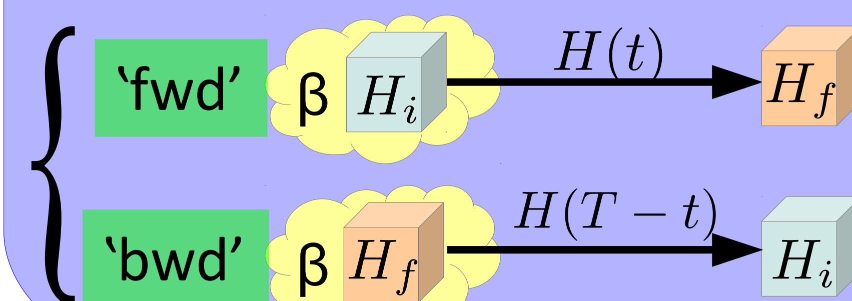
Jarzynski equality [1997]

$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$

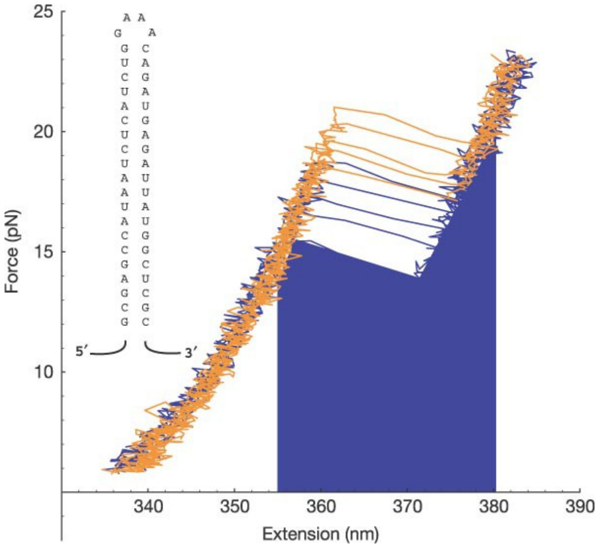


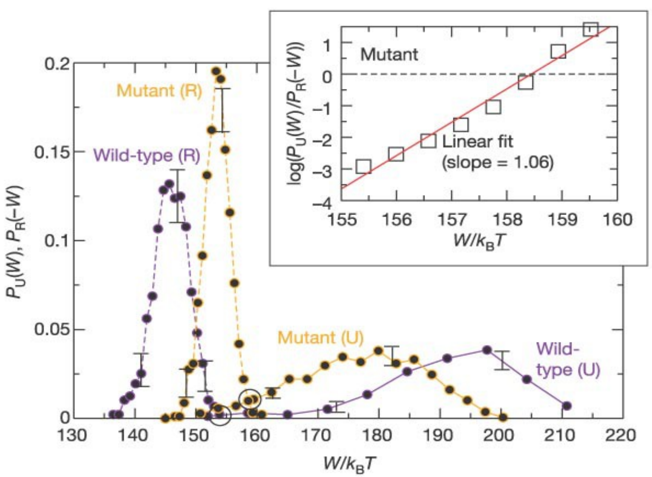
Crooks relation [1998]

$$\frac{P_f(w)}{P_b(-w)} = e^{\beta(w - \Delta F)}$$



Better than a bound

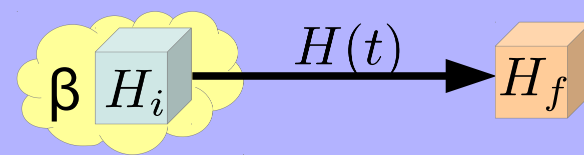




Collin et al., Nature (2005)

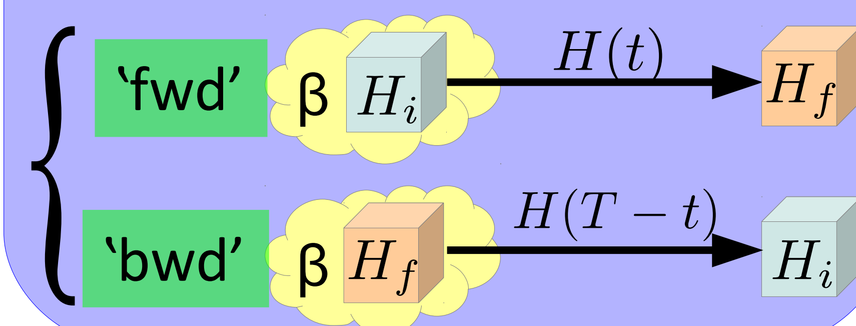
Jarzynski equality [1997]

$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$



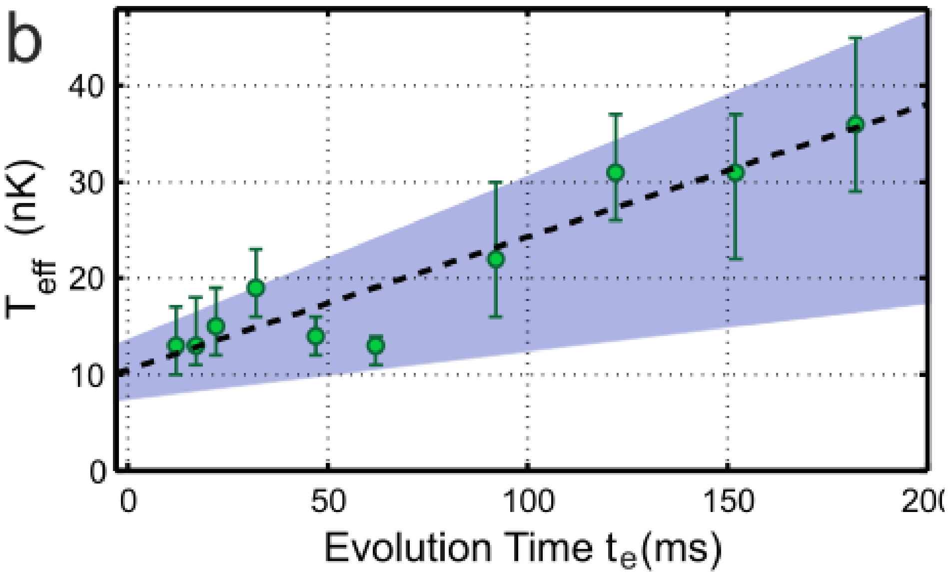
Crooks relation [1999]

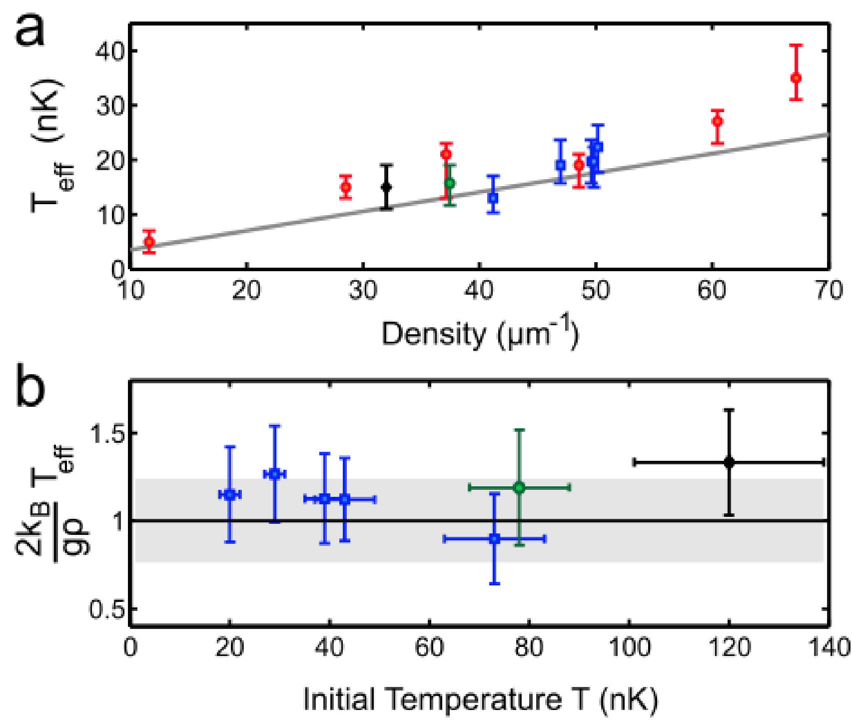
$$\frac{P_f(w)}{P_b(-w)} = e^{\beta(w-\Delta F)}$$



A quench in 1D

Split 1D gas quasi-adiabatically *[Schmiedm./Vienna, 2012]*





Theory:

-Rapid splitting -> Population of many (uncoupled) eigenmodes

-Thereafter: Dephasing of modes <=> Apparent thermalization to Teff

-Teff determined by energy input in splitting process, indep. of initial T

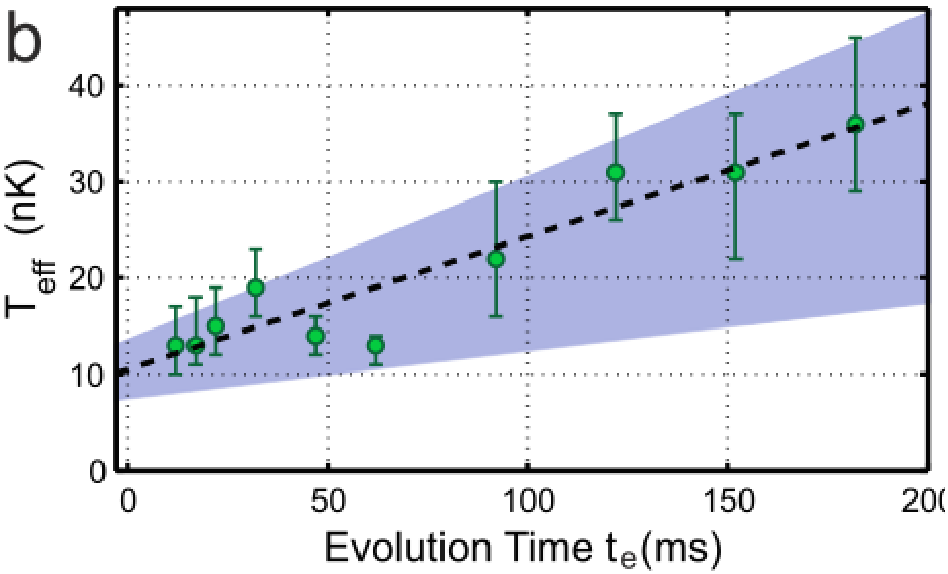
Expt: Fitted Teff~15mK indep. of initial T=78mK (=initial energy!)

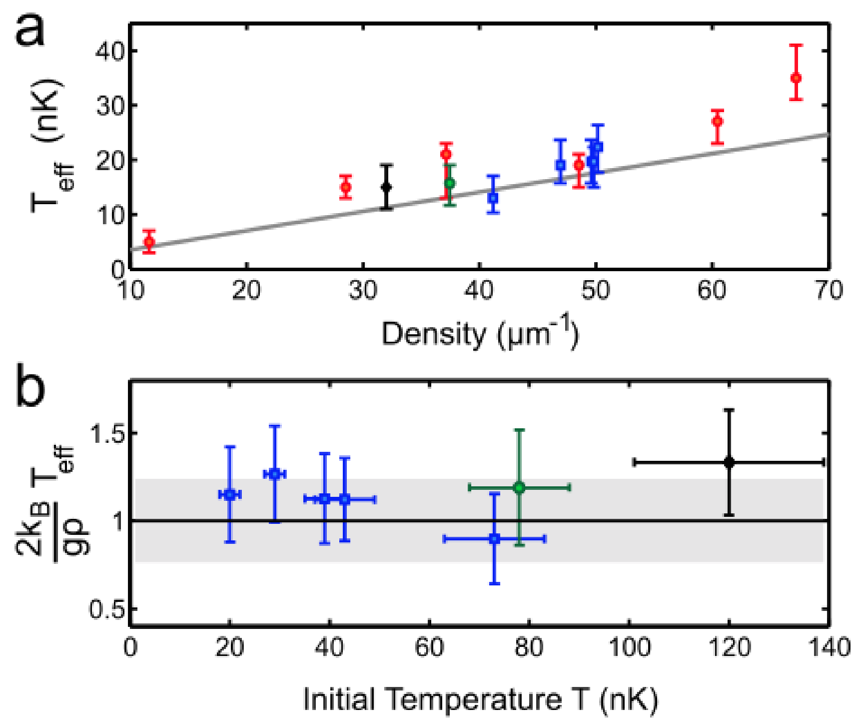
Relaxation to Teff≠T: Pre-thermalization

A quench in 1D

Split 1D gas quasi-adiabatically

[Schmiedm./Vienna, 2012]





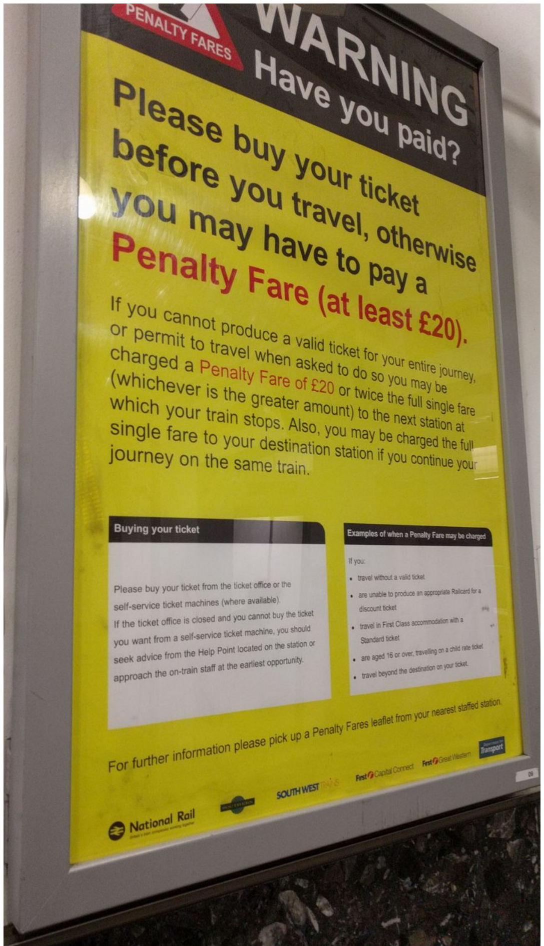
* Fit 2-point correlation functions -> Extract Teff
* Apparent thermalization with Teff independent of initial T!

‘Relaxation’ to Teff≠T: Pre-thermalization

The character of the Law

Human Law

- Can be violated
- Fines, prisons, lawyers, judges, police



Non-equilibrium thermodynamics

2nd Law in terms of work

$$w \geq \Delta F$$

Jarzynski equation (JE)

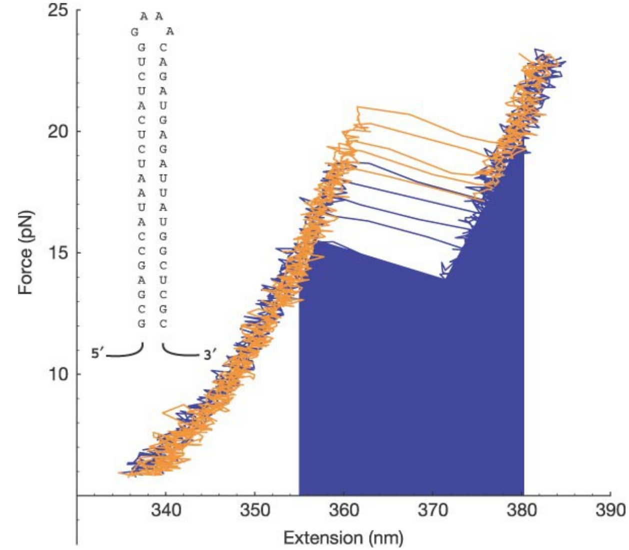
[PRL & PRE (1997)]

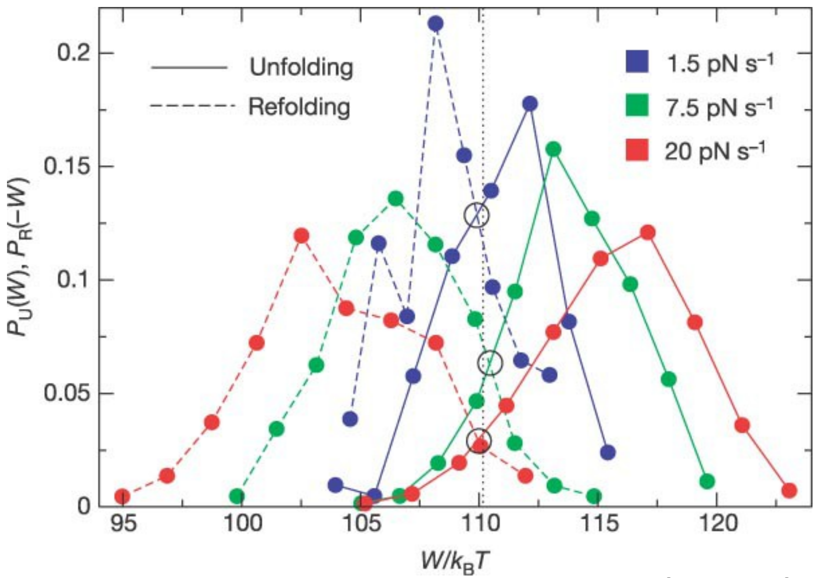
$$\langle e^{-\beta w} \rangle = \int e^{-\beta w} P(w) dw$$
$$= e^{-\beta \Delta F}$$

Crooks fluctuation theorem (CFT)

[PRE (1999)]

$$\frac{P_f(w)}{P_b(-w)} = e^{\beta(w - \Delta F)}$$





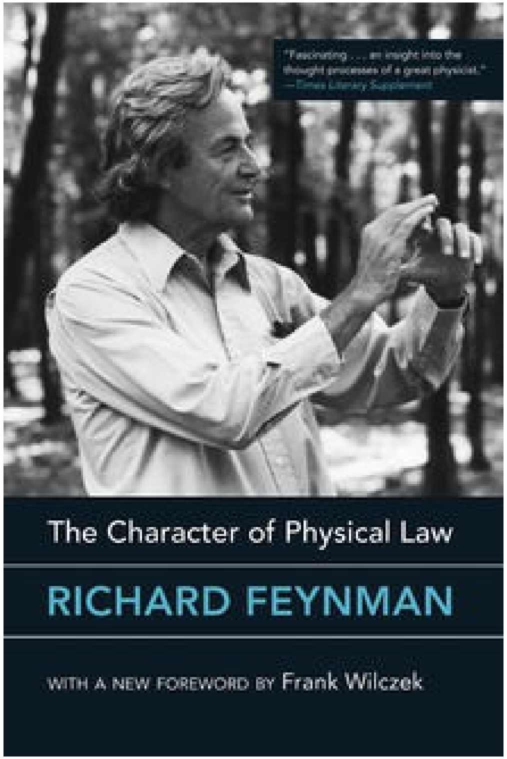
Collin et al., Nature (2005)

The character of the Law

Human Law

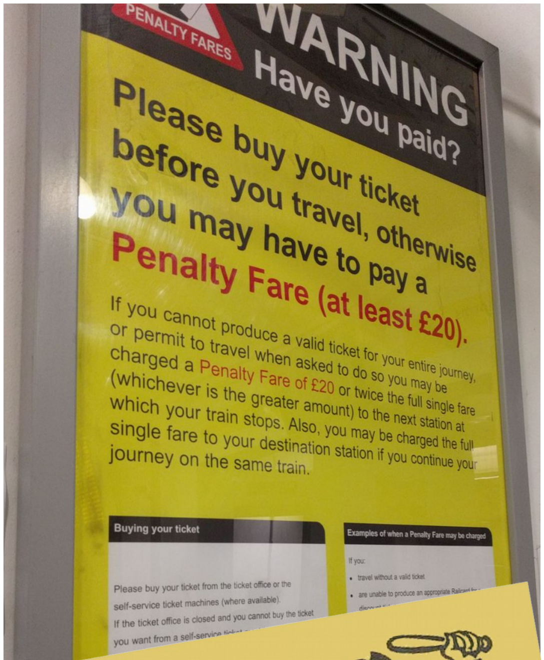
Physical Law

- Can't be violated
- No fines, lawyers…



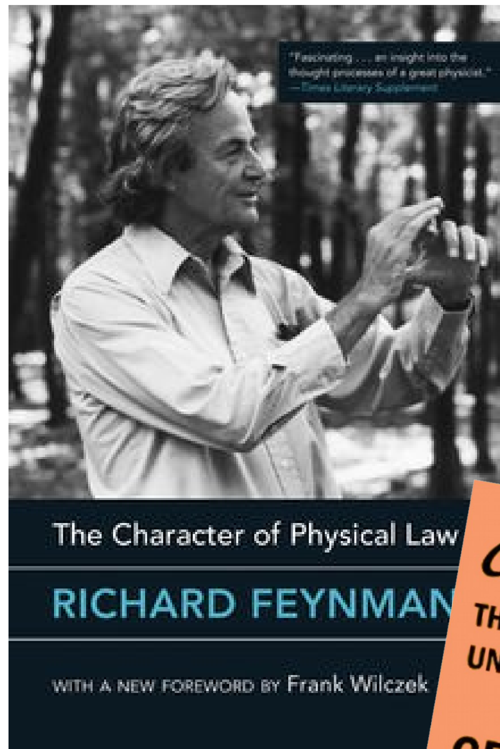
The character of the Law

Human Law



Physical Law

- Can't be violated
- No fines, police…



ΔS>=0

- Or it can…
- For free!

"Now, in the *second* law of thermodynamics . . ."